ANALYSIS OF NEUTRON-TEMPERATURE OSCILATIONS IN NEUTRON MULTIPLYING SYSTEMS WITH DELAYED NEUTRONS

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External sources with a given frequency excite neutron and temperature oscillations. Analytical treatment and analysis of oscillations were performed under conditions of both the weak coupling and strong coupling between the oscillation branches.The neutron oscillation branch describes the wave propagation along the coolant flow with amplitude rising in the range of high flow velocity and with amplitude decreasing in the range of low velocity and high frequencies, as well as, the oscillation propagation in the opposite direction. The temperature oscillations undergo a weak decrease of amplitudes. The strongly coupled neutron-temperature oscillations are excited, as the microscopic fission cross-sections are dependent on the "effective" temperature of thermal neutrons.The influence of delayed neutrons is studied. Comparison between the analytical treatment data and the analysis of experimental measurements confirms these conclusions.

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1. INTRODUCTION

The in-reactor wave processes can possess features characteristic for phenomena of spatial damping and amplification. Therefore, to develop in-reactor control systems it is necessary to investigate physical processes, taking place in the neutron-multiplying system, and their wave properties characteristic for fission reactors [1] - [6].

The goal of the paper is to analyze the propagation and spatial damping (amplification) of neutron field and cool-ant oscillations excited by external sources. Oscillations in the coolant-containing neutron-multiplying systems (cooled reactors) may be described by the set of nonequilibrium statistical physics equations. The paper presents the analysis of wave processes of neutron-temperature oscillation transport in the cool-ant-containing neutron-multiplying system.

The coolant moving through the core makes a convective heat and neutron transport and causes some non-equilibrium that can lead to the oscillation amplification in the dissipative media. This effect is not evident for our system because the neutron diffusion and capture by nuclei act as stabilizing factors. Besides, the choice of a steady state being stable in time does not guarantee the spatial damping of oscillations unlike the linear and nonlinear theory of waves in weakly nonequilibrium media.

The previous papers have considered a problem of excitation of neutron field oscillations by the acoustic wave [7]. In addition, there investigated was a problem of neutron field modulation by the external localized source with a given frequency through the acoustic, neutron and temperature channels [8]. The paper [20] briefly considers the conditions for propagation of neutron-temperature oscillations in the multiplying systems without taking into account the delayed neutrons and temperature dependence of diffusion coefficients. In the present paper, within the framework of the model applied, we comprehensively studied and analyzed the conditions for propagation, damping and amplification of oscillations. The kinetics of centers, emitting the delayed neutrons and the dependence of linearized diffusion terms on the temperature perturbations, are taken into account.

To solve the engineering tasks one uses an inconsistent approximation in definition and solving the problem. The influence of heat release in the fission reactions on the neutron diffusion is related with the neutron density via some transfer function introduced in the design formula as an unknown and should be determined from the experimental analysis of quantities being measured. To the equation of coolant heat balance one adds the equation of a fuel heat balance with an unknown heat-transfer coefficient. Thus, there is no self-consistent description in the engineering tasks. Probably, such an approximation is quite acceptable in the engineering interpretation of the models of noise diagnostics including some data from the experiments of [3] - [5] (more complete list of references see in [9]). Note that the first works on the neutron physics of multiplying systems were per-

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formed in the understandable language of theoretical physics [10]-[15].

2. SELF-CONSISTENT SYSTEM OF EQUATIONS

The influence of hydrodynamic parameter variations on the neutron density oscillations in the multiplying system with coolant and delayed neutrons requires the description by the methods of non-equilibrium statistical physics. To analyze this influence let us consider the set of equations for diffusion of thermal neutrons with their multiplication and capture and the set of hydrodynamic equations for the coolant being a moderator of fast neutrons [15].

$$\frac{\partial N}{\partial t}+\nabla(\vec{v}N)=\nabla(D\nabla N)+D_a\triangle N+\nu_c(K_\beta-1)N+S, \quad (1)$$

$$\rho c_p(\partial T/\partial t+\vec{v}\cdot\nabla T)=\nabla(\kappa\nabla T)+ +Q(\vec{r},t;N)+q_T, \quad (2)$$

$$\rho(\partial\vec{v}/\partial t+(\vec{v}\cdot\nabla)\vec{v})=-\nabla P+\eta\triangle\vec{v}, \quad (3)$$

$$\partial\rho/\partial t+\mathrm{div}(\rho\vec{v}), \qquad P=P(\rho,T). \quad (4)$$

In diffusion equation (1) the term $\nabla(\vec{v}N) = \vec{v}\nabla N + N\nabla\vec{v}$, obtained in [16], represents the conservation of neutron density during the neutron convection with a velocity of the coolant taking into account its compressibility. Then, $K_\beta = K(1-\beta)$, where $K = \nu\varphi\theta$ is the neutron multiplication factor including three factors from which ν is the number of generated neutrons in the one fission act, ψ is the probability to avoid the neutron capture in the process of its slowing-down. The third factor, the coefficient of thermal neutron utilization $\theta = \nu_f/(\nu_f+\nu_a)$, is equal to the relation of the inverse time ("frequency") of neutron capture by fission nuclei $\nu_f = \sigma_f N_F v$ to the sum of frequencies of neutron absorption by fission nuclei and in other processes $\nu_a = \Sigma_i\sigma_a^i N_a^i v$. Here i is the index of nuclear kind; σ_f is the nuclear fission reaction cross-section; N_f is the nuclear concentration and v is the average velocity of thermal neutrons. The effective coefficient of neutron multiplication, K_β , includes a factor where β is the sum of probabilities for formation of all the sources of delayed neutron emission source. The coefficient of diffusion $D = (1/3)vl$ and the diffusion-type transport coefficient $D_a = K\tau\nu_c$, related with the age of slowing-down neutrons, in the linear approximation form an effective coefficient of diffusion equal to $D_{\mathrm{ef}} = D + K\tau\nu_c$. As is shown by the authors of [14], the age parameter is equal to $\tau = l^2\ln(E_0/E)$, where $l^2 = 1/(3\xi_s\Sigma_{\mathrm{s}}\Sigma_{\mathrm{tr}})$ or to $1/l = \Sigma_\alpha N_\alpha\sigma_s^\alpha$ in terms of [12] for the scattering by different-type nuclei with concentrations N_α and scattering cross-sections σ_s^α. In these formulae $\xi_s \approx 1$, and Σ_{tr} is the macroscopic transport cross-section $\Sigma_{\mathrm{tr}} \approx \Sigma_{\mathrm{s}}$, approximately equal to the scattering cross-section $\Sigma_{\mathrm{s}} = N_s\sigma_s$, where σ_s is the microscopic scattering cross-section, N_s is the density of scattering nuclei. In diffusion equation (1) S is the density of thermal neutron sources, either external or delayer neutrons, for which $S = \Sigma_i\nu_i N_i$, where N_i is the concentration of i-type fission products radiating delayed neutrons and $1/\tau_i = \nu_i$ is the inverse "keeping" or delay time of neutron radiation. In the last case the set of equations under consideration is complemented by the following equation

$$\partial N_i/\partial t = \beta_f^i N - N_i/\tau_i, \quad (5)$$

where $\beta_f^i = \nu\beta^i\nu_f$ and $\nu\beta^i$ is "the multiplication of probability of the i-type radiation source formation in the process of fission on the probability of neutron emission by these fission product" [13] and ν_f is denoted above.

The hydrodynamic equations for the coolant are with standard notations. Note that $Q = Q(\vec{r},t)$ is the thermal-power density per time unit of fission nuclear reactions. For the heat release we introduce $EN = Q/(\rho c_p)$, where N denotes the neutron density. The last relation in the line (4) is the coolant state equation.

3. MODEL CHOICE

Propagation of neutron-temperature oscillations occurs when the multiplying medium is in the stationary or quasi-stationary state. The simplest model of the stationary state is a one-dimensional model with a coolant having the constant density ρ_0 and velocity U, neutron density N_0, temperature T_0 and concentration of centers radiating delayed neutrons N_{i0}. These quantities satisfy the equations (here we neglect the inhomogeneity in the velocity and pressure supposing their changes being insignificant)

$$U\frac{dN_0}{dz} = \frac{d}{dz}\left(D\frac{dN_0}{dz}\right) + D_a\frac{d^2N_0}{dz^2} + \nu_c(K_\beta-1)N_0 + S,$$

$$U\frac{dT_0}{dz} = EN_0 + \chi\frac{d^2T_0}{dz^2}, \quad \beta_f^i N_0 = \frac{N_{i0}}{\tau_i}.$$

The choice of the model is accompanied by the investigation of the stationary state stability. (Mathematical aspects of the theory on the existence, uniqueness and stability of stationary states in nuclear reactors, as well as, in a slab model are under consideration in the monograph [17]).

Let us formulate the set of equations for perturbations of stationary values of system parameters. Assume that $N = N_0 + n(z,t)$, $N_i = N_{0i} + n_i(z,t)$, $T = T_0 + T_1(z,t)$, $\rho = \rho_0 + \rho_1(z,t)$ and $\vec{v} = (0,0,U+U_1)$. We will take the linear terms in each of equations (1) - (5) considering the dependence of coefficients from the system parameters. The linear dependence of the capture frequency on the medium density is natural if the relative concentrations of nuclei are not varying, i.e. when $\rho_j = N_j m_j/\rho = \mathrm{const}$, where N_j and m_j , are the density and mass of j-type nuclei. Under these conditions all the macroscopic cross-sections of capture and scattering and inverse times of neutron capture are directly proportional to the medium density $\Sigma_j = \sigma_j N_j = r\Sigma_{j0}$, $1/T_c = \nu_c = \nu_{co}r$, where $r = \rho/\rho_0$. Here and below the index 0 denotes the values of quantities in the stationary state.

The terms, describing the multiplication of neutrons and their capture in equation (1) gain the dependence on the medium density and on the "effective" neutron temperature T_n in the form $\nu_c(K_\beta - 1) = r\vartheta_n^\delta \nu_{c0}(K_{\beta 0})$, where $\vartheta_n = T_n/T_0$.

The contribution of temperature perturbations to the multiplication and capture of neutrons in the same equation (1) gives the term

$$T_1 d\big(\nu_c(K_\beta - 1)\big)\,/dT = T_1\big(\nu_c(K_\beta - 1) + \nu_a\big)\delta/T_0, \quad (6)$$

where $\nu_c = \nu_a + \nu_f$ is the frequency of all the capture types. The frequency of thermal neutron absorption by the nuclei without fission ν_a, in view of the dependence of absorption cross-sections on the average thermal velocity in the form of $\sigma_\alpha \sim 1/v$, does not depend on the energy. In formula (6) assumed was a dependence of the frequency of capture by fission nuclei on the "effective" neutron temperature in the form of $\nu_f \sim T^\delta$. There used was equation (38) from Appendix 1 and the value of the derivative $(\partial\nu_f/\partial T_f)\cdot\partial T_n/\partial T = \nu_f(T_f)\delta/T$, containing, as a multiplier, the inverse medium temperature $1/T$. (Details on the physical situation concerning the "effective" temperature in nuclear reactors see in Appendix 1).

Here we give the results obtained in Appendix 2 on the temperature perturbation contribution into the diffusion transport of neutrons. The dependence of diffusion coefficients on density variations $\rho_1 = \rho - \rho_0$ by the equation of motion (9) we express in terms of temperature perturbations neglecting a medium local acceleration. Then the linearized terms of diffusion, being dependent on the temperature perturbations in the one-dimensional approximation, are added giving the sum

$$\hat{L_T}\left\{\frac{\partial}{\partial z}\left(D\frac{\partial N}{\partial z}\right) + D_a\frac{\partial^2 N}{\partial z^2}\right\} =$$
$$= D'_{G,T}\left(N_0''T_1 + N_0'\frac{\partial T_1}{\partial z}\right) + D'_{Ga,T}N_0''T_1 + R^{\text{ext}}, \quad (7)$$

where:
$R^{\text{ext}} = \{D_0\partial\left(P^{\text{ext}}N_0'\right)/\partial z + N_0''D_{a0}P^{\text{ext}}\}/(\rho s^2)$, $N_0' = dN_0/dz$, $N_0'' = d^2N_0/dz^2$, $D'_{G,T} = D_0 G/T_0$, $D'_{Ga,T} = D_{a0}G_a/T_0$. Parameters G and G_a are given in Appendix 2 after formulae (39) and (40) and contain not only the temperature perturbation contribution but also the medium density perturbation contribution. In the expression given the dependence of stationary values of T_0, D_0 and G on the "nonuniform" coordinate z is neglected. The neglect condition is fulfilled for perturbations having the scales of characteristic lengths significantly less than the characteristic dimensions of uniformities in the stationary state.

The linearized equations of coolant motion and continuity

$$\rho_0\partial U_1/\partial t = -s^2\partial\rho_1/\partial z - \big(P_T'\big)_\rho \partial T_1/\partial z - \partial P^{\text{ext}}/\partial z, \quad (8)$$

$$\partial\rho_1/\partial t + \partial(\rho_1 U + \rho_0 U_1)/\partial z = 0, \quad (9)$$

require explications. As is shown in [9], the perturbations in coolant local acceleration and coolant density inertia cause the excitation of acoustic waves. However, the sound branch of oscillations is separated from the neutron-temperature oscillations and the above-mentioned time derivatives are neglected. Equation (8) includes the force action of the external pressure gradient on the coolant.

The perturbations in the medium density ρ_1 and in the coolant velocity will be removed from the set of perturbations in the linearized system of equations by equations (8) and (9) by neglecting the local medium acceleration and the term in the continuity equation $\partial\rho/\partial t$. At the same time, the perturbations of neutron density, temperature and concentration of sources radiating delayed neutrons satisfy the set of equations

$$-i\widehat{\omega}_D n(z,t) - \widehat{H}T_1 - \sum_i N_i\nu_i = \widehat{R}P^{\text{ext}}\big/E \equiv b_1, \quad (10)$$

$$En(z,t) - i\big(\widehat{\omega}_\chi + i\Gamma\big)T_1 = EN_0P^{\text{ext}}\big/\rho s^2 - q_T \equiv b_2, \quad (11)$$

$$\partial n_i/\partial t = \beta_f^i n - \nu_i n_i. \quad (12)$$

Here R^{ext} is given in the line after formula (7) and the operator quantities and some parameters are equal to the following

$$\begin{aligned} -i\widehat{\omega}_D &= \partial/\partial t + U\partial/\partial z - D_{\text{ef}}\partial^2/\partial z^2 - \nu_c(K_\beta - 1), \\ -i\widehat{\omega}_\chi &= \partial/\partial t + U\partial/\partial z + E_T'N_0, \end{aligned} \quad (13)$$

$$\widehat{H}E = -\Gamma\widehat{\Omega} + ED'_{G,T}\big(N_0'' + N_0'\partial/\partial z\big) + EN_0(\lambda + \nu_a)\delta/T_0, \quad (14)$$

$$\Gamma = \gamma\big(P_T'\big)_\rho \frac{EN_0}{\rho s^2},\ \ \lambda = \nu_c(K_\beta - 1),\ \ \hat{\Omega} = \lambda + ikU. \quad (15)$$

The set of equations (10)-(12) describes both the multiplying medium stationary state stability and the propagation of coupled neutron-temperature oscillations excited by the external sources of force P^{ext} and thermal action q_T.

For the conditions of oscillation propagation an asymptotic solution is found in the form of quasi-classical exponents or exponents with expanded indexes $\exp\left(i\int^{z_0} k(z')dz'\right) \approx \exp(ik(z_1)z_0)$, where $z_1 = \varepsilon z_0$ is a "slow" coordinate for $\varepsilon \ll 1$, and $z_0 = z$ is the "fast" one [18]. The coupling of neutron and temperature perturbations is performed with the help of two factors. The first takes place in the thermal balance equation in the form of the heat releasing from the nuclear fission reactions proportional to the thermal neutron density. The second factor introduces the temperature perturbation into the neutron diffusion equation and is proportional to the sum of four terms:

$$H = -\Gamma\frac{\Omega}{E} + D'_{G,T}\big(N_0'' + ikN_0'\big) + D'_{Ga,T}N_0'' + N_0(\lambda + \nu_a)\frac{\delta}{T_0}, \quad (16)$$

where the notations correspond to these used in formulae (13)-(15). The first term in (16) on the right part expresses the medium density perturbation contribution into the macroscopic capture cross-section

$\Sigma_c = \sum_i N_c^i \sigma_c^i \sim \rho$. As is noted above, the medium density-and-temperature perturbations are interrelated by equation (8) with a left part being equal to zero. The second and the third terms in (16) describe the contribution of dependence of diffusion coefficients on the medium density and medium temperature given in Appendix 2 (see the text with formulae (39) and (40)). The last term in (16) is obtained by taking into account the microscopic nuclear fission cross-sections as a function of the thermal neutron "effective" temperature.

4. CHARACTERISTIC EQUATION FOR NEUTRON MULTIPLYING MEDIA WITH DELAYED NEUTRONS

The propagation of waves with a given frequency oscillations ω from the external sources is determined by the values of complex wave numbers k from the characteristic (or dispersion) equation obtained as a solvability condition for a homogeneous system of equations corresponding to equations (10)-(12) in the k-representation. The characteristic equation has the form

$$\Big(\omega_D - i\sum_j \nu_j \beta_f^j/(\nu_j - i\omega)\Big)(\omega_\chi + i\Gamma) = a_1 + a_2 + A, \quad (17)$$

where the quantities

$$\omega_D = \omega - kU + ik^2 D_{\text{ef}} - i(K_\beta - 1)\nu_c, \quad (18)$$

$$\omega_\chi = \omega - kU + ik^2\chi - iE_T' N_0,$$

are the same as in formulae (13)-(15) with the substitution $\partial/\partial t \Rightarrow -i\omega$, $\partial/\partial z \Rightarrow ik$. In the right part of (17), the sum is presented by the coupling factors

$$a_1 = \Gamma(\lambda + ikU), \;\; a_2 = -D'_{G,T}(N_0^{''} + ikN_0'), \quad (19)$$

$$A = -EN_0(\lambda + \nu_a)\delta/T_0, \;\; \delta < 0. \quad (20)$$

In the left part of equation (17) in the parenthesis the term with an index of summation is a contribution to the propagation of neutron oscillations of delayed neutrons from all the sources of their radiation.

Equation (17) without delayed neutron contribution is obtained in [8]. We will start the analysis of the solutions from the study of the effect equation of delayed neutrons. For low oscillation frequencies ω, which are much less than the inverse time of delayed neutron retention time $\omega \ll 1/\tau_i = \nu_i$, the sum $\sum_i \nu_i \beta_f^i \,/\, (\nu_i - i\omega) \approx \sum_i \beta_f^i = K\nu_c\beta$ is well simplified (here we used the notations $K = \nu\theta$, $\theta = \nu_f \,/\, \nu_c$, $\beta = \sum_i \beta_i$ and $\beta_f^i = \nu\beta_i\nu_f$).

For high frequencies $\omega \gg 1/\tau_i = \nu_i$, as compared to the inverse times $1/\tau_i$, both the reactive contribution and the dissipative contribution are insignificant due to the smallness of $\nu_i/\omega \ll 1$: $\Sigma_i \nu_i \beta_f^i \,/\, (\nu_i - \omega) \approx K\nu_c\big(i\Sigma_i\beta_i\nu_i \,/\, \omega + \Sigma_i\beta_i(\nu_i/\omega)^2\big)$.

If the frequency ω is close to one of the frequencies ν_i then the influence of delayed neutrons may be significant. The frequency band $f = \omega/2\pi$ in this case ranges to the tenth of hertz for the times of fission reaction half-life equal to 2 s and 0.45 s [11]. The contribution of a j-type emitter into the quantity ω_D in equation (17) is estimated as $(1+i)\;\beta_j K\nu_c/(2\omega)$ and can be significant for the case of a great probability β_j of formation in the process of j-type emitter fission. The neutron multiplication factor includes only prompt neutrons and their part from the total number of thermal neutrons is decreased by the sum of probabilities for all-type emitter formation $\beta = \sum_j \beta_j$ (a part of thermal neutrons arrives from the delayed ones).

As is seen from the above analysis, the contribution of delayed neutrons includes the reactive and dissipative parts. In the case of general statement, it can be combined in the dispersion equation with the terms of the same type. Introduce the notations

$$\omega_\beta = \omega + \omega K\nu_c \sum_i \nu_i\beta_i/\big(\omega^2 + \nu_i^2\big), \quad (21)$$

$$K_{\beta\omega} = K_\beta + K\beta_\omega, \quad \beta_\omega = \sum_i \nu_i^2\beta_i/\big(\omega^2 + \nu_i^2\big), \quad (22)$$

with which the investigation of limiting cases is easier. The dispersion function of proper oscillations together with delayed neutrons, taking into account the introduced notations, is written in the following form

$$D_N(\omega, k) = \omega_D - i\sum_i \nu_i\beta_f^i \,/\, (\nu_i - i\omega) =$$
$$= \omega_\beta - kU + ik^2 D_{\text{ef}} - i\nu_c(K_{\beta\omega} - 1). \quad (23)$$

5. PROPAGATION OF OSCILLATIONS IN NEUTRON MULTIPLYING MEDIUM WITH DELAYED NEUTRONS

Let us present the results of dispersion equation solution and their analysis. Of importance are not only the conditions for perturbation theory application in the range of values of the multiplying medium but also its physical characteristics, e.g. dependence on the temperature of medium of the nuclear fission microscopic cross-sections and on the frequencies of capture by these thermal neutron nuclei. It is possible to estimate the influence of these processes in the noise diagnostics of the multiplying system by studying the conditions of neutron-temperature oscillation propagation.

In the disperse equation with taking into account the introduced notations

$$D_N(\omega, k)\big(\omega_\chi + i\Gamma\big) = a_1 + a_2 + A, \quad (24)$$

the sum of partial coupling coefficients of neutron-and-temperature perturbations is given in formula (17) with notations on the line of (18), D_N is in formulae (23). The solution of this equation we search by the perturbation theory in the approximation of weak coupling between neutron and temperature oscillations. The wave number of neutron oscillations is found in the form $k = k_N + \delta k_N$, where $|k_N| \gg |\delta k_N|$.

In the zeroth approximation $k = k_{1,2}$ are the solutions of quadratic equation

$$\omega_\beta - kU + ik^2 D_{\text{ef}} - (K_\beta - 1)\nu_c = 0. \tag{25}$$

The real and imaginary parts of the complex roots of these equations are obtained in [8]

$$k_{1,2} = \left(1 / \sqrt{D_{\text{ef}}}\right)\Big[-iU / \left(2\sqrt{D_{\text{ef}}}\right) \pm$$

$$\pm\sqrt{-U^2/(4D_{\text{ef}}) + i\omega_\beta + \nu_c(K_\beta - 1)}\Big], \tag{26}$$

are written as

$$\text{Re}k_{1,2} = \pm\frac{1}{\sqrt{2}}\left[\sqrt{\omega_\beta + \sqrt{\omega_\beta^2 + B^2}} + \sqrt{\sqrt{\omega_\beta^2 + B^2} - \omega_\beta}\right], \tag{27}$$

$$\text{Im}k_{1,2} = \frac{1}{\sqrt{D_{\text{ef}}}}\Bigg\{-ib \pm$$

$$\pm\frac{1}{2}\left[\sqrt{\omega_\beta + \sqrt{\omega_\beta^2 + B^2}} - \sqrt{\sqrt{\omega_\beta^2 + B^2} - \omega_\beta}\right]\Bigg\}. \tag{28}$$

Here the notations $b = U/\left(2\sqrt{D_{\text{ef}}}\right)$ and $B = \nu_c\left(K_\beta - 1\right) - b^2$ are introduced.

When an above-critical state can be realized and when $B > 0$ ($B \gg \omega_\beta$ or $B \ll \omega_\beta$) it is easy to find approximated expressions using the perturbation theory. In particular, in the case of low values of the squared frequency $\omega_\beta^2 \ll B^2$ the real and imaginary parts of wave numbers take the values

$$\text{Re}k_{1,2} = \pm\sqrt{B/D_{\text{ef}}}\left(1 + \omega^2 / (8B^2)\right), \tag{29}$$

$$\text{Im}k_{1,2} = \left(1 / \sqrt{D_{\text{ef}}}\right)\left(-b + \omega / (2\sqrt{B})\right),$$

where the notations of formulae (27) and (28) are taken. The frequency dependence of the real part of the wave number about its low values looks like a branch of a parabola with vertical axis and the imaginary part of the wave number is a linear function.

As the frequency increases with its high values $\omega_\beta^2 \gg B^2$, the approximated values of the real and imaginary parts of wave numbers are equal

$$\text{Re}k_{1,2} = \pm\sqrt{\omega_\beta/D_{\text{ef}}}\left(1 + B / (2\omega_\beta)\right),$$

$$\text{Im}k_{1,2} = \left(1 / \sqrt{D_{\text{ef}}}\right)\left(-b \pm \sqrt{2\omega_\beta}\left(1 - B / (2\omega_\beta)\right)\right). \tag{30}$$

Besides, the fundamental approximation of modules $\text{Re}k_{1,2}$ expresses their growth depending on the frequency by the parabolic law with a horizontal parabola axis. As the frequency $\text{Im}k_1$ increases and becomes positive, and $\text{Im}k_2$ decreases and remains negative.

The expressions for $\text{Re}k_{1,2}$ and $\text{Im}k_{1,2}$ determine the complex wave numbers of proper neutron oscillations including the diffusion kinetics of thermal neutrons with capture and multiplication by fission nuclei, as well as, the processes of delayed neutron radiation. The waves are propagating in opposite directions. The wave, propagating in the direction opposite to the direction of the coolant flow rate, undergoes the spatial damping at all the values of parameters being in formula (28). The neutron wave, moving along the flow having a low rate and a high oscillation frequency, undergoes the amplitude decreasing too.

When the flow rate increases or the frequency decreases , this wave changes the propagation character from the weakening to its amplification. Indeed, the condition of amplification $\text{Im}k_1 < 0$ for the wave with $\text{Re}k_1 > 0$ leads to the inequality $\omega_\beta^2 < 4b^2\left(B + b^2\right)$ or in the reference variables

$$\omega_\beta^2 < \nu_c\left(K_\beta - 1\right)\left(U^2/D_{\text{ef}}\right). \tag{31}$$

If the frequency values exceed the inverse decay time of all the fission products being the sources of delayed neutron radiation $\omega \gg 1/\tau_i$, condition (31) simplifies and takes the form

$$\omega^2 < \nu_c\left(K(1-\beta) - 1\right)\left(U^2/D_{\text{ef}}\right), \quad \beta = \sum_i \beta_i,$$

where is the sum of probabilities for formation of these sources.

To find additives to the wave number of neutron oscillations (being small because of a weak coupling between the neutron field oscillations and coolant hydrodynamic oscillations), in formula (20) used is the expansion $D_N(\omega, k) = D_N(\omega, k_N) + \delta D_N$, where $D_N(\omega, k_N) = 0$ and $\delta D_N = (-U + 2ik_N D_{\text{ef}})\delta k_N$, $\omega_\chi(k_N) = \omega - k_N U + i\xi$ (last term is small, $\xi = \Gamma + k_N^2 - E_T' N_0 > 0$ and $|E_T' N_0/U| \ll |k_N|$, where $E_T' N_0 = E_T N_0 \delta/T$). Substituting this expansion into equation (24) we find the sought correction to the wave number

$$\delta k_N = \left(a(k_N)\right)/\left[\omega_\chi(k_N)\left(U - 2ik_N D_{\text{ef}}\right)\right], \tag{32}$$

containing the dependence on the delayed neutrons in the quantity k_N, one of the complex roots in formula (26). In formula (32) the terms $a(k_N) = a_1 + a_2(k_N) + A$ are determined in lines (19-20). Propagation of temperature oscillations in the basic approximation occurs with a wave number of the convective transport $k_T = \omega/U$. By analogy with the above case, the correction is found from the equation of neutron oscillations written in the form $k - \omega/U = a/[UD_N(\omega, k)] + i\xi/U$. Assuming that both terms in the right part are small and $k = k_T + \delta_T$, $|k_T| \gg |\delta k_T|$ by the perturbation theory of we find

$$\delta k_T = i\xi / U - a(k_T) / \left[UD_N(\omega, k_T)\right], \tag{33}$$

where $D_N(\omega, k_T) = \omega_\beta - \omega + ik_T^2 D_{\text{ef}} - i\nu_c(K_{\beta\omega} - 1)$. The first term in (33) leads to the oscillation damping unlike the second term, which under conditions of supercriticality $\nu_c(K_\beta - 1) > k_T D_{\text{ef}}$ gives the correction decreasing the oscillation damping.

Note here the value of the factor of coupling between the neutron and the temperature oscillations. The dependence of the frequency of thermal neutron capture by the fission nuclei $\nu_f \sim T_n^\delta$, even with low values of the index $|\delta|$, leads to the strong coupling between the temperature perturbations and the neutron density. In dispersion equation (24) the term A performs a strong coupling, and is proportional not to the supercriticality coefficient $\nu_c(K_\beta - 1)$, but simply $\nu_a K$, to $\nu_c(K_\beta - 1)\delta/T$ that exceeds by several orders of magnitude.

By virtue of a mentioned property, characteristic for nuclear-physical kinetics of thermal neutron capture by the fission nuclei, it is interesting to carry out the analysis of two limiting situations with the strong coupling between the proper neutron wave oscillations and the coolant oscillations. For this purpose in dispersion equation (24) we transpose into the left part the terms independent on the wave number k,

$$(\omega + i\xi)\big(\omega_\beta - i\nu_c(K_\beta - 1)\big) - \alpha = k\varphi(k, \omega), \quad (34)$$

where $a = \alpha + i\gamma$; $\alpha = a_1 + \mathrm{Re}a_2 + A$ is the real part of the sum of coupling factors and the polynomial is equal to the equation

$$\varphi(k, \omega) = U\big[\omega_\beta + i\gamma - i\nu_c(K_\beta - 1) - \omega - i\xi\big] + \quad (35)$$
$$+ikD_{\mathrm{ef}}(\omega + i\xi) - kU^2 + ik^2 UD_{\mathrm{ef}}.$$

After the transformation, the dispersion equation takes the form

$$\omega\omega_\beta - \alpha + 2\xi\lambda + i\omega\lambda = k\varphi(k, \omega), \quad (36)$$

where the parameter $\lambda = \nu_c(K_\beta - 1)$ is proportional to the supercriticality of the neutron multiplying system. Of a particular interest are the strongly coupled neutron-temperature oscillations with high values of the frequency and coupling factors. Therefore, when searching the solution of a dispersion equation one supposes that the oscillation frequency contribution is much higher than the contribution into the dispersion equation of diffusion, convection and balance between the multiplication and capture of neutrons. It means that the wave numbers of oscillations (at least, one of the branches) will be low and at a given frequency of the external oscillation source undergoes weak spatial variations. Such a behavior is important for the oscillation recording in the core of the multiplying medium.

The left part of equation (36) is small at low values of the parameters $\delta_\alpha = \omega\omega_\beta - \alpha$ and $\xi\lambda$. Then, assuming that $|k|$ is low, in the right part it is possible to restrict oneself by the term linear on k, namely: $k\varphi(0)$, where $\varphi(0) = U\big[\omega_\beta + i\gamma - i\nu_c(K_\beta - 1) - \omega - i\xi\big]$. Neglecting the low terms we have

$$k_1 = q_1 + \kappa_1 = \big(\delta_\alpha + i\xi\omega_\beta - i\omega\lambda\big) \;/\; \varphi(0). \quad (37)$$

Besides the roots determined there are another two roots with the same assumptions about the parameter smallness. These roots do not contain at all the smallness of the above-mentioned parameters and are close to zeros k_2 and k_3 of quadratic trinomial (35), $\varphi(k_{2,3}) = 0$, with minor corrections. However, here we do not given them and restrict ourselves by the following interpretation of conditions of calculation validity. The equality to zero of the parameter $\delta_\alpha = \omega\omega_\beta - \alpha = 0$ with high values of its parameters, as well as the equality $\xi\omega_\beta - \omega\lambda = 0$ in root expression (37) does not mean "automatically" the amplification or nonpassage of oscillations with frequencies from the vicinity of their values ω_1 and ω_2, $q(\omega_1) = 0$ and $\kappa(\omega_2) = 0$. There required is an adequate problem definition and choice of parameter values such as, for example, in guide [19] about amplification or nonpassage of waves in the stable and absolutely- or convectively unstable medium. For example, for a medium for the problem with initially boundary conditions it is a choice of "immediately including" external source in the form of $g(x, t) \sim \delta(x)\exp(-i\omega_0 t)$ at $t > 0$ and $g(x, t) = 0$ at $t < 0$ corresponding to the initially boundary problem with the point localization and given frequency ω_0. Far from the source, the steady-state condition at $t \to \infty$ with amplification or nonpassage of oscillations at $|x| \to \infty$ is possible only in the problem " on the convective unstable (or stable) system. In the case of absolute instability the perturbation is increasing unrestrictedly in all the points of space, so that it is impossible at all to reach a steady-state condition" [19] p.332. Note that the low wave number values violate the condition of applicability of the quasi-classical approximation $\big|(d/dz)(1/k(z)\big| = \big|(1/k(z)^2(dk(z)/dz)\big| \ll 1$, within the framework of which the treatment accepted in this paper is valid. Therefore, a special asymptotic expansion [21] of the desired solution and numerical computation are required.

Besides, it is of interest to consider the case with high modulo wave numbers. We solve the problem on a spatial behavior of strongly coupled neutron and temperature oscillations. Their coupling occurs through the density and temperature oscillations of both the media and the neutron field. To be exact: the neutron density variation in the term being the released heat contribution causes temperature variations in the medium. Under conditions of neglecting the inertia and weak convective transfer of a hydrodynamic medium pulse to these temperature variations "fitted" are the medium density variations together with the density of fuel nuclei. In the neutron diffusion equation among the terms of neutron self-action we take into account only the diffusion of neutrons and neglect their multiplication and nuclear capture (the matter concerns the spatial behavior of excitations over the stationary state of the active multiplying medium not the stationary state itself). In the right part of equation (36) the largest wave-number cubic term remains. Neglecting the small terms we obtain from (34) the equation

$$\delta_\alpha \equiv \omega\omega_\beta - \alpha = ik^3 UD_{\mathrm{ef}}\,.$$

When $\delta_\alpha > 0$ the solution of this equation is $k_j = \rho\exp\big(i2\pi j/3 - i\pi/6\big)$, where j is the integer and $\rho = \sqrt[3]{\delta_\alpha/(UD_{\mathrm{ef}})}$ or $k_0 = \rho\exp\big(-i\pi/6\big)$, $k_1 = i\rho$ and $k_{-1} = \rho\exp(-i5\pi/6)$.

It is possible to consider the definition of a boundary problem with oscillations being stationary in time at a given frequency. Then the position in the complex plane of a single root k_0 for oscillations in the region where $z > 0$, on the right of their excitation source site, expresses the amplification of these oscillations. Two other roots determine the damped oscillations in the infinite medium. Similar considerations are convenient if $\delta_\alpha < 0$. In the problems on the excitation of oscillations in finite systems, for construction of solutions with in-time stable behavior, one uses all three roots of the dispersion equations. The application of a method of variation of arbitrary constants using the exponents makes it possible to solve the problems with an arbitrary dependence on the spatial variable (see also in [19] § 65 on the finite system instability).

6. CONCLUSIONS

The conditions of excitation and propagation of neutron-temperature oscillations in the multiplying system with delayed neutrons are studied. It is obtained and investigated the characteristic equation that is equation of the third degree in relation to the wave number of complex kind. The roots behavior of equation depending on external source frequency and other parameters of the system is studied, in particular, values of coefficient of reproduction of neutrons, their "effective" temperature, times of capture and of coolant velocity.

The calculation analysis data show that in the case of a weak coupling between the neutron field and temperature the dependence of the wave number real part on the frequency, near its low values, looks like a branch of a parabola with vertical axis and the imaginary part of the wave number is a linear function. As the frequency of external source oscillations increases, the values of the wave number real part, as well as, the neutron oscillation spatial damping are increasing too (at high frequencies it is as square root of ω). The signal amplitudes are noticeably decreasing when the frequency increases.

Numerical analysis shows [22], that for standard parameter values of WWER-1000 type reactors, the neutron wave frequencies can be within the range of values from fractions and units of Hz to several tens of Hz with taking into account the time of thermal neutron capture by fission nuclei as a function of their "effective" temperature.

If the frequency oscillations is close to one of the inverse times of delayed neutron emission their influence may be significant. Found out the noticeable increase of real values of wave numbers and coefficients of the spatial damping of neutron branch of oscillations at diminishing of coefficient of diffusion of neutrons. Under similar conditions, the neutron fields can experience the transport by the convective temperature mode undergoing a slight damping with a weak connection of neutron and temperature perturbations.

Dependence of time of capture of thermal-neutron from their "effective" temperature by nuclei of division plays a key role at the analysis of conditions of propagation of neutron-temperature oscillations. As a result of its account the strongly coupled neutron-temperature oscillation are formed.

The mentioned dependence is a problem of nuclear kinetics of the neutron field evolution in time. Namely, it is necessary to formulate the methods describing the different non-stationary physical processes during slowing-down of fast neutrons from the nuclear fission reactions to the thermal energies simultaneously with their scattering and nuclear capture. The conditions for propagation of analyzed oscillations revealed in experiment [22] allows one to solve the problem of the "effective" temperature existence and dependence of the time of thermal neutron capture by fission nuclei on this temperature.

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APPENDIX 1

"EFFECTIVE" TEMPERATURE OF THERMAL NEUTRON

The question concerning the dependence of the fission nuclear capture frequency ν_f on the "effective" neutron temperature T_n and its relation with the medium temperature T is very complicated. One knows that in the reactor core the neutron temperature is by 50-100 degree higher than the medium temperature. In the region of thermal neutron energies, a steady-state connection was established [13] from the condition of a balance between the flow of neutrons slowing down in the scattering processes and the flow of thermal neutrons captured by nuclei. The foregoing is expressed by the equation

$$T_n = \big(1 + \Delta \cdot \Sigma_a \;/\; \Sigma_s\big)T. \tag{38}$$

Here represented are the medium temperature T, the "effective" neutron temperature T_n and the macroscopic capture cross-sections $\sum_a = \sigma_a N_a$ and scattering cross-sections $\sum_s = \sigma_s N_s$, where σ_a and σ_s are the corresponding microscopic cross-sections of nuclei with concentrations N_a and N_s. The parameter $\triangle$ depends on the moderator type (water, graphite etc.) and equals to the tenth part of unity. Phase densities (their energy distributions) of slowing-down neutrons are not Maxwellian ones. It has been found [13] that the value of "effective" temperature, at which the neutron distribution is maximum, exceeds the medium temperature .

The evolution processes of neutron slowing-down flow during $10^{-5}\,s$ and their capture time is less than $10^{-4}\,s$ (the lifetimes of thermal neutrons). As the periods of neutron slowing-down exceeds $2 \cdot 10^{-2}\,s$, it can be assumed that, in the oscillation processes too, the "effective temperature of neutrons is related with the medium temperature by the relationship of (38)-type established for the stationary conditions.

APPENDIX 2

Linearization of diffusion terms Let us go on to the problem of linearization in the neutron diffusion equation. Classical diffusion of thermal neutrons and their transport, represented by the term of a diffusion type and related with the age of neutrons being slowing-down, are written in equation (1) in the form of two terms with the coefficients D and D_a. Regard the explicit dependence of the temperature and density in the diffusion constants. By changing them for dimensionless ones (relatively to the values in the stationary state) $\vartheta = T/T_a$ and these introduced above ϑ_n and r we obtain, $D = (1/3)v/(\sigma_{tr}N_{tr}) = \vartheta^{1/2+\mu}D_0/r$, $D_a = \nu_c K\tau = \vartheta_n^{\delta}D_{a0}/r$. The values of derivatives of diffusion constant are equal to $\partial D/\partial\rho = -D/\rho$, $\partial D/\partial T = -(\mu + 1/2)D/T$ and $\partial D_a/\partial\rho = -D_a/\rho$, $\partial D_a/\partial T = \delta - 1/\ln(E_0/T)D_a/T$. Here $\mu \approx 0.1$ represents the weak dependence of the scattering cross-section on the medium temperature $\sigma_s \approx \sigma_{tr} \sim \vartheta^{-\mu}$. The above-mentioned formulae takes into account the dependence of the inverse time of thermal neutron capture by fission nuclei on the density and medium temperature T and the dependence on the "effective" temperature T_n. The result of diffusion term $\hat{L}$ linearization takes the form

$$\widehat{L}\nabla(D\nabla N) = \nabla\{D_0\nabla n + [T_1 G/T_0 + P^{\text{ext}}/(\rho s^2)]D_0\nabla N_0\}. \quad (39)$$

Here introduced is the notation for the factor $G = 1/2 + \mu + \left(\gamma_c T_0 / (\varrho s^2)\right)(P_T')_\rho$, where the first two terms show the dependence of the diffusion constant on the temperature, and the last term is obtained by changing its density perturbation for $\rho_1 = -(T_1/s^2)\gamma_c(P_T')_\rho - P^{\text{ext}}/s^2$ via the perturbation of temperature T_1 in accordance with equation (9) without an inertial term and with zero boundary conditions $\rho_1(\vec{r}_0) = T_1(\vec{r}_0) = P^{\text{ext}}(\vec{r}_0) = 0$. Linearization of the diffusion-type term, related with the slowing-down neutron age, leads to the expression (a zero index denotes, as before, the stationary values of quantities)

$$\widehat{L}D_a\triangle N) = D_0\triangle n + \left(D_{a0}G_a\frac{T_1}{T_0} + \frac{D_0}{\rho s^2}P^{\text{ext}}\right)\triangle N_0, \quad (40)$$

where the factor $G_a = \delta + \left(\gamma_c T_0 \,/\, \rho s^2\right)(P_T')_\rho - 1/\ln(E_0/T)$ contains the second terms obtained from the parameter of neutron age $\tau = l^2\ln(E_0/T)$. When in the fission reaction the energy of neutron generated is $E_0 \approx 5\,MeV$ and the temperature $T \approx 0.05\,eV$ this term is equal to $\ln(E_0/T \simeq 1/20$. The result of diffusion term linearization in the one-dimensional approximation is given in formulae (39), (40) and (7) of the paper text.

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АНАЛИЗ НЕЙТРОННО-ТЕМПЕРАТУРНИХ КОЛЕБАНИЙ В РАЗМНОЖАЮЩИХ СИСТЕМАХ С ЗАПАЗДЫВАЮЩИМИ НЕЙТРОНАМИ

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Внешние источники с заданной частотой в нейтронной размножающей системе возбуждают нейтронные и температурные колебания. Проведены аналитическое рассмотрение и анализ колебаний в условиях как слабой, так и сильной связей между ветвями. Нейтронная ветвь колебаний описывает их распространение вдоль течения теплоносителя с нарастанием их амплитуды в области больших скоростей его движения, а также с затуханием амплитуды – в противоположном направлении. Конвективная ветвь колебаний испытывает слабое затухание амплитуды. При зависимости микроскопических сечений деления ядер от температуры тепловых нейтронов возбуждаются сильно связанные нейтронно-температурные колебания. Учтено влияние запаздывающих нейтронов. Сравнение аналитического рассмотрения с анализом экспериментальных измерений подтверждает эти выводы.

АНАЛІЗ НЕЙТРОННО-ТЕМПЕРАТУРНИХ КОЛИВАНЬ У РОЗМНОЖУВАЛЬНИХ СИСТЕМАХ З ЗАПІЗНІЛИМИ НЕЙТРОНАМИ

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Зовнішні джерела з заданою частотою в нейтронній розмножувальній системі збуджують нейтронні і температурні коливання. Проведено аналітичний розгляд і аналіз коливань в умовах як слабкого, так і сильного зв'язків між гілками. Нейтронна гілка коливань описує їх розповсюдження уздовж течії теплоносія з наростанням амплітуди в області великих швидкостей його руху, а також, з затуханням амплітуди – в протилежному напрямку. Конвективна гілка температурних коливань зазнає слабкого затухання амплітуди. При залежності мікроскопічних перетинів поділу ядер від температури теплових нейтронів збуджуються сильно зв'язані нейтронно-температурні коливання. Враховано вплив запізнілих нейтронів. Аналітичний розгляд і аналіз експериментальних вимірювань підтверджують ці висновки.