# ELASTIC SCATTERING OF DEUTERONS BY DEUTERONS <br> AT $E_{d} \leq 85 \mathrm{MeV}$ 

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#### Abstract

The differential cross sections of elastic $d d$-scattering at $E_{d}=36.9 \mathrm{MeV}$ in the angular range $30^{\circ} \leq \theta_{\text {c.m. }} \leq 116^{\circ}$ are measured. For describing of main peak at $\theta_{c . m} \leq 60^{\circ}$ we used the diffraction nuclear model taking into account the structure of the colliding nuclei. Satisfactory agreement of the present results with published data at energies of $12.1 \mathrm{MeV} \leq E_{d} \leq 85 \mathrm{MeV}$ was obtained. For the theoretical interpretation of the angular distributions the identity of the colliding deuterons is taken into account.


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## 1. INTRODUCTION

The deuteron is the simplest bound nuclear system of nucleons in which nuclear interaction takes place, and processes with participation of deuterons are valuable and convenient method to study some aspects of nuclear forces and the structure of the deuteron. Containing only two nucleons it is better investigated in many aspects in comparison with the majority of the rest nuclei, and allows to study some more fine details of the nuclear $N N$ interaction, thus complementing information got from the nucleon-nucleon scattering. Elastic scattering of deuterons by deuterons at energies $12 \leq E_{d} \leq 100 \mathrm{MeV}$ devoted a small number of studies [1-9]. Generally the angular distributions of elastically scattered deuterons were investigated both experimentally and theoretically at energies up to 25 MeV . This is conditioned by the complexity of the theoretical interpretation of the data at $E_{d} \geq$ 30 MeV , and the paucity of experimental results. In this work experimental data on elastic scattering of deuterons with energy $E_{d}=36.9 \mathrm{MeV}$ by deuterons and results of the elastic $d d$-scattering analysis at energies of $12<E_{d}<85 \mathrm{MeV}$ are presented. We used diffraction nuclear model with accounting of NN interaction for theoretical interpretation of data on elastic scattering at angles $\theta_{c . m} . \leq 60^{\circ}$. Diffraction approximation considers collisions of two deuterons as two classical balls with accounting their identity was used to explain structural peculiarities of angular distributions at different energies.

## 2. EXPERIMENT

Experimental study of $d d$-scattering was carried out on the $U-240$ cyclotron in the KINR of NAS
of Ukraine on the external deuteron beam with the energy $E_{d}=36.9 \mathrm{MeV}$. Measurements were fulfilled with $C D_{2}$ (deuterated polyethylene) and ${ }^{12} C$ targets. The statistical accuracy of measurements was $1 . . .2 \%$ and absolute values of cross sections were determined with an accuracy of $\sim 5$ percent. Measurements were carried out using installation and procedures, described earlier in works [10-12]. In the Fig. 1 our angular distributions of deuterons scattered by deuterons in c.m. system are presented with published data in the energy range $12<E_{d}<85 \mathrm{MeV}$.


Fig. 1 Angular distributions of the elastic scattering deuterons by deuterons at energies: 12.1 [1], 12.3 [9], 25.3 [4], 36.9 (our data) and $50 \ldots 85 \mathrm{MeV}$ [5] in c.m.s. Energies of deuterons are presented in l.s.

The differential cross section at energy $E_{d}=12.1 \mathrm{MeV}$ reduces gradually with the growth

[^0]of the scattering angle up to $\theta_{c . m} \approx 90^{\circ}$, and then increased also gradually with the growth of the scattering angle. With the growth of the deuteron energy, starting from $E_{d}=25 \mathrm{MeV}$ (according available experimental data) a strong energy dependence in cross section is observed (to up $\theta_{\text {c.m. }} \sim 60^{\circ}$ ). A structure in angular distributions ( $60^{\circ} \leq \theta_{c . m} \leq 120^{\circ}$ ), i.e. indications on the minimum at $\theta_{\text {c.m. }} \approx 60^{\circ}$ and the broad maximum, centered at the angle $\theta_{c . m} \approx 90^{\circ}$ starts to display. The similar trend is seen more clearly with the further growth of incoming deuteron energy [5]. But with increasing energy up to $E_{d}=85 \mathrm{MeV}$ noticeable decrease in the cross sections observed (up to 2 mb ) and structure is almost absent in the angular range $60^{\circ} \leq \theta_{\text {c.m. }} \leq 120^{\circ}$.

## 3. THEORY

At the energy of incoming deuteron in l.s. $E_{d} \approx$ 40 MeV , as in our experiment, collision of two deuterons can be considered as quasi-classical one. In this case product of the relative wave vector $k$ and the radius of nuclear interaction R will exceed a unity a few times. That is why the diffraction approximation $[10,11]$ for small scattering angles in l.s. $\theta \leq(k R)^{-1} \ll 1$ can be used.

### 3.1. THE MICROSCOPIC DIFFRACTION MODEL

The microscopic diffraction model [11, 12, 13] was used to describe deuteron-deuteron collisions. An interaction of each nucleon of the incoming deuteron with each nucleon of the target deuteron is taken into account. The nucleon-nucleon profile function was chosen in the form of Gaussian:

$$
\begin{equation*}
\omega_{i j}=\omega\left(\left|\rho_{i j}\right|\right)=\alpha \exp \left(b^{2} \rho_{i j}^{2}\right), \quad \alpha=\alpha_{1}-\iota \alpha_{2} \tag{1}
\end{equation*}
$$

where $\vec{\rho}_{i j}$ is the component of vector $\vec{r}_{i j}=\vec{r}_{i}+\vec{r}_{j}$ perpendicular to the incoming deuteron wave vector $\vec{k}_{d}$ in l.s., while $\vec{r}_{i}$ is a radius -vector for $i$-nucleon of deuteron target $(i=1,2)$, and $\vec{r}_{j}$ is a radius-vector of the nucleon j of the incoming deuteron $(j=3,4)$. Functions $\omega_{i j}$ are connected with appropriate scattering matrixes $\Omega_{i j}$ by a simple relation: $\Omega_{i j}=1-\omega_{i j}$. Values of real interaction parameters $a_{1}, a_{2}$ and $b$ in (1) were taken to be approximately the same as in works [13, 14].

The deuteron-deuteron elastic scattering amplitude in the diffraction approximation cab be build in the form as it was done in $[11,12]$ for scattering of a deuteron on a triton ( $\theta$ is the scattering angle in the c.m. system)

$$
\begin{align*}
A(\theta)=\int d^{(3)} \vec{r} \int & d^{(3)} \vec{s} \int d^{(2)} \overrightarrow{R_{\perp}} \varphi^{*}(\vec{r}) \varphi_{c \vec{h} i^{*}}\left(\overrightarrow{R_{\perp}}\right) \times \\
& \times \Omega_{13} \Omega_{14} \Omega_{23} \Omega_{24} \varphi(\vec{r}) \varphi(\vec{s}) \varphi_{0}\left(\overrightarrow{R_{\perp}}\right) \tag{2}
\end{align*}
$$

where inner (structural) wave functions of the deuteron-target $\varphi(\vec{r})$ and the incoming deuteron $\varphi(\vec{s})$ depend on relative radius-vectors $\vec{r}=\overrightarrow{r_{12}}=\overrightarrow{r_{1}}-\overrightarrow{r_{2}}$
and $\vec{s}=\overrightarrow{r_{34}}=\overrightarrow{r_{3}}-\overrightarrow{r_{4}}$ and wave function $\varphi_{0}\left(\overrightarrow{R_{\perp}}\right)$ and $\varphi_{\vec{\varphi}}\left(\overrightarrow{R_{\perp}}\right)$ describing the relative movement of deuterons before and after scattering, depend on he component $\overrightarrow{R_{\perp}}$ of the radius-vector $\vec{R}$, connecting centers of mass of the two deuterons, perpendicular to the relative wave vector $\vec{k}=\frac{1}{2} \overrightarrow{k_{d}}$ and $\vec{\chi}$ is the perpendicular to the vector $\vec{k}$ component of the momentum of the scattered deuteron, while $\vec{\chi}=-\vec{q}$, where $\vec{q}$ is the transferred momentum [15]. So far as the amplitude in [2] contains rather high multiplicity of integration, we will use for calculation the wave functions of the simplest form $[11,12,13]$

$$
\begin{equation*}
\varphi(\vec{s})=\left(\frac{2 \lambda^{2}}{\pi}\right), \quad \lambda=0.267 \mathrm{fm}^{-1} \tag{3}
\end{equation*}
$$

$$
\begin{array}{r}
\varphi_{\circ}\left(\overrightarrow{R_{\perp}}\right)=1, \quad \varphi_{\vec{\chi}}\left(\overrightarrow{R_{\perp}}\right)=e^{i \vec{\chi} \overrightarrow{R_{\perp}}}, \quad \vec{\chi}=-\vec{q}, \\
\chi=2 k \sin \frac{\theta}{2} . \tag{4}
\end{array}
$$

For our incoming deuteron energy the $d d$-scattering will take place in the $c . m$. system mainly in small angles range $\theta \leq 60^{\circ}$ (for l.s. $\theta \leq 30^{\circ}$ ). Therefore, to calculate amplitudes and cross sections for kinematical conditions of our experiment we can use the impulse approximation, which will be proved to be correct, when comparing calculated cross sections with our experimental data. Then the substitution of (1), (3) and (4) in (2) for diffraction elastic scattering of the deuteron by the deuteron we will get the expression in the explicit form:
$A(\theta)=\frac{4 \pi a}{b^{2}} \exp \left[-\chi^{2}\left(\frac{1}{4 b^{2}}+\frac{1}{16 \lambda^{2}}\right)\right], \quad \chi=2 k \sin \frac{\theta}{2}$,
and for the appropriate cross section of the diffraction scattering we will get formula

$$
\begin{align*}
& \frac{d \sigma}{d \Omega}=\frac{k^{2}}{(2 \pi)^{2}}|A(\theta)|^{2}=\frac{4|a|^{2} k^{2}}{b^{4}} \times \\
& \quad \times \exp \left[-2 k^{2}\left(\frac{1}{b^{2}}+\frac{1}{4 \lambda^{2}}\right) \sin ^{2} \frac{\theta}{2}\right] . \tag{6}
\end{align*}
$$

Accounting the identity of colliding deuterons we need to take the superposition of amplitude $A(\theta)$ and $A(\theta)$ [16] instead of the amplitude $A(\theta)$ in (5) and the cross section (for integer spins equal 1 , as in our case) will be now

$$
\begin{array}{r}
\frac{d \sigma}{d \Omega}=\frac{k^{2}}{(2 \pi)^{2}}\left\{|A(\theta)|^{2}+|A(\pi-\theta)|^{2}+\right. \\
\left.+\frac{3}{2} \operatorname{Re}\left[A(\theta) A^{*}(\pi-\theta)\right]\right\} \tag{7}
\end{array}
$$

that leads to the following formula for the deuteron elastic scattering cross section

$$
\begin{array}{r}
\frac{d \sigma}{d \Omega}=\frac{4|a|^{2} k^{2}}{b^{4}}\left\{\exp \left[-2 k^{2}\left(\frac{1}{b^{2}}+\frac{1}{4 \lambda^{2}}\right) \sin ^{2} \frac{\theta}{2}\right]+\right. \\
+\exp \left[-2 k^{2}\left(\frac{1}{b^{2}}+\frac{1}{4 \lambda^{2}}\right) \cos ^{2} \frac{\theta}{2}\right]+ \\
+  \tag{8}\\
\left.+\frac{2}{3} \exp \left[-k^{2}\left(\frac{1}{b^{2}}+\frac{1}{4 \lambda^{2}}\right)\right]\right\}, \quad k_{2}=M E_{d}
\end{array}
$$

where $M$ is nucleon mass. The way it should be, cross section is symmetric in the $c . m$. system relatively the angle $\theta=90^{\circ}$, in particular, the values of cross section at angles $\theta=0^{\circ}$ and $\theta=180^{\circ}$ will be equal.

The first term in the right part of the formula (8) is the cross section of elastic scattering (6) of the incoming deuteron by the deuteron-target and as $k^{2}\left(\frac{1}{b^{2}}+\frac{1}{4 \lambda^{2}}\right) \gg 1$, it gives the main contribution in the cross section (8) only for small angles of scattering $\theta \leq 60^{\circ}$. The second terms in (8) is the cross section of the deuteron-target knocking-out, and it contributes significantly only for angles $\theta$ near to $180^{\circ}$, i.e. when $\theta \geq 120^{\circ}$. The third terms in (8) for our model wave functions (3) and (4) does not depend on the angle $\theta$ and is an interference (quantum mechanical) term and, as it has to be in quasi-classical approximation, it is negligibly small for our energy, thus we can retain in the cross section (8) only the first two (classical) terms in a good approximation. As our measurements of cross sections were limited only by angles $\theta<120^{\circ}$, the second term in (8) will have also only limited application for the description of the experiment for such scattering angles.

### 3.2 THE NON PENETRATING SPHERES APPROXIMATION

The behavior of the observed cross section of $d d-$ scattering for angles $\theta \gg(k R)^{-1}$ in the diffraction approximation is not longer described with model functions (3) and (4), that is why for angles $60^{\circ} \leq$ $\theta \leq 120^{\circ}$, where the observed cross section is very small, one can try to use a version of quasi-classical approximation, in which deuterons are treated as two identical non penetrating collided balls [11, 12, 17]. In this approximation one can describe qualitatively the cross section for whole angle region $60^{\circ} \leq \theta \leq 180^{\circ}$. At $k R \gg 1$ for all angles $\theta_{\text {c.m. }} \leq 180^{\circ}$ the quasiclassical amplitude of two colliding identical particles will look like:

$$
\begin{align*}
F(\theta)= & i \frac{R}{2 \sin \frac{\theta}{2}} J_{1}\left(2 k R \sin \frac{\theta}{2}\right)- \\
& -\frac{i R}{2} \exp \left(-2 i k R \sin \frac{\theta}{2}\right)  \tag{9}\\
F 1(\pi-\theta)= & i \frac{R}{2 \cos \frac{\theta}{2}} J_{1}\left(2 k R \cos \frac{\theta}{2}\right)- \\
& -\frac{i R}{2} \exp \left(-2 i k R \cos \frac{\theta}{2}\right), \tag{10}
\end{align*}
$$

where first part - diffraction quantum amplitude $f_{\text {dif }}(\theta)$, second - amplitude of classical isotropic scattering. The differential scattering cross section can be written as

$$
\begin{array}{r}
\frac{d \sigma}{d \Omega}=(|F 1(\theta)|)^{2}+ \\
+\frac{2}{3} R e\left[F(\theta) \frac{(|F 1(\theta)|)^{2}}{F 1(\theta)}\right] \tag{11}
\end{array}
$$

For small angle interval $\theta \quad \ll(k R)^{-\frac{1}{3}}$ the module of diffractional part of the amplitude
$f_{d i f}(\theta)$ exceeds significantly the classical one $f_{c l}:\left|f_{d i f}(\theta)\right| \gg\left|f_{c l}(\theta)\right|$, and for the angular interval $\theta \gg(k R)^{\frac{1}{3}} \quad$, on the contrary, $\left|f_{c l}(\theta)\right| \gg\left|f_{d i f}(\theta)\right|$. For the small angle region near $\theta \approx(k R)^{-\frac{1}{3}}$ amplitudes $f_{\text {dif }}(\theta)$ and $f_{c l}(\theta)$ are of the same order of magnitude and to find simple and explicit expression for the amplitude is not easy, but the contribution for this angular region to the integral cross section is negligibly small for $k R \gg 1$. The explicit expression for amplitudes $f_{\text {dif }}(\theta)$ and $f_{c l}(\theta)$ were received in [17] for different conditions, that is why it is worth to examine separately differential cross sections of elastic scattering for two mentioned angular regions, not to sum up amplitudes $f_{d i f}(\theta)$ and $f_{c l}(\theta)$. Then the cross section of diffraction scattering, taking into account (9) and (10) can be written as

$$
\begin{align*}
\frac{d \sigma}{d \Omega}=\frac{R^{2}}{4}\{ & \frac{J_{1}^{2}\left(2 k R \sin \frac{\theta}{2}\right)}{\sin ^{2} \frac{\theta}{2}}+\frac{J_{1}^{2}\left(2 k R \cos \frac{\theta}{2}\right)}{\cos ^{2} \frac{\theta}{2}}+ \\
& \left.+\frac{2}{3} \frac{J_{1}\left(2 k R \sin \frac{\theta}{2}\right)}{\sin \frac{\theta}{2}} \frac{J_{1}\left(2 k R \cos \frac{\theta}{2}\right)}{\cos \frac{\theta}{2}}\right\} \tag{12}
\end{align*}
$$

In the quasi-classical approximation $k R \gg 1$ the first term in the braces of (12) gives the main contribution for $\theta \leq(k R)^{\frac{1}{3}}$, the second term contributes for $\theta \geq \pi-(k R)^{\frac{1}{3}}$, the third term, as a rule, can be neglected as being an interference quantum contribution in this approximation [18]. The classical cross section describes the isotropic distribution of scattered particles and has a very simple form

$$
\begin{equation*}
\frac{d \sigma_{c l}}{d \Omega}=\frac{R^{2}}{4} \tag{13}
\end{equation*}
$$

## 4. RESULTS OF CALCULATIONS AND COMPARISON WITH EXPERIMENTAL DATA

Experimental dependencies of angular distributions $\frac{d \sigma}{d \Omega}$ of scattered deuterons in $d d$ - collisions in the c.m. system for a number of incoming deuteron energies $E_{d}$ in laboratory system, presented in the Fig. 1 and Fig.2, give the possibility to determine the general structure of the distributions, some trends and, in particular, the influence of incoming energy $E_{d}$ on them. Diffraction nuclear model that takes into account $N N$-interaction between the nucleons and the identity of the colliding deuterons as two identical bosons was used for theoretical description of elastic $d d$-scattering angular distributions in the region of the main peak. Calculations were carried out for energy of incoming deuterons $12.1 \leq E_{d} \leq 85 \mathrm{MeV}$. In follow we presented the results of our calculations of elastic $d d$-scattering differential cross sections $\frac{d \sigma}{d \Omega}$ for energies of incoming deuterons 12.1, 36.9, 60 and 70 MeV which reflects the most characteristic features of angular distributions in this energy range. The theoretical curves obtained using a microscopic diffraction model by formulae (1) - (8) are shown in Fig. 2 (as a thick solid curve 1).


Fig.2. Comparison experimental and theoretical angular distributions of elastic dd-scattering at energies: $12 \mathrm{MeV}[1,8,9]$, 36.9 MeV (our data), 60 and 70 MeV [5]. Points - experiment. Curves 1 - calculation by microscopic diffraction model: (a) $\alpha_{1}=0.7, \alpha_{2}=0.5, b^{2}=0.355 \mathrm{fm}^{-2}$; (b) $\alpha_{1}=0.5, \alpha_{2}=0.5, b^{2}=$ $0.45 \mathrm{fm}^{-2} ;(c) \alpha_{1}=0.6, \alpha_{2}=0.6, b^{2}=0.52 \mathrm{fm}^{-2} ;(d) \alpha_{1}=0.45, \alpha_{2}=0.49, b^{2}=0.49 \mathrm{fm}^{-2} ;$ curves 2 diffraction scattering: (a) $R=5.5$, (b) $R=3.8$ (2a $-R=4.4$ ), (c) $R=3.32$, (d) $R=3.0$; curves 3 - calculation by classical approximation: (a) $R=2.79$, (b) $R=5.5$; curve 4 -calculation by formula take into account interference of diffraction and classical amplitudes: (a) $R=5.5$

According to mentioned above, such model when using wave function (3) and (4) can lead to adequate description of experiments only for comparatively small deuteron scattering angles in the c.m. system $\theta \leq(k R)^{-\frac{1}{3}} \leq 60^{\circ}$, i.e. in the region of the main (by height) first maximum of the cross section at $\theta=0^{\circ}$. If the deuterons (recoils) would be registered in experiments also for angles $\theta>120^{\circ}$, than, as it was marked earlier, we would have also the second observed high maximum at $\theta=180^{\circ}$, which would be same in the high (and the form) as the maximum at $\theta=0^{\circ}$ due to identity of colliding deuterons. That is why, as it could be expected, satisfactory agreement with experimental distributions of scattered deuterons for all mentioned energies of incoming deuterons was achieved for angle $\theta \leq(k R)^{-\frac{1}{3}} \approx 60^{\circ}$. Good agreement is observed also for angle interval $110^{\circ} \leq \theta_{\text {c.m. }} \leq 145^{\circ}$ at deuteron energy $E_{d}=12 \mathrm{MeV}$. For angle interval $60^{\circ} \leq \theta_{\text {c.m. }} \leq 120^{\circ}$, where this model should not work, qualitative description of the experiment is ob-
served, i.e. the shape of experimental angular distribution is reproduced. As follows from fig. 1 minimum near $\theta_{\text {c.m. }}=60^{\circ}$ is observed both for our measured angular distribution of deuterons scattered by deuterons at $E_{d}=36.9 \mathrm{MeV}$ and for other data at $E_{d}=25.3 \mathrm{MeV}$ and $E_{d}=50 \ldots 85 \mathrm{MeV}$. But for the lowest energies (12.1...12.3 MeV) such structure in angular distribution is not seen. Characteristic peculiarities of behavior of cross sections of elastic $d d$-scattering for different energies can be also qualitatively described with the help of diffraction model of two colliding nonstructural hard balls-deuterons with the account of their identity. The angular distributions of the scattering cross section $\frac{d \sigma_{d i f}}{d \Omega}$ were calculated by the formula (12) for all energies and are shown in Fig. 2 by dashed curves. Here as well as for the microscopic diffraction model, conditions of model approximations can be satisfied only for angles $60^{\circ} \leq \theta_{\text {c.m. }} \leq 120^{\circ}$ and $120^{\circ} \leq \theta_{\text {c.m. }} \leq 180^{\circ}$. The theoretical cross sections calculated by the formula (12) lead to almost the same satisfactory description
of the experiment, as well as microscopic diffraction model in this angular region. This model clearly shows the presence of extremums in angular distributions at $\theta \approx 60^{\circ}$ and $\theta \approx 120^{\circ}$ for energies of incoming deuterons $E_{d} \geq 25 \mathrm{MeV}$, and for smaller energies this model describes the change of angular dependence for angular interval $60^{\circ} \leq \theta_{\text {c.m. }} \leq 120^{\circ}$. Positions of extremums in the angular distribution of cross sections are qualitatively connected with the value of a boundary angle $\theta \approx(k R)^{-\frac{1}{3}}$ (see (9) and (10), and as well [17]), being dependent weakly on deuteron energy $E_{d}$ for energy interval $25.3 \mathrm{MeV} \leq E_{d} \leq 51.5 \mathrm{MeV}$. In the intermediate region of angles $60^{\circ} \leq \theta_{c . m} \leq 120^{\circ}$, where conditions of applicability of the model are already violated to some extent, not only qualitative but also quantitative description of the experiment is achieved for $E_{d}=36.9 \ldots 70 \mathrm{MeV}$ for the region of small by height secondary diffraction maximum. As in the region of medium angles $60^{\circ} \leq \theta_{\text {c.m. }} \leq 120^{\circ}$ conditions of applicability of used approximations for our deuteron energies are still violated than to describe cross sections in this wide angular region, where cross sections are getting very small almost permanent values, we used the simplified phenomenological model for colliding nonpenetrating balls (with isotropic distributions of particles over angles) $[1,11,12,17]$. According to this model observed cross sections are described approximately by the formula (13) with the radius of nuclear interaction $R$ [10, 12] depending on energy. Appropriate dependences of cross sections at the energy $E_{d}=36.9 \mathrm{MeV}$ are given in the Fig. 2 by dense solid line 3. Analyzing the theoretical angular distribution of elastic $d d$-scattering at $E_{d}=12,1(12,3) \mathrm{MeV}$, we can note that the microscopic (8) and diffraction (12) models qualitatively describe the shape of the angular distribution and indicate the presence of a minimum at $\theta_{\text {c.m. }}=90^{\circ}$ (and its absence at $\theta_{c . m}=60^{\circ}$ ). Analyzing the experimental angular distributions (see Fig.1), we can note that the cross section at the minimum $\left(\theta_{c . m .}=90^{\circ}\right)$ at $E_{d}=12,1 \mathrm{MeV}$ roughly an order of magnitude higher then the cross sections in the secondary maximum $\left(\theta_{c . m}=90^{\circ}\right)$ at energies $E_{d} \geq 36,9 \mathrm{MeV}$. Calculations of the elastic scattering were carried out by the formula (11) taking into account interference of classical and diffraction amplitudes. The results are shown in Fig. 2 (a) by dotted curve. Interference term in expression (11), which is usually ignored, was significant at low energies $E_{d}$, that perhaps explains the nature of the appearance of a minimum at $\theta_{\text {c.m. }}=90^{\circ}$ at a given energy. It follows from the above analysis that both for our measured angular distribution of deuterons scattered by deuterons at $E_{d}=36.9 \mathrm{MeV}$ and for data of other works at $E_{d}=25.3 \mathrm{MeV}[4]$ and $E_{d}=50 \ldots 85 \mathrm{MeV}$ [5] a minimum is observed near the angle $\theta_{\text {c.m. }}=60^{\circ}$. Position of this minimum is in accord with given earlier formulae for differential cross sections $\frac{d \sigma}{d \Omega}$. It has a diffraction nature for energies $E_{d} \approx 25 \ldots 50 \mathrm{MeV}$ as well as the secondary not strictly expressed diffraction maximum at $\theta_{\text {c.m. }} \approx 90^{\circ}$. Eerier (see, for example, [8]) the
arising of the mentioned structure in cross sections $\frac{d \sigma}{d \Omega}$ was treated theoretically by methods sometimes being some artificial and not very evident, while the observed extremums in cross sections, as we think, arise and are explained naturally with the help of simple diffraction mechanism.

## 5. SUMMARY

1. The angular distribution of deuterons scattered by deuterons at energy $E_{d}=36.9 \mathrm{MeV}$ in the angle interval in the c.m. system $30^{\circ} \leq \theta_{\text {c.m. }} \leq 116^{\circ}$ was measured.
2. Comparison of the present results with published data on elastic scattering of deuterons by deuterons at energies of $12.1 \mathrm{MeV} \leq E_{d} \leq 85 \mathrm{MeV}$ was conducted. It is shown that the energy dependence of angular distributions of $d d$-scattering characterized by decreasing cross sections with increasing energy $E_{d}$ and by noticeable structural changes.
3. Diffraction nuclear model which takes into account $N N$-interaction between the nucleons and the diffraction model of two colliding structureless hard spheres-deuterons were used to describe the angular distributions of elastic $d d$-scattering. Both models take into account the identity of the colliding deuterons, which naturally explains the observed symmetry of the differential cross sections for the angle $\theta_{\text {c.m. }}=90^{\circ}$.
4. Our experimental data, as well as measured differential cross sections of elastic scattering from other works for energies of incoming deuterons $12.1 \mathrm{MeV} \leq E_{d} \leq 85 \mathrm{MeV}$ were satisfactory described with the help of microscopic diffraction nuclear model which takes into account $N N$-interaction between the nucleons.
5. Structural peculiarities of angular dependences of elastic $d d$-scattering cross sections were qualitatively explained with the help of the diffraction model of two colliding hard balls-deuterons with the account of their identity and qualitative and quantitative explanation of the diffraction structure of the cross sections was achieved for scattering angles of $60^{\circ} \leq \theta_{c . m .} \leq 120^{\circ}$ at energies of $12.1 \mathrm{MeV} \leq E_{d} \leq$ 85 MeV .

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## УПРУГОЕ РАССЕЯНИЕ ДЕЙТРОНОВ НА ДЕЙТРОНАХ ПРИ $E_{d} \leq 85$ МэВ

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Измерены дифференциальные сечения упругого рассеяния дейтронов с энергией $E_{d}=36,9 \mathrm{M}$ В ядрами дейтерия в диапазоне углов $30^{\circ} \leq \theta_{c . m} . \leq 116^{\circ}$. Для описания основного максимума $\theta_{c . m} \leq 60^{\circ}$ использовалась дифракционная ядерная модель, учитывающая структуру сталкивающихся ядер. Получено удовлетворительное согласие с экспериментом для углов $\theta_{c . m} \leq 60^{\circ}$ как с нашими экспериментальными данными, так и с данными других работ для энергий $12 \mathrm{M} \mathrm{B} \leq E_{d} \leq 85 \mathrm{M}$ В. При теоретической интерпретации угловых распределений учтена тождественность сталкивающихся дейтронов.

# ПРУЖНЕ РОЗСІЯННЯ ДЕЙТРОНІВ НА ДЕЙТРОНАХ ПРИ $E_{d} \leq 85 \mathrm{MeB}$ 

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Було виміряно диференціальні перерізи пружного розсіювання дейтронів з енергією $E_{d}=36,9 \mathrm{MeB}$ ядрами дейтерію в діапазоні кутів $30^{\circ} \leq \theta_{\text {c.m. }} \leq 116^{\circ}$. Для опису основго максимуму $\theta_{\text {c.m. }} \leq 60^{\circ}$ використовувалась дифракційна ядерна модель, яка враховує структуру ядер, що зіштовхуються. Отримано задовільне узгодження з експериментом для кутів $\theta_{c . m} \leq 60^{\circ}$ і з нашими експериментальними даними, і з даними інших робіт для енергій $12 \mathrm{MeB} \leq E_{d} \leq 85 \mathrm{MeB}$. Для теоретичної інтерпретації кутових розподілів було враховано тотожність дейтронів, що зіштовхуються.


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