# MEASUREMENT POSSIBILITY OF SMALL AMPLITUDES CONTRIBUTION IN THE REACTIONS OF TWO-PARTICLE ${ }^{4} \mathrm{He}$ PHOTODISINTEGRATION 

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(Received August 15, 2008)


#### Abstract

The two-body photodisintegration of ${ }^{4} \mathrm{He}$ was investigated. The dependence of expansion coefficients hard error on the histogram bin, the angle measurement precision and coordinate position where fitting function is calculated, was studied. It is shown that the logarithm of the relative error in coefficients is in the linear dependence on the logarithm of a number of events. The relation between the number of events, necessary for obtaining the relative errors of $5 \%$, $10 \%$ and $20 \%$, and the value of coefficients was found. The results are compared with the experimental data. In the future experiment the estimation of the upper limits in coefficients can be set as a main goal.


PACS: 25.20.Dc; 25.20-x

## 1. INTRODUCTION

This research was undertaken to study the possibility of the small amplitudes measurement in coming light nuclei photodisintegration experiment which will be performed in the NSC KIPT. In the experiment a streamer chamber in the magnet, provided with a digital information pickup will be used. When investigating the reactions caused by bremsstrahlung a problem arises because of different numbers of events in the various energy ranges. It is known that above the gigantic resonance a total reaction cross-section decreases almost exponentially with energy increasing. The number of photons per 1 MeV interval is inversely proportional to the energy increase. The numbers of events falling on the equal energy intervals near the gigantic resonance region and at 100 MeV differs almost by two orders of magnitude. The track techniques record all events simultaneously. To get results in the selected energy range, an experimenter must measure the kinematical parameters of all events and then select the required ones.

## 2. EXPERIMENT

In approach of the electric dipole, electric quadrupole and magnetic dipole transitions the differential cross-section of $\gamma+{ }^{4} \mathrm{He} \rightarrow p+{ }^{3} \mathrm{H}$ and $\gamma+{ }^{4} \mathrm{He} \rightarrow n+{ }^{3} \mathrm{He}$ reactions can be expressed by means of trigonometric functions [1]:

$$
\begin{align*}
& d \sigma / d \Omega=a \sin ^{2} \theta+b \sin ^{2} \theta \cos \theta  \tag{1}\\
& \quad+c \sin ^{2} \theta \cos ^{2} \theta+d+f \cos \theta
\end{align*}
$$

where the $\theta$ - S.C.M. polar angle. The five expansion coefficients include information about amplitudes. The coefficients $a, b$ and $c$ correspond to the
channel with the spin $\mathrm{S}=0$. They give main contribution to the cross-section. The coefficients $d$ and $f$ correspond to the channel with $\mathrm{S}=1$. Their contribution is insignificant $[1,2,3]$. Coefficients $d$ and $f$ increase with energy increasing [1, 2, 3]. Hence, they should be measured at energies above the gigantic resonance, but in this region the cross-section is small. The yield is measured in the experiments by track techniques:

$$
\begin{gather*}
d N(\theta)=2 \pi\left(a \sin ^{2} \theta+b \sin ^{2} \theta \cos \theta+\right.  \tag{2}\\
\left.\left.c \sin ^{2} \theta \cos ^{2} \theta+d+f \cos \theta\right)\right) \sin \theta d \theta
\end{gather*}
$$

Below this function is used for simulation of angle distributions. The streamer chamber, placed in the magnetic field, is a spectrometer with a wide spatial capture angle, but actually it is not a $4 \pi$-detector. A pulse of each particle, forming a small angle $\alpha$ with the


Fig.1. The scheme of the interaction area vector of magnetic-field strength ( mfs ), is measured with a poor precision. Therefore, a limit angle $\alpha_{0}$ is introduced. The event is rejected if the angle of any particle pulse with mfs vector is less than $\alpha_{0}$. In

[^0]Fig. 1 the center of unit sphere is coincided with the interaction point. The OX axis is directed along the photon pulse, OZ - along the mfs vector. Two cones with the opening angle of $2 \alpha_{0}$ restrict the area of rejected events. The geometrical correction is introduced for the rejected events. It was calculated on the assumption of the azimuth isotropy of differential cross-sections. This assumption is correct for a non polarized photon beam:

$$
\begin{equation*}
\psi=\pi /(2 \arcsin (\sin \beta / \sin \theta)), \tag{3}
\end{equation*}
$$

if $\sin \beta / \sin \theta<1$ and $\psi=1$ if $\sin \beta / \sin \theta \geq 1$, where $\beta=\pi / 2-\alpha$ is the angle between the pulse and the plane being perpendicular to the mfs vector, $\theta$ - is the polar angle. The generation of random numbers, distributed according to formula (2), is performed using the standard technique [4]. As an example, the coefficients chosen are close to the experimental ones $[1,2,3]: a=1, b=0.75, c=0.43, d=0.075, f=0.025$. Kinematic parameters of reaction products were recalculated to L-system having supposition that photon's energy, caused reaction, is 40 MeV . The limit angle between the particle pulse and the mfs vector $\alpha_{0}=55^{\circ}$. The rejection was performed for particles output at smaller angles. At such limitation only a half of events are counted. The error of angle measurement $\delta \theta=2$ is also taken into account. Kinematic parameters of the residual events were recalculated to c.m.system. The generated events were sorted by the angle $\theta$. The histogram with a step of $10^{\circ}$ is built.


Fig.2. The coefficients as a function of the event of numbers. Points - fitting results, Lines - initial values

The number of events entered into the angle interval, is divided by the solid angle value. The coefficients were calculated by the least-square method from the fit of expression (1) to the experimental data
histogramed with a bin value of $10^{\circ}$ [5]. It is adopted for fitting that the distribution center on the step coincides with the step midpoint. As the result of the fitting five coefficients and their errors are obtained: $a \pm \delta a, b \pm \delta b, c \pm \delta c, d \pm \delta d$ and $f \pm \delta f$. After dividing them by $a$, the coefficients of expansion (1) are obtained. They are shown in Fig. 2 as a function of the number of events N per the histogram. The coefficients $a$ and $b$ are in accord with the initial $a$ and $b$ at $\mathrm{N}=500, c$ - at $\mathrm{N}=1000$. The small coefficients approximate to the initial ones at much greater N , systematically exceeding the initial coefficients by $30 \%-50 \%$.

## 3. PRESENTATION AND PROCESSING OF THE RESULTS

To reveal the reason of overstating the small coefficient, the method of building the histograms and their approximations was carefully studied. The distributions were generated at $d=f=0$, but were fitted with five coefficients. The coefficients f and d are shown in Fig.3. The intersection of the two dotted lines corresponds to the case $d=f=0$ accepted upon generation of events. The number of playable events $\mathrm{N}=200000$. The distribution is presented by the histogram with a step of $10^{\circ}$. The square is the result of fitting. The centroid of distribution is assumed to be coinciding with a histogram step midpoint. The obtained coefficients differ markedly from the initial ones.


Fig.3. The coefficients $f$ and $d$ in different cases of the result presentation

The trigonometric functions, by which the crosssection (1) is expanded, are represented graphically by lines of a complex shape. By reducing the histogram step their structure is displayed in more detail. The circle in Fig. 3 represents the result of fitting the histogram with a step of $5^{\circ}$. The generation of events was performed with taken into account the error of angle measurement. The centroid of distribution is assumed to be coinciding with a histogram step midpoint. This time coefficients become closer to the initial ones. Function (2) is nonlinearly changing in each of histogram steps, what is why the centroid of distribution does not always coincide with the step midpoint. The coordinate of the centroid of distribution was determined in each of
steps. The triangle corresponds to the histogram which is formed with $5^{\circ}$ step. The coordinates of the each point coincide with the center of distribution weight on the step. Within the errors, coefficients coincide with the initial ones. The results shown by rhombus were obtained with the same assumptions as the results shown by triangles, but without taking into account the error of angle measurement. The influence of a histogram representation method on coefficient values was discussed in [6], but no results which can be compared with ours were obtained. In the experiments $[1,2,3]$ a histogram step is $10^{\circ}$, the position of each point is taken in the step midpoint.


Fig.4. The comparison between the calculation result and the experimental data

In Fig.4, $a$ the experimental values of $d[1]$ (shown by black circles) are compared with the calculated ones. The computation is performed with the experimental statistical supply in each point independently eight times to show the spread of points. The generation of events is provided at $d=f=0$, the fitting was performed by four coefficients. Like in the experiment the histogram step is $10^{\circ}$. The calculated values are close to the experimental data. In the Fig. $4, b$ the calculation results are compared with the experimental values of $d[2,3]$ shown by the black circles for ${ }^{4} \mathrm{He}(\gamma, p)^{3} \mathrm{H}$ reaction and by the white circles for ${ }^{4} H e(\gamma, n)^{3} H e$ reaction. The calculation was carried out at the same suggestions as in Fig.4, $a$. The comparison shows that experimental results are not reliable.

## 4. RELATIVE COEFFICIENT ERRORS

The dependence of the relative error of coefficients on the number of events is obtained for the distribution of (1) with the following set of coefficients: $a=1, b=0.86, c=0.43, d=0.05$ and $f=0.025$. The histogram step is $5^{\circ}$ and the error of angle measurement
is taken into account. The position of each point coincides with the centroid of distribution on the step. The error of relative coefficients as a function of the histogram statistical supply is shown in Fig.5.


Fig.5. The relative errors of coefficients versus the number of events

There is a linear dependence in the double logarithmic scale. The lines are the result of the fitting with least-square method and the dotted lines are respective precisions. The intersection of lines gives a required number of events to provide the corresponding precision. The events number N , necessary to measure a coefficient with a given precision, increases with coefficient value decreasing. The number of events as a function of $d$ and $f$ values is shown in Fig.6.


Fig.6. The number of events versus the $d$ and $f$ values

In the case of $d=f$ the numbers of event for each of coefficient, corresponding to the same precision, differ in few percents. Therefore, in the graph shown are the averaged lines for both coefficients. Using the energy dependence of the total cross-section [1] and the bremsstrahlung spectrum, the event distribution in the energy intervals, being very non uniform, was obtained. The K factor in the Table is equal to the ratio between the all events detected by the chamber and the number of events, recorded in the specified energy interval. As is mentioned above, the efficiency of event recording in the given reactions is $50 \%$. So, to measure the expan-
sion coefficients with the required precision in the specified energy range, the $\mathrm{M}=2 \mathrm{NK}$ events should be processed. Let us consider an example where $\mathrm{d}=0.05$.

| $E_{\gamma}, \mathrm{MeV}$ |  |  | K | $E_{\gamma}, \mathrm{MeV}, \mathrm{K}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 30 | 2.1 | 60 | 70 | 35.4 |  |
| 30 | 40 | 3.4 | 70 | 80 | 56.8 |  |
| 40 | 50 | 8.2 | 80 | 90 | 100.6 |  |
| 50 | 60 | 17.9 | 90 | 100 | 194.0 |  |

According to Fig. 6 for the precision of 20\%, it is necessary to obtain $N \approx 7500$ events and $M \approx 32000$ for the $20 \ldots 30 \mathrm{MeV}$ range and $M \approx 1600000$ for the $80 \ldots 90 \mathrm{MeV}$ range.

## 5. SUMMARY

The simulation of two-body reactions of ${ }^{4} \mathrm{He}$ photodisintegration is a helpful tool to estimate the capability of small amplitude contribution measurements in the experiment with the use of a streamer chamber. By this method it is possible to reveal a cause of a hard error in the estimation of small coefficients of expansion (1), if the experimental data are represented as a histogram with the step of $10^{\circ}$. The hard errors are eliminated, if the histogram step is less than $5^{\circ}$ and the position of each point of the fitted distribution must coincides with the coordinates of the centroid of distribution in each step. For the given precision and in the defined energy interval the simulation model has allowed to get the dependency of the number of events N falling on the histogram on the value of small coefficients and to estimate the number of events M, recorded by the streamer cham-
ber in the entire energy range of photons. Comparison of the small coefficients, obtained by simulation, with the experimental data $[1,2,3]$ evidences on the low reliability of experimental coefficients. The values of the experimental coefficients can be accounted for the hard errors, committed when processing the experimental results. The upper limit estimation of the coefficient values can be set as the main goal in the planned experiment. The authors wish to express deep gratitude to Professor P.V. Sorokin for the fruitful discussion of the problem.

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# О ВОЗМОЖНОСТИ ИЗМЕРЕНИЯ ВКЛАДА МАЛЫХ АМПЛИТУД В РЕАКЦИЯХ ДВУХЧАСТИЧНОГО ФОТОРАСЩЕПЛЕНИЯ ЯДРА ${ }^{4}$ Не 

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Исследована зависимость систематической погрешности коэффициентов разложения дифференциальных сечений двухчастичного фоторасщепления ядра ${ }^{4} \mathrm{He}$ по тригонометрическим функциям от шага гистограммы, точности измерения угла и положения координаты, в которой вычисляется подгоняемая функция, на шаге гистограммы. Показано, что логарифм относительной погрешности коэффициентов линейно зависит от логарифма числа событий. Найдена зависимость числа событий, необходимых для получения относительных погрешностей 5,10 и $20 \%$ для коэффициентов, соответствующих малым амплитудам, от величины коэффициентов. Выполнено сравнение с экспериментом. В планируемом эксперименте можно ставить задачу оценки верхней границы коэффициентов.

# ПРО МОЖЛИВІСТЬ ВИМІРУ ВНЕСКУ МАЛИХ АМПЛІТУД У РЕАКЦІЯХ ДВОХЧАСТКОВОГО ФОТОРОЗЩЕПЛЕННЯ ЯДРА ${ }^{4} \mathrm{He}$ 

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Досліджено залежність систематичної погрішності коефіцієнтів розкладання диференціальних перетинів двохчастичного фоторозщеплення ядра ${ }^{4}$ Не по тригонометричних функціях від кроку гістограми, точності виміру кута і положення координати, у якій обчислюється функція, що підганяється, на кроці гістограми. Показано, що логарифм відносної погрішності коефіцієнтів лінійно залежить від логарифма числа подій. Знайдено залежність числа подій, необхідних для одержання відносних погрішностей 5,10 та $20 \%$ для коефіцієнтів, що відповідають малим амплітудам, від величини коефіцієнтів. Виконано порівняння з експериментом. У планованому експерименті можна ставити задачу оцінки верхньої границі коефіцієнтів.


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