THE TIME OF SIMULTANEOUS TUNNELING OF IDENTICAL PARTICLES THROUGH THE RECTANGULAR QUANTUM BARRIER

L.S. Martseniuk*[*], *V.S. Olhovsky

Institute for Nuclear Research, National Academy of Sciences of Ukraine, 03680, Kiev, Ukraine

(Received April 6, 2010)

Work is devoted to studying the influence of exchange processes on a time of simultaneous crossing by identical particles of a rectangular quantum barrier. It is shown, that such processes essentially influence on the parameters of tunneling. The size of addition to time of identical particles tunneling, arising up because of their exchange interaction in a field of a rectangular quantum barrier is first counted.

PACS: 03.65.-w, 03.65.Xp, 03.65.Nk

1. INTRODUCTION

At research of processes of two identical particles dispersion on each other it is necessary to take into account their exchange interaction which essentially influences on the parameters of dispersion. Such account for coulomb potential has been lead earlier [1-3]. The Mott's formula is describing the dispersion of identical particles in a field of coulomb potential, it contains additional element, arising up owing to effect of exchange interaction. Parameters of the tunneling process which can be considered as a limiting case of dispersion at a zero corner of dispersion also can depend on characteristics of exchange processes between identical particles. In the given work research of influence of exchange interaction of identical particles on a time of their simultaneous tunneling through a rectangular quantum barrier is lead. As follows from [1], a wave function describing the collision of two particles in the system of center of masses can be presented by the following expression:

$$u(r)_{r\to\infty} \to e^{i\hbar z} + r^{-1}\ f(\theta,\varphi)e^{ikr}\,, \qquad (1)$$

where r, θ, φ - are spherical coordinates of vector r. For identical particles, taking into account the necessity of symmetrisation of wave function of dispersion, asymptotical expressions for symmetric and antisymmetric functions of waves it is necessary to write down as follows:

$$\begin{aligned}\Psi \;=\;& (e^{ikz} \pm e^{-ikz}) + \\ & [f(\theta,\varphi) \pm f(\pi-\theta,\varphi+\pi)]r^{-1}e^{ikr}\,, \quad (2)\end{aligned}$$

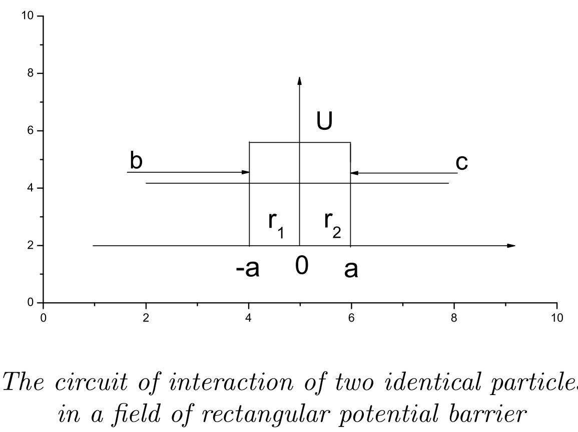
where "+" corresponds to symmetric function, "-" to antisymmetric function. The effective section of dispersion, according to [1], will be described by:

$$\begin{aligned}\sigma(\theta,\varphi) \;=\;& |f(\theta,\varphi)|^2 + |f(\pi-\theta,\varphi+\pi)|^2 \pm \\ & 2Re[f(\theta,\varphi)f^*(\pi-\theta,\varphi+\pi)]\,. \quad (3)\end{aligned}$$

As follows from this expression, at the account of identity of particles at their dispersion in the field of quantum barrier, to the section of dispersion for two particles it is necessary to bring in addition, determinate by the third element in a formula (3). Just the same difference in the formula of dispersion and determines a change time for identical particles by virtue of their co-operation.

2. ESTIMATION THE TIME OF IDENTICAL PARTICLES TUNNELING THROUGH THE RECTANGULAR QUANTUM BARRIER

The chart of tunneling of two identical particles in one dimension variant is represented on the Figure



The circuit of interaction of two identical particles in a field of rectangular potential barrier

In a theory of dispersion are shown that if present a dispersive potential, the asymptotical form of wave function looks like [2]:

[*] Corresponding author E-mail address: prolisok77@yandex.ru

$$r\Psi = \sum_l P_l(\cos\theta)\ g_l(r) \cong \sum_l A_l P_l(\cos(\theta)\sin(kr+\Delta)\,, \quad (4)$$

where$\Delta_l = \delta_l - \frac{l\pi}{2}$ It ensues from the theory of dispersion with the use of method of partsial waves [2], that expression for transversal dispersion of not identical particles looks like

$$\sigma = |f(\theta)|^2 = \frac{1}{k^2}|\sum_l \frac{(2l+1)}{2}P_l(\cos\theta)(e^{2i\delta^l}-1)|^2\,. \quad (5)$$

This formula shows dependence of transversal section on a phase δ. At $l=0$ angular dependence is absent, and have the following expression:

$$\sigma = \frac{1}{k^2}\frac{[e^{2i\delta_l}-1]^2}{4}\,. \quad (6)$$

Taking into account this expression will have:

$$Re(f_b f_c^*) = \frac{1}{4k^2}(e^{2i\delta}-1)(e^{-2i\delta}-1) = \frac{1}{2k^2}(1-\cos 2\delta)\,. \quad (7)$$

Thus, function, describing dispersion of two identical particles will have the following kind:

$$\Psi = (e^{ikz} \pm e^{-ikz}) + r^{-1}e^{ikr}\sqrt{\frac{1}{2k^2}(1-\cos 2\delta)} = (e^{ikz} \pm e^{-ikz}) + r^{-1}e^{ikr}\frac{1}{k\sqrt{2}}\sin\delta\,. \quad (8)$$

For small values δ (as specified in [2] at small values δ a phase is also small): $\sin\delta \cong \delta \cong e^{\delta}-1$

$$\Psi = (e^{ikz} \pm e^{-ikz}) + r^{-1}e^{ikr}\frac{1}{k\sqrt{2}}(e^{\delta}-1)\,. \quad (9)$$

Thus, for dispersion of symmetric identical particles the additional change of phase appeared approximately equal δ (while in ordinary case - for not identical particles, his value 2δ , [2]. Addition to time of tunneling taking into account exchange interaction of identical particles, described a symmetric wave function, in accordance with [2, 5] makes:

$$\tau_f = \frac{1}{\nu_g}\frac{d}{dk}\delta = \frac{m}{\hbar k}\frac{d}{dk}\delta\,. \quad (10)$$

Expressions for δ are led in educational literature for various forms of potential. Having substituted these expressions in (10) it is possible to define the interesting us size of time of tunneling. In [2] such expressions are brought for $\tan\delta$

$$\tan\delta = \frac{\frac{k}{a}-\tan ka}{1+\frac{k}{a}\tan ka} \cong \frac{\frac{k}{a}-ka}{1+\frac{k}{a}ka}\,, \quad (11)$$

where:$a = k_1\cot k_1 a$,$k_1^2 = (E-U)\frac{2m}{\hbar^2}$ for quantum hole and $k_1^2 = (U-E)\frac{2m}{\hbar^2}$ -for a barrier in high U; $k^2 = \frac{2m}{\hbar^2}E$. From expression for k and k_1 follow, that $\frac{dk_1}{dk} = \frac{k}{k_1}$. Then:

$$\frac{d\sigma}{dk} = \frac{1}{1+(\frac{\frac{k}{a}-ka}{1+\frac{k}{a}ka})^2} \times \frac{(1+\frac{k^2}{\alpha}a)(\frac{\alpha-k\frac{d\alpha}{dk}}{\alpha^2}-a)-(\frac{k}{\alpha}-ka)[(\frac{\alpha-k\frac{d\alpha}{dk}}{\alpha^2})ka+\frac{ka}{\alpha}]}{(1+\frac{k^2a}{\alpha})^2}\,, \quad (12)$$

where:

$$\frac{d\alpha}{dk} = \frac{dk_1}{dk}(\cot k_1 a - k\frac{a}{\sin^2 k_1 a}) = \frac{k}{k_1}\frac{\sin 2k_1 a + 2k_1 a}{2\sin^2 k_1 a}\,. \quad (13)$$

Then to addition of tunneling time get next expression:

$$\Delta\tau_f = \frac{1}{\nu_g}\frac{d}{dk}\delta = \frac{m}{\hbar k}\frac{(\alpha+k^2a)(\alpha-\alpha^2a-k\frac{d\alpha}{dk})-(k-\alpha ak)(2\alpha ak-k\frac{d\alpha}{dk})}{\alpha^3[(1+\frac{k}{\alpha}ka)^2+(\frac{k}{\alpha}-ka)^2]}\,. \quad (14)$$

3. CONCLUSIONS

Additional time of tunnelling for identical particles, simultaneously crossing through a quantum potential barrier (a potential hole) in opposite directions is designed, taking into account their exchange interaction. Tunnelling was considered as a limiting case of dispersion at a corner of dispersion aspiring to zero. It is shown, that the exchange interaction changes the time of tunnelling. Such analysis of the temporal descriptions of simultaneous tunnelling processes is conducted first.

References

1. L. Schiff. *Quantum mechanics.* M: "F.L.", 1959, 473 p. (in Russian).

2. D. Bohm. *Quantum theory.* M: "Science", 1965, 727 p. (in Russian).

3. L.D. Landau and E.M. Lifshitz. *Quantum mechanics, course of Theoretical Physics*, v. 3. M: "Nauka", 1989, p.768 [in Russian; eng. variant: Oxford, Uk, Pergamon, 1982].

4. V.G. Levich, J.A. Vdovin, V.A. Miamlin. *Course of theoretical physics.* v. 2. M: Science, 1971, p.936 (in Russian).

5. G. Privitera, G. Salesi, V.S. Olkhovsky, E. Recami. Tunneling times: An elementary introduction // *Rivista del Nuovo Cimento.* 1991, 26 (4), p.55.

ВРЕМЯ СИНХРОННОГО ТУННЕЛИРОВАНИЯ ТОЖДЕСТВЕННЫХ ЧАСТИЦ ЧЕРЕЗ ПРЯМОУГОЛЬНЫЙ КВАНТОВЫЙ БАРЬЕР

Л.С. Марценюк, В.С. Ольховский

Работа посвящена теоретическому исследованию влияния обменного взаимодействия тождественных частиц на временные характеристики их одновременного туннелирования через прямоугольный потенциальный барьер. Показано, что, поскольку включение обменной энергии изменяет фазовые характеристики волновых функций, описывающих процесс туннелирования, то обменные процессы влияют на временные характеристики этого процесса. Впервые было рассчитано значение дополнительного времени туннелирования тождественных частиц с учетом их обменного взаимодействия.

ЧАС СИНХРОННОГО ТУНЕЛЮВАННЯ ТОТОЖНИХ ЧАСТИНОК ЧЕРЕЗ ПРЯМОКУТНИЙ КВАНТОВИЙ БАР'ЄР

Л.С. Марценюк, В.С. Ольховський

Робота присвячена теоретичному дослідженню впливу обмінної взаємодії тотожних частинок на часові характеристики їх синхронного тунелювання через прямокутний потенційний бар'єр. Показано, що, оскільки включення обмінної енергії змінює фазові характеристики хвильових функцій, що описують процес тунелювання, то обмінні процеси впливають на часові характеристики цього процесу. Вперше було розраховано значення додаткового часу тунелювання тотожних частинок з урахуванням їх обмінної взаємодії.