THE IDEAL GAS OF HYDROGEN-LIKE ATOMS RESPONSE TO THE PERTURBATION BY THE EXTERNAL ELECTROMAGNETIC FIELD

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The response of the ideal gas consisting of hydrogen-like atoms to the perturbation by the external electromagnetic field in low temperature region is studied. Consideration is based on using the Green functions formalism [1] and the second quantization method in the presence of bound states of particles [2]. As the most interesting phenomenon, the perturbation of system in Bose-condensation state is studied. The dispersion characteristics of such system at frequencies close to the energy interval between alkali atoms energy levels are investigated.

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1. INTRODUCTION

In the process of describing a behavior of many-particle systems a class of problems appear, that are concerned with the system response to the perturbing action of the external, in particular, electromagnetic field. Widespread approach to solving such kind of problems is based on using the Green functions formalism (see in that case e.g. Ref. [1]).

It is well known that the most convenient method of describing physical processes in quantum many-particle theory is the second quantization method. Thus, within the framework of the second quantization it is the simplest way to formulate an approach to a description of the system response to the perturbation by the external field that is based on Green functions. However, if we try to realize such an approach, we can come across an essential difficulty, connected with the possible occurrence of the particle bound states.

As it has been shown in [2] for a system that consists of two types of fermions (e.g. ions and electrons) and bound states (atoms or molecules) in low kinetic energies region we can use the approximate formulation of the second quantization method. It makes the mathematical description of such kind of systems rather simple, but preserves the required information concerning internal degrees of freedom for the bound states.

If the creation and annihilation operators for the different kind of particles are constructed, it is not difficult to broaden this theory on the external fields' existence situation. In this case the physical quantities operators (such as charge and current density operators) and the Maxwell-Lorentz system of equations that includes contribution of neutral bound states can be constructed (see Ref. [3]).

In order to simplify the following mathematical description it is convenient to consider such system (ideal low-temperature hydrogen-like plasma) in the equilibrium state (including its photon component). In this case (as it has been shown in Ref. [4]) we can neglect of free fermions contribution in the different physical processes that are observed in the system at extremely low temperatures.

2. BASIC OPERATORS

As we consider the system in low temperature region, the creation and annihilation operators for the hydrogen-like atoms (as the bound states of two different particles, see Ref. [2]) can be written as

$$\begin{aligned} &\hat{\varphi}^{+}(\mathbf{x}_1,\mathbf{x}_2)=\sum_{\alpha}\varphi_{\alpha}^{*}(\mathbf{x})\hat{\eta}_{\alpha}^{+}(\mathbf{X}),\\ &\hat{\varphi}(\mathbf{x}_1,\mathbf{x}_2)=\sum_{\alpha}\varphi_{\alpha}(\mathbf{x})\hat{\eta}_{\alpha}(\mathbf{X}),\\ &\mathbf{x}=\mathbf{x}_1-\mathbf{x}_2,\quad \mathbf{X}=\tfrac{m_1\mathbf{x}_1+m_2\mathbf{x}_2}{m_1+m_2}. \end{aligned} \tag{1}$$

Here m_1, $\mathbf{x}_1$ and m_2, $\mathbf{x}_2$ are the atomic core and the outer electron masses and coordinates, respectively, α is the set of quantum numbers of the atom in the certain state, characterized by the wave function $\varphi_{\alpha}(\mathbf{x})$, $\hat{\eta}_{\alpha}(\mathbf{X})$ is the annihilation operator of boson in α state.

The expression for the Hamiltonian of non-interacting atoms can be written as:

$$\mathcal{H}_p=V^{-1}\sum_{\alpha}\sum_{\mathbf{p}}\varepsilon_{\alpha}(\mathbf{p})\hat{\eta}_{\alpha}^{+}(\mathbf{p})\hat{\eta}_{\alpha}(\mathbf{p}), \tag{2}$$

where $\varepsilon_{\alpha}(\mathbf{p})=\varepsilon_{\alpha}+\mathbf{p}^2/2M$, ε_{α} is the energy of an atom in α quantum state ($\varepsilon_{\alpha}<0$), M is the bound state mass, V is the system volume. Using Eqs. (1)-(2) one can find the expressions for the charge and current density operators (accordingly to Ref. [1]), which in the Heisenberg representation take the form:

$$\begin{aligned} \hat{\sigma}(\mathbf{x})&=V^{-1}\sum_{\mathbf{p},\mathbf{p}'}\sum_{\alpha,\beta}e^{-i\mathbf{x}(\mathbf{p}-\mathbf{p}')}e^{-it(\varepsilon_{\alpha}(\mathbf{p})-\varepsilon_{\beta}(\mathbf{p}'))}\\ &\times\sigma_{\alpha\beta}(\mathbf{p}-\mathbf{p}')\hat{\eta}_{\alpha}^{+}(\mathbf{p})\hat{\eta}_{\beta}(\mathbf{p}'),\\ \hat{\mathbf{j}}(\mathbf{x})&=V^{-1}\sum_{\mathbf{p},\mathbf{p}'}\sum_{\alpha,\beta}e^{-i\mathbf{x}(\mathbf{p}-\mathbf{p}')}e^{-it(\varepsilon_{\alpha}(\mathbf{p})-\varepsilon_{\beta}(\mathbf{p}'))}\\ &\times\Big(\tfrac{(\mathbf{p}+\mathbf{p}')}{2M}\sigma_{\alpha\beta}(\mathbf{p}-\mathbf{p}')+\mathbf{I}_{\alpha\beta}(\mathbf{p}-\mathbf{p}')\Big)\hat{\eta}_{\alpha}^{+}(\mathbf{p})\hat{\eta}_{\beta}(\mathbf{p}'). \end{aligned} \tag{3}$$

Here for the compactness we have introduced the charge density $\sigma_{\alpha\beta}(\mathbf{k})$ and the current density $\mathbf{I}_{\alpha\beta}(\mathbf{k})$ matrix elements, which can be represented in terms of the atomic wave functions by the following formulas (see also Ref. [2]):

$$\sigma_{\alpha\beta}(\mathbf{k}) = e\int d\mathbf{y}\varphi_\alpha^*(\mathbf{y})\varphi_\beta(\mathbf{y})$$
$$\times\left[\exp\left(i\tfrac{m_2}{M}\mathbf{ky}\right) - \exp(-i\tfrac{m_1}{M}\mathbf{ky})\right],$$
$$\mathbf{I}_{\alpha\beta}(\mathbf{k}) = e\int d\mathbf{y}\left(\varphi_\alpha^*(\mathbf{y})\frac{\partial\varphi_\beta(\mathbf{y})}{\partial\mathbf{y}} + \frac{\partial\varphi_\alpha^*(\mathbf{y})}{\partial\mathbf{y}}\varphi_\beta(\mathbf{y})\right) \quad (4)$$
$$\times\left[\frac{1}{m_1}\exp i\tfrac{m_2}{M}\mathbf{ky} + \frac{1}{m_2}\exp(-i\tfrac{m_1}{M}\mathbf{ky})\right],$$

where e corresponds to the absolute value of electron's charge.

3. MAXWELL EQUATIONS AND GREEN FUNCTIONS

We have introduced the operators of charge and current density of ideal gas that consists of neutral atoms, and then it is not difficult to build the Maxwell-Lorentz system of equations that can describe the system response to the perturbation by the external electromagnetic field. These equations for the mean values of operators have the following form:

$$\frac{\partial\mathbf{H}}{\partial t} = -c\,\mathrm{rot}\,\mathbf{E},\quad \frac{\partial\mathbf{E}}{\partial t} = c\,\mathrm{rot}\,\mathbf{H} - 4\pi(\mathbf{J}+\mathbf{J}^{(e)}),$$
$$\mathrm{div}\,\mathbf{H} = 0,\qquad \mathrm{div}\,\mathbf{E} = 4\pi(\sigma+\sigma^{(e)}), \quad (5)$$

where values $\sigma^{(e)}$ and $\mathbf{J}^{(e)}$ are the external charge and current densities. If to assume that the perturbation of the system by the external field source is rather small, and the Hamiltonian of interaction is linear in respect to the external electromagnetic field, the mean values for the charge and current densities in Eq. (5) can be represented in terms of the Green functions (see Ref. [1], [3])

$$\sigma(\mathbf{x},t) = \int_{-\infty}^{\infty} dt'\int d^3x'\Big[-G_i^{(+)}(\mathbf{x}-\mathbf{x}',t-t')$$
$$\times\frac{1}{c}A_i^{(e)}(\mathbf{x}',t') + G^{(+)}(\mathbf{x}-\mathbf{x}',t-t')\varphi^{(e)}(\mathbf{x}',t')\Big],$$
$$J_k(\mathbf{x},t) = \int_{-\infty}^{\infty} dt'\int d^3x'\Big[-G_{kl}^{(+)}(\mathbf{x}-\mathbf{x}',t-t') \quad (6)$$
$$\times\frac{1}{c}A_l^{(e)}(\mathbf{x}',t') + G_k^{(+)}(\mathbf{x}-\mathbf{x}',t-t')\varphi^{(e)}(\mathbf{x}',t')\Big].$$

Here $G^{(+)}, G_k^{(+)}, G_{kl}^{(+)}$ are retarded scalar, vector and tensor Green functions respectively. Introducing the equilibrium state distribution functions for the ideal gas of hydrogen-like (alkali) atoms

$$f_\alpha(\mathbf{p}) = \left\{\exp\left[(\varepsilon_\alpha(\mathbf{p}) - \mu_\alpha)/T\right] - 1\right\}^{-1}, \quad (7)$$

where μ_α is the atomic chemical potential, T is the temperature in the energy units, the Fourier transforms of these Green functions, according to Ref. [3], can be written as follows:

$$G^{(+)}(\mathbf{k},\omega) = V^{-1}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\sigma_{\alpha\beta}(\mathbf{k})\sigma_{\beta\alpha}(-\mathbf{k})$$
$$\times\frac{f_\alpha(\mathbf{p}-\mathbf{k}) - f_\beta(\mathbf{p})}{\varepsilon_\alpha(\mathbf{p}) - \varepsilon_\beta(\mathbf{p}-\mathbf{k}) + \omega + i0},$$
$$G_l^{(+)}(\mathbf{k},\omega) = V^{-1}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\Big[\frac{(2\mathbf{p}-\mathbf{k})}{2M}\sigma_{\alpha\beta}(\mathbf{k})$$
$$+\mathbf{I}_{\alpha\beta}(\mathbf{k})\Big]_l\frac{\sigma_{\beta\alpha}(-\mathbf{k})\left[f_\alpha(\mathbf{p}-\mathbf{k}) - f_\beta(\mathbf{p})\right]}{\varepsilon_\alpha(\mathbf{p}) - \varepsilon_\beta(\mathbf{p}-\mathbf{k}) + \omega + i0}, \quad (8)$$
$$G_{lj}^{(+)}(\mathbf{k},\omega) = V^{-1}\sum_{\mathbf{p}}\sum_{\alpha,\beta}\Big[\frac{(2\mathbf{p}-\mathbf{k})}{2M}\sigma_{\alpha\beta}(\mathbf{k})$$
$$+\mathbf{I}_{\alpha\beta}(\mathbf{k})\Big]_l\left[\frac{(2\mathbf{p}-\mathbf{k})}{2M}\sigma_{\beta\alpha}(-\mathbf{k}) + \mathbf{I}_{\beta\alpha}(-\mathbf{k})\right]_j$$
$$\times\frac{f_\alpha(\mathbf{p}-\mathbf{k}) - f_\beta(\mathbf{p})}{\varepsilon_\alpha(\mathbf{p}) - \varepsilon_\beta(\mathbf{p}-\mathbf{k}) + \omega + i0}.$$

4. THE IDEAL GAS IN THE BEC STATE

As the hydrogen-like atoms obey the Bose-statistics, our system can also exhibit different properties peculiar only to Bose-gases. The most interesting feature of such systems (for our opinion) is a capability to produce the Bose-Einstein condensate (BEC) at extremely low temperatures (see more in that case in Ref. [4]). Moreover, analyzing most of experiments with dilute gases of alkali atoms in BEC state (see e.g. Refs. [5], [6]) it is clear that interaction of such system with an external field plays one of the main roles in the investigated phenomena.

4.1. MACROSCOPIC PARAMETERS

To find the macroscopic parameters of the system in BEC state ($T << T_c$) we shall consider the case of zero temperature that is equivalent to the assumption when we can neglect the over-condensate particles contribution. Therefore, one can state that the bound states distribution functions $f_\alpha(\mathbf{p})$ are proportional to the Dirac delta-function. According to (8) after integration over momentum $\mathbf{p}$ the scalar Green function of the ideal gas in BEC state will take the form:

$$G^{(+)}(\mathbf{k},\omega) = (2\pi)^{-3}\sum_{\alpha,\beta}\sigma_{\alpha\beta}(\mathbf{k})\sigma_{\beta\alpha}(-\mathbf{k})$$
$$\times\left[\frac{n_\alpha}{\Delta\varepsilon_{\alpha\beta} - \varepsilon_k + \omega + i0} - \frac{n_\beta}{\Delta\varepsilon_{\alpha\beta} + \varepsilon_k + \omega + i0}\right], \quad (9)$$

where n_α is the density of condensed atoms in the certain state, $\Delta\varepsilon_{\alpha\beta} = \varepsilon_\alpha - \varepsilon_\beta$ is the energy interval and the quantity $\varepsilon_k = k^2/2M$ (we shall neglect it below). Note, that analogously to Eq. (9) the vector and tensor Green functions (see Eq. (8)) for the system in BEC state can be found.

If the Green functions are known, it is not difficult to find out the macroscopic parameters (see, for example Ref. [1]), characterizing the system response to the perturbation by the external electromagnetic field (e.g. the laser radiation). The expressions for the permittivity and magnetic permeability in terms of the Green functions can be written as follows:

$$\epsilon^{-1}(\mathbf{k},\omega) = 1 + \frac{4\pi}{k^2} G^{(+)}(\mathbf{k},\omega),$$
$$\mu^{-1}(\mathbf{k},\omega) = 1 + \frac{4\pi\omega}{ic^2k^2}(\sigma^l - \sigma^t), \tag{10}$$

where σ^l and σ^t are longitudinal and transversal conductivity coefficients, respectively:

$$\sigma^l(\mathbf{k},\omega) = \frac{i\omega G^{(+)}(\mathbf{k},\omega)}{k^2 + 4\pi G^{(+)}(\mathbf{k},\omega)},$$
$$\sigma^t(\mathbf{k},\omega) = \frac{k^2c^2 - \omega^2}{i\omega} \frac{\mathcal{G}^{(+)}(\mathbf{k},\omega)}{(\omega^2 - k^2c^2) + 4\pi \mathcal{G}^{(+)}(\mathbf{k},\omega)},$$
$$\mathcal{G}^{(+)}(\mathbf{k},\omega) = \frac{1}{2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) G_{ij}^{(+)}(\mathbf{k},\omega),$$

where δ_{ij} is the Kronekker symbol.

4.2. LIGHT DELAY PHENOMENON

As it is known from the experiments (see Ref. [6]), if to set the frequency of the laser (as an external field source) close to some of the atomic levels, the group velocity of such signal can be slow down to the extremely little values. We assume that the developed theory also can describe such kind of peculiarities.

Let us demonstrate it on the system that is close by the energy structure to alkali atom's levels (see Fig. 1).

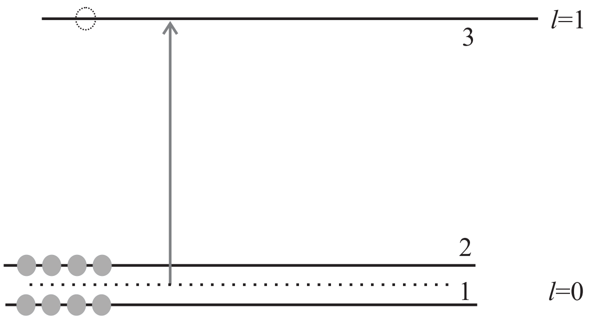

Fig. 1. *Three-level system. Atoms are occupying mostly the first and the second states (hyperfine structure levels); the third state (non-occupied) corresponds to the dipole-exited state. The laser frequency (arrow vertical line) is dephased relatively to the energy intervals* $\Delta\varepsilon_{13}$ *and* $\Delta\varepsilon_{23}$

In that case (when we can neglect the contribution of the other states of atoms), using Eqs. (9)-(10), one can find the following expression for the permittivity:

$$\epsilon^{-1}(\mathbf{k},\omega) = 1 + \frac{2}{(2\pi)^2 k^2} \times \left[\frac{|\sigma_{13}(\mathbf{k})|^2 n_1}{\omega - \Delta\varepsilon_{13} + i\gamma_{13}} + \frac{|\sigma_{23}(\mathbf{k})|^2 n_2}{\omega - \Delta\varepsilon_{23} + i\gamma_{23}} \right], \tag{11}$$

where the quantities γ_{13} and γ_{23} are linewidths related to the probability of the spontaneous transition from the dipole-excited state to the lower and upper states of hyperfine structure levels, respectively. As it is easy to see from Eq. (11), the permittivity $\epsilon(\mathbf{k},\omega)$ in general case is the complex quantity $\epsilon(\mathbf{k},\omega) = \epsilon'(\mathbf{k},\omega) + i\epsilon''(\mathbf{k},\omega)$.

Using well-known relations for the refractive index n and for the dissipation factor ς

$$n = \frac{1}{\sqrt{2}}\sqrt{\sqrt{\epsilon'^2 + \epsilon''^2} + \epsilon'},$$
$$\varsigma = \frac{1}{\sqrt{2}}\sqrt{\sqrt{\epsilon'^2 + \epsilon''^2} - \epsilon'}$$

one can find the refractive index and the intensity of passed light dependencies (see Fig. 2,3)

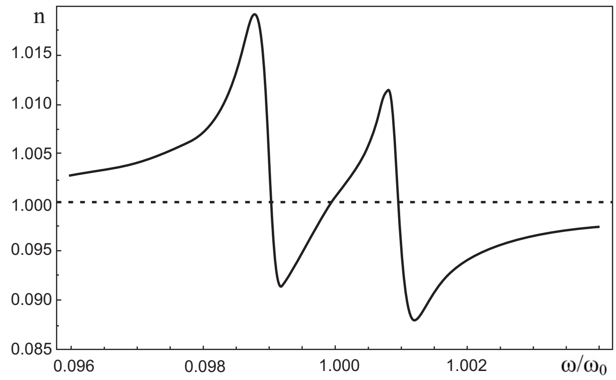

Fig. 2. *Refractive index behavior for the three-level system.* $\omega_0 = \Delta\varepsilon_{13} + \Delta\varepsilon_{12}/2$ *. Left and right steep slopes (* $x = \omega/\omega_0 = 0{,}099; 1{,}001$ *) correspond to the frequencies* $\omega = \Delta\varepsilon_{13}$ *and* $\omega = \Delta\varepsilon_{23}$*, respectively*

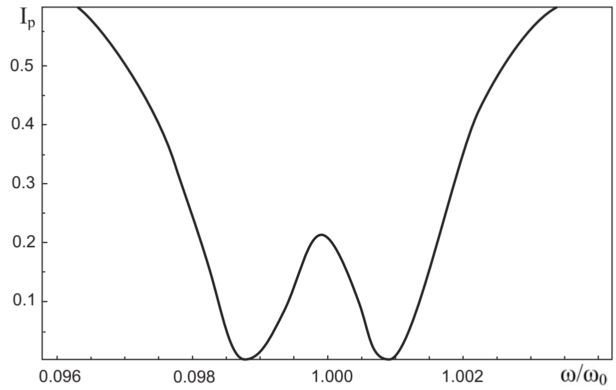

Fig. 3. *Relative intensity of the passed light dependence on the relative frequency. At the frequencies* $\omega = \Delta\varepsilon_{13}$ *and* $\omega = \Delta\varepsilon_{23}$ *the dissipation is large and light doesn't propagate. At frequencies that is dephased relatively to the energy intervals, the transparency window can exist*

In the regions where the dissipation is not large one can use the expression for the group velocity:

$$v_g = \frac{c}{n + \omega(dn/d\omega)}. \tag{12}$$

From the Fig. 2, Fig. 3 and Eq. (12) it is clear that the group velocity depends greatly from the steepness of the central slope. If the levels marked by 1 and 2 (levels of hyperfine structure for alkali atoms) are situated sufficiently close to each other, the light can be slow down to the extremely little values (see Ref. [6]). Note, that such phenomenon cannot be observed for the frequencies $\omega = \Delta\varepsilon_{13}$ and $\omega = \Delta\varepsilon_{23}$ because of large dissipation (see Fig. 3).

Finally, one can conclude that if the frequency of the illuminating laser is dephased relatively to the energy intervals, and ground state levels (marked here by numbers 1 and 2) are situated close to each other, the light delay phenomenon can be observed.

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ОТКЛИК ИДЕАЛЬНОГО ГАЗА ВОДОРОДОПОДОБНЫХ АТОМОВ НА ВОЗМУЩЕНИЕ ВНЕШНИМ ЭЛЕКТРОМАГНИТНЫМ ПОЛЕМ

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Изучен отклик идеального газа, состоящего из водородоподобных атомов, на возмущение внешним электромагнитным полем в низкотемпературной области. Исследования основаны на формализме функций Грина [1] и методе вторичного квантования в присутствии связанных состояний частиц [2]. В качестве наиболее интересного явления изучено возмущение системы, находящейся в состоянии бозе-конденсации. Исследованы дисперсионные характеристики такой системы в области частот, близких к интервалам энергий между уровнями атомов щелочных металлов.

ВІДГУК ІДЕАЛЬНОГО ГАЗУ ВОДНЕВОПОДІБНИХ АТОМІВ НА ЗБУРЕННЯ ЗОВНІШНІМ ЕЛЕКТРОМАГНІТНИМ ПОЛЕМ

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Вивчено відгук ідеального газу, що складається із водневоподібних атомів, на збурення зовнішнім електромагнітним полем у низькотемпературній області. Дослідження базуються на використанні формалізму функцій Гріна [1] та методу вторинного квантування, що враховує можливість утворення зв’язаних станів частинок [2]. Як найбільш цікаве явище вивчено збурення системи, яка знаходиться у стані бозе-конденсації. Досліджені дисперсійні характеристики бозе-конденсату в області частот, близьких до інтервалів енергій між рівнями атомів лужних металів.