DYNAMIC THEORY OF CONDENSED MATTERS WITH INTERNAL STRUCTURE

M.Y. Kovalevsky[1,2], *L.V. Logvinova*[1], *and V.T. Matskevych*[2]

[1]*Belgorod State University, Belgorod, Russia;*
[2] *National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;*
e-mail: matskevych@kipt.kharkov.ua

The results of investigations of condensed matters with internal structures are presented. Additional macroscopic parameters connected with this internal structure in the case of uniaxial and biaxial nematic liquid crystals and biological tissue are introduced as the definite functions of distortion tensor. Thermodynamics of such states is formulated and non-linear equations of hydrodynamics in alternating external field for these condensed matters are derived. Acoustic spectra of collective excitations are found out and their angular dependences are investigated. Low-frequency asymptotics of two-time Green functions are obtained.

PACS: 61.30Cz, 83.10.Bb

1. INTRODUCTION

Nowadays investigation of liquid crystalline matters is of great interest. These condensed states possess the property of liquid – fluidity and anisotropy – property of solid state. The essential feature of liquid crystals is presence of internal anisotropic ordered structure of mesoscopic or nanoscopic sizes, which is shown on macroscopic level as the certain physical phenomena and processes.

The purpose of the given work is the investigation of dynamics and establishment of spectra of collective excitations in uniaxial and biaxial nematic liquid crystals with the structural elements size and shape taken into account in the presence of external alternating fields. The basis of investigation is use of the conception of reduced description of multiparticle states, application and development of Hamiltonian mechanics for condensed matters with structure.

As the application of the approach developed by us the model of blood dynamics which takes into account erythrocytes size and shape on the macroscopic level is constructed. Hemodynamic equations with regard to the dissipation processes are derived. It is clarified that taking into account size and shape of the erythrocytes leads to the appearance of the two new kinetic coefficients of percolation, besides the coefficient of heat conduction and two coefficients of viscosity. Acoustic spectrum of collective excitations is obtained and structure of the damping factor of sound is clarified.

2. UNIAXIAL NEMATIC

2.1. ROD-LIKE MOLECULES

Using Hamiltonian approach of Refs. [1, 2], we obtained dynamic equations of uniaxial nematic with rod-like molecules in an external alternating field. The Hamiltonian of the system consists of Hamiltonian of medium and interaction with an external field

$$H = \mathrm{H} + V, \qquad \mathrm{H} = \int d^3x \varepsilon\left(\zeta_a(x), n_i(x), \nabla n_i(x), l(x)\right),$$
$$V = \int d^3x \xi(x,t) B\left(\zeta_a(x), n_i(x), \nabla n_i(x), l(x)\right).$$

Here $\zeta_a \equiv \sigma, \pi_i, \rho$ are the densities of mass ρ, momentum π_i, entropy σ and energy ε; n_i is unit vector of spatial anisotropy, l is length of a molecule, $\xi(x,t)$ is external field, $B(\zeta_a, n_i, \nabla n_i, l)$ is arbitrary local physical quantity, which at large enough times becomes function of reduced description parameters. For dynamic quantities equations are obtained

$$\begin{aligned}
&\dot\rho = -\nabla_i \pi_i + \eta_\rho, \ \dot\pi_i = -\nabla_k t_{ik} + \eta_{\pi_i}, \\
&\dot\sigma = -\nabla_k(\sigma v_k) + \eta_\sigma, \\
&\dot l = -v_s \nabla_s l - l\delta^{\perp}_{ij}(\vec n)\nabla_j v_i + \eta_l, \\
&\dot n_j = -v_s \nabla_s n_j + \delta^{\perp}_{ij}(\vec n) n_k \nabla_k v_i + \eta_{n_j},
\end{aligned} \tag{1}$$

where $v_i \equiv \pi_k / \rho$ is macroscopic velocity, $\delta^{\perp}_{ik}(\vec n) = \delta_{ik} - n_i n_k$ and sources caused by an external field have the following form

$$\begin{aligned}
&\eta_\rho = \rho \frac{\partial B}{\partial \pi_i} \nabla_i \xi, \ \eta_{n_j} = -n_k \delta^{\perp}_{ij}(\vec n) \frac{\partial B}{\partial \pi_i} \nabla_k \xi, \\
&\eta_l = l\delta^{\perp}_{ji}(\vec n) \frac{\partial B}{\partial \pi_i} \nabla_j \xi, \quad \eta_\sigma = \sigma \frac{\partial B}{\partial \pi_i} \nabla_i \xi, \\
&\eta_{\pi_j} = \zeta_a \frac{\partial B}{\partial \zeta_a} \nabla_j \xi + l\delta^{\perp}_{jk}(\vec n) \frac{\partial B}{\partial l} \nabla_k \xi \\
&+ \left(\frac{\partial B}{\partial n_i} - \nabla_\lambda \frac{\partial B}{\partial \nabla_\lambda n_i} \right) \delta^{\perp}_{ji}(\vec n) n_k \nabla_k \xi .
\end{aligned} \tag{2}$$

The expression for the flux density of momentum looks like

$$\begin{aligned}
t_{ik} &= P\delta_{ik} + \frac{\partial \varepsilon}{\partial \pi_k} \pi_i + \frac{\partial \varepsilon}{\partial \nabla_k n_\lambda} \nabla_i n_\lambda \\
&+ \frac{\partial \varepsilon}{\partial l} l \delta^{\perp}_{ik}(\vec n) - n_k \delta^{\perp}_{i\lambda}(\vec n) \left(\frac{\partial \varepsilon}{\partial n_\lambda} - \nabla_j \frac{\partial \varepsilon}{\partial \nabla_j n_\lambda} \right),
\end{aligned}$$

where $P \equiv -\varepsilon + \pi_l \frac{\partial \varepsilon}{\partial \pi_l} + n \frac{\partial \varepsilon}{\partial n} + \sigma \frac{\partial \varepsilon}{\partial \sigma}$ is pressure.

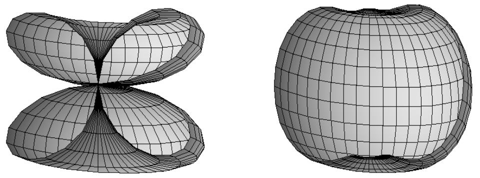
 PROBLEMS OF ATOMIC SCIENCE AND TECHNOLOGY. 2007, N3 (2), p. 380-384.

We investigate spectra of collective excitations on the basis of Eqs (1). Their linearization leas to the two linear acoustic spectra

$$\omega_\pm^2(\vec k)=c_\pm^2\left(\frac{\vec k}{k}\right)k^2,\quad c_\pm(\theta)=\frac{c}{\sqrt2}$$
$$\times\sqrt{1+\lambda\sin^2\theta\pm\left[(1+\lambda\sin^2\theta)^2-\lambda\sin^2 2\theta\right]},$$

where $\lambda\equiv \underline{B}/c^2>0$, $\underline{B}=\frac{Tl^2}{\rho}\frac{\partial^2\varepsilon}{\partial l^2}>0$,

$c=(\partial P/\partial\rho)^{1/2}$ is the velocity of sound in isotropic phase; θ polar angle specifying direction of a wave vector $\vec k$ in relation to an anisotropy axis and $T\equiv\partial\varepsilon/\partial\sigma$ temperature.



a *b*

Fig. 1. *Angular dependence of the velocity c_+ at $\lambda=0.1$ (a) and c_- at $\lambda=0.1$ (b)*

From the system of Eqs. (1), (2), using the approach [3], the general expression of the low-frequency asymptotics of Green functions is obtained

$$G_{AB}(\vec k,\omega)=L_i^A(\vec k,\omega)D_{ij}^{-1}(\vec k,\omega)L_j^B(-\vec k,-\omega),\qquad(3)$$

where the following designations are introduced

$$L_i^A(\vec k,\omega)=\underline{\zeta}_a\frac{\partial A}{\partial\underline{\zeta}_a}k_i+l\frac{\partial A}{\partial l}\delta_{ji}^{\perp}(\vec n)k_j+$$
$$+\omega\rho\frac{\partial A}{\partial\pi_i}+\left(\frac{\partial A}{\partial n_j}-ik_\lambda\frac{\partial A}{\partial\nabla_\lambda n_j}\right)\delta_{ij}^{\perp}(\vec n)n_kk_k,$$
$$D_{ij}^{-1}=\frac{1}{2\Delta}\varepsilon_{iab}\varepsilon_{ja'b'}D_{aa'}D_{bb'},$$
$$D_{ij}=\omega^2\delta_{ij}-c^2k_ik_j-c^2\lambda\delta_{il}^{\perp}(\vec n)k_l\delta_{jm}^{\perp}(\vec n)k_m,$$
$$\Delta=\omega^2\left(\omega^2-c_+^2k^2\right)\left(\omega^2-c_-^2k^2\right).\qquad(4)$$

Given low-frequency asymptotics do not contain peculiarities like $1/k^2, 1/k$. The reason is connected with the fact that as against cases considered before (see Refs. [3, 4]), set of reduced description parameters does not include density of generator of broken symmetry. In given case this quantity is orbital moment, which is conjugated in relation to anisotropy axis. In given works set of hydrodynamic parameters contained both density of generator of broken symmetry (mass or spin density), and corresponding conjugated quantity, as resulted at the end to above-stated peculiarities.

2.2. DISC-LIKE MOLECULES

The dynamic behavior of condensed matter with such form of molecules is carried out similarly with the earlier considered case. The dynamic equations of uniaxial nematic with disk-like molecules in an external alternative field look like

$$\dot\rho=-\nabla_i\pi_i+\eta_\rho,\ \dot\pi_i=-\nabla_kt_{ik}+\eta_{\pi_i},$$
$$\dot\sigma=-\nabla_k(\sigma v_k)+\eta_\sigma,$$
$$\dot d=-v_s\nabla_sd-dn_kn_l\nabla_kv_l+\eta_d,$$
$$\dot n_j=-v_s\nabla_sn_j-n_i\delta_{j\lambda}^{\perp}(\vec n)\nabla_\lambda v_i+\eta_{n_j},\qquad(5)$$

where d is the disc diameter, and the sources look as follows

$$\eta_\rho=\rho\frac{\partial B}{\partial\pi_i}\nabla_i\xi,\ \eta_{n_j}=-n_k\delta_{ij}^{\perp}(\vec n)\frac{\partial B}{\partial\pi_i}\nabla_k\xi,$$
$$\eta_d=dn_in_j\frac{\partial B}{\partial\pi_i}\nabla_j\xi,\quad \eta_\sigma=\sigma\frac{\partial B}{\partial\pi_i}\nabla_i\xi,$$
$$\eta_{\pi_j}=\underline{\zeta}_a\frac{\partial B}{\partial\underline{\zeta}_a}\nabla_j\xi+dn_in_j\frac{\partial B}{\partial d}\nabla_i\xi$$
$$+\left(\frac{\partial B}{\partial n_i}-\nabla_\lambda\frac{\partial B}{\partial\nabla_\lambda n_i}\right)\delta_{ji}^{\perp}(\vec n)n_k\nabla_k\xi.\qquad(6)$$

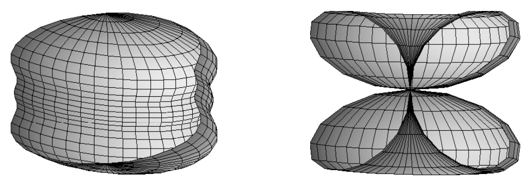
The density of momentum flow looks like

$$t_{ik}=P\delta_{ik}+\frac{\partial\varepsilon}{\partial\pi_k}\pi_i+\frac{\partial\varepsilon}{\partial\nabla_kn_\lambda}\nabla_in_\lambda$$
$$+\frac{\partial\varepsilon}{\partial d}dn_in_k+n_i\delta_{k\lambda}^{\perp}(\vec n)\left(\frac{\partial\varepsilon}{\partial n_\lambda}-\nabla_j\frac{\partial\varepsilon}{\partial\nabla_jn_\lambda}\right).$$

Linearization of Eqs. (5) in this case also leads to the two anisotropic velocities of acoustic waves

$$c_\pm(\theta)=\frac{c}{\sqrt2}$$
$$\times\sqrt{1+\lambda\cos^2\theta\pm\left[(1+\lambda\cos^2\theta)^2-\lambda\sin^2 2\theta\right]},$$

где $\lambda\equiv\underline{C}/c^2>0$, $\underline{C}=\frac{Td^2}{\rho}\frac{\partial^2\varepsilon}{\partial d^2}>0$.



a *b*

Fig. 2. *Angular dependence of the velocity c_+ at $\lambda=3$ **(a)** and c_- at $\lambda=1$ **(b)***

With the help of Eqs. (5), (6) we obtain the general expression of the low-frequency asymptotics of Green functions

$$G_{AB}(\vec k,\omega)=L_i^A(\vec k,\omega)D_{ij}^{-1}(\vec k,\omega)L_j^B(-\vec k,-\omega),\qquad(7)$$

where the following designations are introduced

$$L_i^A(\vec k,\omega)=\left(\underline{\zeta}_a\frac{\partial A}{\partial\underline{\zeta}_a}\right)k_i+d\frac{\partial A}{\partial d}n_in_jk_j$$

$$+\omega\rho\frac{\partial A}{\partial \pi_i}+\left(\frac{\partial A}{\partial n_j}-ik_\lambda\frac{\partial A}{\partial \nabla_\lambda n_j}\right)\delta_{ij}^{\perp}(\vec{n})n_k k_k\,,$$

$$D_{ij}=\omega^2\delta_{ij}-c^2k_ik_j-c^2\lambda(\vec{k}\vec{n})^2n_in_j\,. \qquad (8)$$

Comparing the formulas (4) and (8), we see, that the low-frequency asymptotics of Green functions are different in the case of uniaxial nematics with the rod-like and disc-like molecules. Also it is easy to see, that in the given case Bogolyubov theorem about singularity like $1/k^2$ is violated, because orbital moment does not enter the set of reduced description parameters.

3. BIAXIAL NEMATIC

3.1. ROD-LIKE MOLECULES

In the case of biaxial nematics with the ellipsoid-like molecules set of thermodynamic variables contains two unit and orthogonal vectors of anisotropy and three conformational parameters describing size and shape of a molecule. Acting further similarly to previously considered case of uniaxial molecules, it is easy to obtain equations of ideal hydrodynamics of biaxial nematics with rod-like molecules

$$\dot{\rho}=-\nabla_i\pi_i\,,\quad \dot{\pi}_i=-\nabla_k t_{ik}\,,\qquad \dot{\sigma}=-\nabla_k(\sigma v_k)\,,$$
$$\dot{n}_j=-v_s\nabla_s n_j-F_{i\lambda j}\nabla_\lambda v_i\,,$$
$$\dot{m}_j=-v_s\nabla_s m_j-G_{i\lambda j}\nabla_\lambda v_i\,,$$
$$\dot{u}=-v_s\nabla_s u-F_{ij}\nabla_j v_i\,,$$
$$\dot{v}=-v_s\nabla_s v-G_{ij}\nabla_j v_i\,,$$
$$\dot{p}=-v_s\nabla_s p-H_{kl}\nabla_k v_l\,. \qquad (9)$$

Here n_i, m_i are unit orthogonal vectors of spatial anisotropy, conformational parameters p, u, v determine the sizes of long and short axes of molecule and an angle between them. The expression for the flux density of momentum looks like

$$t_{ik}=P\delta_{ik}+\frac{\partial\varepsilon}{\partial\pi_k}\pi_i+\frac{\partial\varepsilon}{\partial\nabla_k n_\lambda}\nabla_i n_\lambda+\frac{\partial\varepsilon}{\partial\nabla_k m_\lambda}\nabla_i m_\lambda$$
$$+\frac{\partial\varepsilon}{\partial u}F_{ik}+\frac{\partial\varepsilon}{\partial v}G_{ik}+\frac{\partial\varepsilon}{\partial p}H_{ik}+F_{ik\lambda}\left(\frac{\partial\varepsilon}{\partial n_\lambda}-\nabla_j\frac{\partial\varepsilon}{\partial\nabla_j n_\lambda}\right)$$
$$+G_{ik\lambda}\left(\frac{\partial\varepsilon}{\partial m_\lambda}-\nabla_j\frac{\partial\varepsilon}{\partial\nabla_j m_\lambda}\right),$$

where the designations are entered

$$F_{i\lambda j}=-n_\lambda\delta_{ij}^{\perp}(\vec{n})+pm_j\left[n_im_\lambda+n_\lambda m_i\right],$$
$$G_{i\lambda j}=-m_\lambda\delta_{ij}^{\perp}(\vec{m})+(1-p)n_j\left[n_im_\lambda+n_\lambda m_i\right],$$
$$F_{ik}=u\left(\delta_{ik}^{\perp}(\vec{n})+\sqrt{p(1-p)}(n_im_k+n_km_i)\right),$$
$$G_{ik}=v\left(\delta_{ik}^{\perp}(\vec{m})-\sqrt{p(1-p)}(n_im_k+n_km_i)\right),$$
$$H_{ik}=2p(1-p)(n_in_k-m_im_k)\,.$$

Linearization of the Eqs. (9) leads to the dispersion equation

$$\omega^6I_6(k,\theta,\varphi)+\omega^4I_4(k,\theta,\varphi)$$
$$+\omega^2I_2(k,\theta,\varphi)+I_0(k,\theta,\varphi)=0\,. \qquad (10)$$

Here coefficients I_a in terms of azimuthal and polar angles are as follows

$$I_6=1\,,\; I_4=-k^2c^2\left(1+\lambda_i\Phi_i(\theta,\varphi)\right),$$
$$I_2=k^4c^4\lambda_i\Psi_i(\theta,\varphi),$$
$$I_0=-k^6c^6\sum_{i>j}\lambda_i\lambda_j\Psi_{ij}(\theta,\varphi),$$
$$\Phi_1(\theta,\varphi)=1-\sin^2\theta\sin^2\varphi+\frac{1}{4}\sin^2\theta+\frac{1}{2}\sin^2\theta\sin2\varphi,$$
$$\Phi_2(\theta,\varphi)=1-\sin^2\theta\cos^2\varphi+\frac{1}{4}\sin^2\theta-\frac{1}{2}\sin^2\theta\sin2\varphi,$$
$$\Phi_3(\theta,\varphi)=\sin^2\theta\,,$$
$$\Psi_1(\theta,\varphi)=\frac{1}{4}+\sin^2\varphi\cos^2\theta+\frac{1}{2}\sin2\varphi\,,$$
$$\Psi_2(\theta,\varphi)=\frac{1}{4}+\cos^2\varphi\cos^2\theta-\frac{1}{2}\sin2\varphi\,,$$
$$\Psi_3(\theta,\varphi)=\sin^2\theta\left(1-\sin^2\theta\cos^2 2\varphi\right),$$
$$\Psi_{21}(\theta,\varphi)=\frac{1}{2}\sin^4\theta\cos^2\theta\sin^2\left(\frac{\pi}{4}-2\varphi\right),$$
$$\Psi_{31}(\theta,\varphi)=\sin^4\theta\cos^2\theta\sin^4\left(\frac{\pi}{4}-\varphi\right),$$
$$\Psi_{32}(\theta,\varphi)=\sin^4\theta\cos^2\theta\cos^4\left(\frac{\pi}{4}-\varphi\right), \qquad (11)$$

where three dimensionless parameters are entered

$$\lambda_1\equiv\frac{u^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial u^2}\,,\quad \lambda_2\equiv\frac{v^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial v^2}\,,$$
$$\lambda_3\equiv\frac{p^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial p^2}\,.$$

Polar and azimuthal angles θ,φ specifying a direction of the wave vector $\vec{e}=\vec{k}/k$ in relation to anisotropy axes are entered by ratio

$$\vec{e}\vec{m}=\sin\theta\cos\varphi\,,\,\vec{e}\vec{n}=\sin\theta\sin\varphi\,,\;\vec{e}\vec{l}=\cos\theta\,.$$

The vectors $\vec{m},\vec{n},\vec{l}$ form such a rectangular Cartesian coordinate system, that two anisotropy axes $\vec{m},\vec{n}$ coincide with an axes direction in considered nematic liquid crystals in nondeformed state. Eq. (10) can be transformed to as follows

$$y^3+wy+z=0\,,$$

where

$$w=I_2-\frac{1}{3}I_4^2\,,\qquad z=\frac{2}{27}I_4^3-\frac{1}{3}I_2I_4+I_0\,.$$

Three real solutions corresponding to three values of velocity of oscillations

$$y_n=2\sqrt[3]{s}\cos(\varphi+2(n-1)\pi)/3\,,\; n=1,2,3$$

exist when the inequality for discriminant D is valid

$$D\equiv(w/3)^3+(z/2)^2\le0\,,$$

where the designations are entered

$$\cos\varphi=-z/2s\,,\qquad s=\sqrt{-w^3/27}\,.$$

The solutions of the initial Eq. (10) accordingly will take accordingly a form

$$\omega_n^2(k,\theta,\varphi) = y_n - I_4/3 \equiv c_n^2(\theta,\varphi)k^2 .$$

Here $c_n(\theta,\varphi)$ are three velocities of propagation of collective excitations in the given condensed matter.

When $D>0$, then the cubic equation has one real solution and two complexly conjugated. In this case nematic is characterized by one branch of collective excitations, which looks like

$$y_1(k,\theta,\varphi) = \sqrt[3]{-\frac{z}{2}+\sqrt{D}} + \sqrt[3]{-\frac{z}{2}-\sqrt{D}}$$
$$\equiv c_1^2(\theta,\varphi)k^2 + I_4/3.$$

In the area of changing of thermodynamic parameters, where inequalities $\lambda_3 >> \lambda_1, \lambda_2$ are valid, the formulas for velocities of acoustic waves become simpler and get a kind

$$c_1^2 = c^2(1+\lambda_3\Phi_3(\theta,\varphi)),$$
$$c_2^2 = c^2\lambda_3\Psi_3(\theta,\varphi), \qquad c_3 \approx 0 .$$

Thus, in nematic with rod-like molecules with three conformational degrees of freedom taken into account the propagation of one up to three branches of acoustic spectra is possible.

3.2. DISC-LIKE MOLECULES

The equations of ideal hydrodynamics of biaxial nematic with disc-like molecules and with three conformational degrees of freedom look like

$$\dot\rho = -\nabla_i\pi_i , \quad \dot\pi_i = -\nabla_k t_{ik} , \quad \dot\sigma = -\nabla_k(\sigma v_k),$$
$$\dot n_j = -v_s\nabla_s n_j - f_{i\lambda j}\nabla_\lambda v_i ,$$
$$\dot m_j = -v_s\nabla_s m_j - g_{i\lambda j}\nabla_\lambda v_i ,$$
$$\dot q = -v_s\nabla_s u - f_{ij}\nabla_j v_i ,$$
$$\dot t = -v_s\nabla_s v - g_{ij}\nabla_j v_i ,$$
$$\dot p = -v_s\nabla_s p - h_{kl}\nabla_k v_l , \tag{12}$$

where the quantities q, t determine change of length of both axes of anisotropy. The expression for the density of momentum flow looks like

$$t_{ik} = P\delta_{ik} + \frac{\partial\varepsilon}{\partial\pi_k}\pi_i + \frac{\partial\varepsilon}{\partial\nabla_k n_\lambda}\nabla_i n_\lambda + \frac{\partial\varepsilon}{\partial\nabla_k m_\lambda}\nabla_i m_\lambda$$
$$+\frac{\partial\varepsilon}{\partial q}f_{ik} + \frac{\partial\varepsilon}{\partial t}g_{ik} + \frac{\partial\varepsilon}{\partial p}h_{ik} + f_{ik\lambda}\left(\frac{\partial\varepsilon}{\partial n_\lambda} - \nabla_j\frac{\partial\varepsilon}{\partial\nabla_j n_\lambda}\right)$$
$$+ g_{ik\lambda}\left(\frac{\partial\varepsilon}{\partial m_\lambda} - \nabla_j\frac{\partial\varepsilon}{\partial\nabla_j m_\lambda}\right),$$

where the designations are entered

$$f_{i\lambda k} \equiv n_i\delta^{\perp}_{j\lambda}(\vec n) - pm_j(n_i m_\lambda + n_\lambda m_i),$$
$$g_{i\lambda k} \equiv m_i\delta^{\perp}_{j\lambda}(\vec m) - (1-p)n_j(n_i m_\lambda + n_\lambda m_i),$$
$$f_{ik} = q\left(n_i n_k - \sqrt{p(1-p)}(n_i m_k + n_k m_i)\right),$$
$$g_{ik} = t\left(m_i m_k + \sqrt{p(1-p)}(n_i m_k + n_k m_i)\right),$$
$$h_{ik} = 2p(1-p)(m_i m_k - n_i n_k).$$

Linearization of Eqs. (12) leads to dispersion equation like Eq. (10), where the coefficients I_6, I_2, I_0 are the same as well, as in formula (11), and I_4 looks as follows

$$I_4(k,\theta,\varphi) = -k^2c^2(1+\lambda_i\Phi_i(\theta,\varphi)),$$
$$\Phi_1(\theta,\varphi) = \left(\frac{1}{4} + \sin^2\varphi - \sin\varphi\cos\varphi\right)\sin^2\theta,$$
$$\Phi_2(\theta,\varphi) = \left(\frac{1}{4} + \cos^2\varphi + \sin\varphi\cos\varphi\right)\sin^2\theta,$$
$$\Phi_3(\theta,\varphi) = \sin^2\theta = \Phi_3(\theta,\varphi). \tag{13}$$

Here three dimensionless parameters are entered

$$\lambda_1 \equiv \frac{q^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial q^2}, \lambda_2 \equiv \frac{t^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial t^2}, \tag{14}$$
$$\lambda_3 \equiv \frac{p^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial p^2}.$$

The further analysis is performed similarly the previously considered case. It is shown, that also in given case the condensed matter is characterized by one up to three spectra of collective excitations. By virtue of distinction of coefficients I_4 for nematics with disc-like and rod-like molecules angular dependences of these spectra for these cases are different.

4. HEMODYNAMICS

In this section we will use the results of developed dynamic theory to describe movement of blood taking into account erythrocytes size and shape on the macroscopic level. It can be shown, that the equation of dissipative hemodynamics are as follows

$$\dot\rho = -\nabla_k\pi_k , \tag{15}$$
$$\dot\pi_i = -\nabla_k\left(t^0_{ik} + t^1_{ik}\right)$$
$$t^0_{ik} = P\delta_{ik} + \frac{\pi_i\pi_k}{\rho} + \frac{1}{3}bT\frac{\partial\varepsilon}{\partial b}\delta_{ik},$$
$$t^1_{ik} = -\eta_1\left(\nabla_k v_i + \nabla_i v_k - \frac{2}{3}\delta_{ik}\nabla_l v_l\right) - \eta_2\delta_{ik}\nabla_l v_l ,$$
$$\dot\sigma = -\nabla_k\left(j^0_{\sigma k} + j^1_{\sigma k}\right) + I , \qquad j^0_{\sigma k} = \sigma v_k ,$$
$$j^1_{\sigma k} = -\frac{1}{T}\kappa\nabla_k T - \xi_1(\nabla_k T + \Gamma\nabla_k b),$$
$$I = \frac{\kappa}{T^2}(\nabla_l T)^2 + \frac{1}{2T}\eta_1\left(\nabla_k v_i + \nabla_i v_k - \frac{2}{3}\delta_{ik}\nabla_l v_l\right)^2$$
$$+\frac{1}{T}\eta_2(\nabla_l v_l)^2$$
$$+\frac{1}{T}\xi_1(\nabla_k T + \Gamma\nabla_k b)^2 + \frac{1}{T}\xi_2\Gamma^2(\nabla_k b)^2 > 0,$$
$$\dot b = -v_k\nabla_k b - \frac{1}{3}b\nabla_k v_k + \xi_1\nabla_k T\nabla_k b + \frac{1}{3}\xi_1 b\Delta T$$
$$+(\xi_1+\xi_2)\Gamma(\nabla_k b)^2$$
$$+\frac{1}{3}(\xi_1+\xi_2)b\left(\Gamma\Delta b + \Gamma_1(\nabla_k b)^2\right).$$

Here b – erythrocyte diameter; κ – coefficient of heat conduction; η_1 и η_2 – coefficients of the first and second viscosity; ξ_1 и ξ_2 – kinetic coefficients of percolation. For the quantities Γ and Γ_1 the following designations are entered

$$\Gamma \equiv \frac{1}{3} Tb \frac{\partial^2 \varepsilon}{\partial b^2}, \quad \Gamma_1 \equiv \frac{1}{3} Tb \frac{\partial^2 \varepsilon}{\partial b^2} + \frac{1}{3} Tb \frac{\partial^3 \varepsilon}{\partial b^3}.$$

Linearization of Eqs. (15) allows us to obtain the expressions for modes propagating in given condensed matter

$$\omega_{1,2} = 0, \qquad \omega_{3,4} = -i \frac{1}{\rho} \eta_1 k^2,$$

$$\omega_{5,6} = ck + i\gamma k^2,$$

where the damping factor γ is as follows

$$\gamma = \frac{1}{2\rho c^2 - b\Gamma / T} \times \left[\left(\frac{2b\Gamma}{3T} \frac{\partial T}{\partial S} - \rho \frac{\partial P}{\partial S} \frac{\partial T}{\partial \rho} \right) \left(\frac{\kappa}{T} + \xi_1 \right) - \frac{\rho c^2}{3} \left(\eta_2 + \frac{4}{3} \eta_1 \right) \right].$$

Here k is modulus of wave vector and

$$c^2 = \frac{\partial P}{\partial \rho} + \frac{4b^2}{9\rho} T \frac{\partial^2 \varepsilon}{\partial b^2},$$

is speed of sound in a blood. We can see that taking into account size and shape of erythrocytes leads to sound velocity increase. This fact is in accordance with the experimental data obtained before [5].

Note, that in the limit of isotropic liquid ($b = 0, \xi_1 = 0, \xi_2 = 0$), we will obtain known results valid for unstructured liquid [6].

This work is performed at partial support of the grant RFFI № 05-02-16663.

REFERENCES

1. A.P. Ivashin, M.Y. Kovalevsky, L.V. Logvinova //*Int. J. Quantum Chemistry*. 2004, v. 96, p. 636-644.
2. A.P. Ivashin, M.Y. Kovalevsky, L.V. Logvinova //*Theor. Math. Phys*. 2004, v. 140, N 3, p. 1316-1327.
3. N.N. Bogolyubov (jr.), M.Y. Kovalevsky, A.M. Kurbatov, S.V. Peletminsky, A.N. Tarasov //*Uspekhi Fiz. Nauk*. 1989, v. 159, N 4, p. 585-620 (in Russian).
4. M.Y. Kovalevsky, A.A. Rozhkov //*Fiz. Niz. Temp.* 1994, v. 20, N 4, p. 375-383 (in Russian).
5. V.A. Levtov, S.A. Regirer, N.C. Shadrina. *The blood reology*. M.: “Medizina”, 1982, 272 p. (in Russian).
6. A.I. Akhiezer, S.V. Peletminsky. *The methods of statistical physics.* M.: “Nauka”, 1977, 368 p. (in Russian).

ДИНАМИЧЕСКАЯ ТЕОРИЯ КОНДЕНСИРОВАННЫХ СРЕД С ВНУТРЕННЕЙ СТРУКТУРОЙ

М.Ю. Ковалевский, Л.В. Логвинова, В.Т. Мацкевич

Представлены результаты исследований динамики конденсированных сред с внутренней структурой. Дополнительные макроскопические параметры, связанные со структурой, в случае одноосных и двухосных нематических жидких кристаллов и биологических сред введены как определенные функции тензора дисторсии. Сформулирована термодинамика таких состояний и получены нелинейные уравнения гидродинамики во внешнем переменном поле. Найдены акустические спектры коллективных возбуждений и исследованы их угловые зависимости. Получены низкочастотные асимптотики двухвременных функций Грина.

ДИНАМІЧНА ТЕОРІЯ КОНДЕНСОВАНИХ СЕРЕДОВИЩ З ВНУТРІШНЬОЮ СТРУКТУРОЮ

М.Ю. Ковалевський, Л.В. Логвінова, В.Т. Мацкевич

Представлено результати досліджень конденсованих середовищ з внутрішньою структурою. Додаткові макроскопічні параметри, пов’язані з внутрішньою структурою, у випадку одновісних та двовісних нематичних рідких кристалів і біологічних середовищ уведені як визначені функції тензора дисторсії. Сформульована термодинаміка таких станів та отримані нелінійні рівняння гідродинаміки у зовнішньому змінному полі. Знайдені акустичні спектри колективних збуджень та досліджені їх кутові залежності. Отримані низькочастотні асимптотики двочасових функцій Гріна.