REDUCED DESCRIPTION METHOD IN DYNAMIC THEORY OF PARTICLES INTERACTING WITH MEDIUM

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We consider spatially inhomogeneous states of particles, weakly interacting with hydrodynamic medium involving Bogolubov's reduced description method. It has been shown that such a system has both kinetic and hydrodynamic stages of evolution. The coupled system of equations of motion for this evolution stage is obtained. The transition from kinetic to hydrodynamic stage of evolution for particles interacting with medium has been also studied. Consequently we obtained a system of equations, which completely describes the evolution of the system on hydrodynamic stage. These equations can describe such systems as neutrons propagating in hydrodynamic medium without multiplication and capture.

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1. INTRODUCTION

The reduced description method is the most consistent microscopic approach in modern kinetics. Basic concepts of this method, used for description of classical systems are stated in the book [1] by N.N. Bogolubov. The extension of this method to quantum many-particle systems is given in the book [2].

The application of reduced description method for a one-component system gives us a microscopic approach, which allows us to obtain kinetic equations (when the system is described by one-particle distribution function) and hydrodynamic equations (in case when the system is described with a set of hydrodynamic parameters such as temperature, density and velocity).

However some systems may include different subsystems on different stages of evolution. For example, in a two-component system one component can be at the kinetic evolution stage, which means that it is described by a one-particle distribution function, while the other component evolutes hydrodynamicaly. Such a situation occurs when the system consists of strongly interacting particles of one type (hydrodynamic medium) and particles of the other type, which weakly interact with the medium, but do not interact with each other owing to their small number. One of specific examples of such systems are slow neutrons in a hydrodynamic medium.

The microscopic theory, describing spatially homogeneous evolution of particles, in hydrodynamic medium by means of the reduced description method has been developed in [2]. In our work we consider spatially inhomogeneous states of particles weakly interacting with a hydrodynamic medium by using the reduced description method.

2. REDUCED DESCRIPTION METHOD CONCEPTS

At arbitrary time t, our system can be described with a statistical operator $\rho(t)$, which evolves according to Liouville equation

$$\frac{\partial\rho(t)}{\partial t} = -i[\mathcal{H}, \rho(t)], \tag{1}$$

where $\mathcal{H}$ is the system's Hamiltonian. For the closed systems, the solution of this equation can be written as

$$\rho(t) = e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}t}, \tag{2}$$

where ρ_0 is the statistical operator of initial state.

Operator $\rho(t)$ satisfies two fundamental principles according to the reduced description method's concepts: The principle of weakening of spatial correlations represents a simplification of traces of statistical operator $\rho(t)$ and the products of quasi-local operators $a(\mathbf{x})$ and $b(\mathbf{y})$ when their arguments are separated

$$Sp\rho(t)a(\mathbf{x})b(\mathbf{y}) \underset{|\mathbf{x}-\mathbf{y}|\gg r_c}{\rightarrow} Sp\rho(t)a(\mathbf{x})\cdot Sp\rho(t)b(\mathbf{y}). \tag{3}$$

Here r_c is the correlation radius of state $\rho(t)$.

The ergodic relation describes asymptotic form of the statistical operator (and, certainly, traces with this operator) at large time:

$$\rho(t) = e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}t} \underset{t\to\infty}{\rightarrow} w. \tag{4}$$

Here w is the equilibrium Gibbs operator. Actually, the relation (4) represents the fact that the system transforms in a state of statistical equilibrium at large time scales, described with Gibbs statistical operator w.

According to Bogolubov's concept of relaxation time hierarchy, as system evolves towards equilibrium, the set of description parameters becomes more and more simple. Moreover, as the system approaches the equilibrium, the number of parameters, required for description of the system, decreases, and the system's description simplifies. A set of parameters, which describes the system on one evolution stage, is called the set of reduced description parameters of the system.

3. HAMILTONIAN AND DESCRIPTION PARAMETERS

The system considered in our work consists of two sybsystems – medium and particles interacting with the medium. These system's Hamiltonian $\mathcal{H}$ can be written as

$$\mathcal{H} = \mathcal{H}_0 + \hat{V}, \quad \mathcal{H}_0 = \mathcal{H}_m + \mathcal{H}_p. \tag{5}$$

Here $\mathcal{H}_0$ describes the non-interacting subsystems, and operator $\hat{V}$ describes the interaction of subsystems. We suppose the particle density to be small, so we can neglect the interaction among them. Therefore, the operator $\mathcal{H}_p$, which describes the particle subsystem by its own, has a structure of a free particle Hamiltonian. Though, we assume that Hamiltonian $\mathcal{H}_m$, which describes the medium subsystem, contains a strong interaction of medium particles with each other. This leads to a fast relaxation of this subsystem to local equilibrium state. Also the interaction of subsystems is weaker than that of medium particles with each other. So the relaxation time for medium subsystem is substantially lower than that for the both subsystems τ_0, determined by the inter-subsystem interaction. So we see, that the relaxation time of the whole system is determined by the weaker interaction $\hat{V}$. According to the basic concepts of the reduced description method, we accept that at times $t \gg \tau_0$ the additive motion integral's densities can be taken as reduced description parameters. This means that the statistical operator $\rho(t)$, which describes the systems on evolution stage when time is greater than specific relaxation time τ_0, has functional dependence on the additive integrals densities $\zeta_A(\mathbf{x})$.

Hydrodynamic medium has five additive motion integrals relatively to Hamiltonian $\mathcal{H}_0$: these are mass, energy and momentum (for details, see [2]). So the hydrodynamic medium will be described in terms of five variables $\zeta_\alpha(\mathbf{x})$, $\alpha = 0, i, 4$, where $\zeta_0(\mathbf{x}) \equiv \varepsilon(\mathbf{x})$ is the medium energy density, $\zeta_i(\mathbf{x}) \equiv \pi_i(\mathbf{x})$ is the medium momentum density, $\zeta_4(\mathbf{x}) \equiv \rho^{(m)}(\mathbf{x})$ is the medium mass density. For each variable an operator $\hat{\zeta}_\alpha(\mathbf{x})$, $\alpha = 1, i, 4$ can be introduced. These operators can be expressed in terms of creation $\varphi^+(\mathbf{x})$ and annihilation $\varphi(\mathbf{x})$ operators of medium particles:

$$\begin{aligned}\hat{\zeta}_0(\mathbf{x}) \equiv \hat{\varepsilon}(\mathbf{x}) &= \frac{1}{2m_m}\nabla\varphi^+(\mathbf{x})\nabla\varphi(\mathbf{x}) \\ &+ \frac{1}{2}\int d^3R V_m(\mathbf{R})\varphi^+(\mathbf{x}+\mathbf{R})\varphi^+(\mathbf{x})\varphi(\mathbf{x})\varphi(\mathbf{x}+\mathbf{R}),\end{aligned} \tag{6}$$

$$\hat{\zeta}_i(\mathbf{x}) \equiv \hat{\pi}_i(\mathbf{x}) = \frac{i}{2}\left(\frac{\partial\varphi^+(\mathbf{x})}{\partial x_i}\varphi(\mathbf{x}) - \varphi^+(\mathbf{x})\frac{\partial\varphi(\mathbf{x})}{\partial x_i}\right), \tag{7}$$

$$\hat{\zeta}_4(\mathbf{x}) \equiv \hat{\rho}^{(m)}(\mathbf{x}) = m_m\varphi^+(\mathbf{x})\varphi(\mathbf{x}). \tag{8}$$

Here m_m is the medium particle mass. Medium motion integrals operators can be expressed as following:

$$\hat{M} = \int d^3x\hat{\rho}^{(m)}(\mathbf{x}), \hat{P}_i = \int d^3x\hat{\pi}_i(\mathbf{x}), \hat{\mathcal{H}}_m = \int d^3x\hat{\varepsilon}(\mathbf{x}). \tag{9}$$

As mentioned before, $\mathcal{H}_p$ has a structure of free particles Hamiltonian:

$$\mathcal{H}_p = \frac{1}{2m}\int d^3x\nabla\hat{\psi}^+(\mathbf{x})\nabla\hat{\psi}(\mathbf{x}). \tag{10}$$

Here $\hat{\psi}^+(x)$ and $\hat{\psi}(x)$ are correspondingly creation and annihilation operators of particles, and m is the particle mass. As a description parameter for particles we choose the Wigners distribution function [2,3]

$$\hat{f}_\mathbf{p}(\mathbf{x}) \equiv \int d^3x' e^{-i\mathbf{p}\mathbf{x}'}\hat{\psi}^+\left(\mathbf{x} - \frac{\mathbf{x}'}{2}\right)\hat{\psi}\left(\mathbf{x} + \frac{\mathbf{x}'}{2}\right), \tag{11}$$

which is a density of motion integral for free particles.

We introduce a generalizing symbol $\hat{\zeta}_\alpha(\mathbf{x})$ for description parameters, where "A" possesses values $A = \{\alpha, \mathbf{p}\}$:

$$\hat{\zeta}_A(\mathbf{x})\big|_{A=\alpha} = \hat{\zeta}_\alpha(\mathbf{x}), \; \hat{\zeta}_A(\mathbf{x})\big|_{A=\mathbf{p}} = \hat{f}_\mathbf{p}(\mathbf{x}). \tag{12}$$

We note, that medium and particles creation and annihilation operators commute:

$$\begin{aligned}&\left[\hat{\psi}^+(\mathbf{x}), \hat{\varphi}^+(\mathbf{x})\right] = 0, \left[\hat{\psi}(\mathbf{x}), \hat{\varphi}(\mathbf{x})\right] = 0, \\ &\left[\hat{\psi}(\mathbf{x}), \hat{\varphi}^+(\mathbf{x})\right] = 0, \left[\hat{\psi}^+(\mathbf{x}), \hat{\varphi}(\mathbf{x})\right] = 0.\end{aligned} \tag{13}$$

Time derivative of additive motion integrals densities can be expressed in terms of additive motion integrals flows [2], [4]:

$$\dot{\hat{\zeta}}_A(\mathbf{x}) = i[\mathcal{H}_0, \hat{\zeta}_A(\mathbf{x})] = -\frac{\partial\hat{\zeta}_{Ak}(\mathbf{x})}{\partial x_k}. \tag{14}$$

Flow operators $\hat{\zeta}_{Ak}(\mathbf{x})$ can be expressed in terms of description parameters $\hat{\zeta}_A(\mathbf{x})$ with rather complicated integral expressions (see more in [2]), but expression for $\hat{\zeta}_{\mathbf{p}k}(\mathbf{x})$ is very simple:

$$\hat{\zeta}_{\mathbf{p}k} \equiv \hat{f}_{\mathbf{p}k}(\mathbf{x}) = \frac{p_k}{m}\hat{f}_\mathbf{p}(\mathbf{x}). \tag{15}$$

4. MOTION EQUATION

Using the concepts of Bogolubov's reduced description method we can obtain motion equations for reduced description parameters. As mentioned, the statistical operator $\rho(t)$, on evolution stage when time is greater than specific relaxation time τ_0, has functional dependence on description parameters $\zeta_A(\mathbf{x})$:

$$\rho(t) = e^{-i\mathcal{H}t}\rho e^{i\mathcal{H}t} \underset{t \gg \tau_0}{\rightarrow} \sigma(\zeta(\mathbf{x}', t; \rho)), \tag{16}$$

and operator $\rho(t)$ on the initial state (statistical operator ρ) is included in the reduced description parameters. Operator σ is called the coarse-grained statistical operator and has functional dependence on description parameters $\zeta_A(\mathbf{x})$. Coarse-grained statistical operator σ must satisfy the relation

$$\zeta_A(\mathbf{x}) \equiv Sp\sigma(\zeta)\hat{\zeta}_A(\mathbf{x}). \tag{17}$$

Using relations (16) and (17), Liouville equation (1) and some fundamental properties of Hamiltonian (5) we can obtain a motion equation for reduced description parameters

$$\dot{\zeta}_A(\mathbf{x}) = iSp\sigma\left(\zeta(\mathbf{x}')\right)\left[\hat{V},\hat{\zeta}_A(\mathbf{x})\right] - \frac{\partial}{\partial x_k} Sp\sigma\left(\zeta(\mathbf{x}')\right)\hat{\zeta}_{Ak}(\mathbf{x}), \tag{18}$$

and integral equation for coarse-grained statistical operator σ :

$$\begin{aligned}\sigma\left(\zeta(\mathbf{x}')\right) = \rho - i\int_{-\infty}^{0} d\tau\, e^{i\mathcal{H}_0\tau}\Big\{\left[\mathcal{H}_0,\rho\right] \\ - \int d^3x \frac{\delta\sigma\left(\zeta(\mathbf{x}')\right)}{\delta\zeta_A(\mathbf{x})}\frac{\partial}{\partial x_k} Sp\sigma\hat{\zeta}_{Ak}(\mathbf{x}) \\ + i\int d^3x \frac{\delta\sigma\left(\zeta(\mathbf{x}')\right)}{\delta\zeta_A(\mathbf{x})} Sp\sigma + \left[\hat{V},\hat{\zeta}_A(\mathbf{x})\right]\Big\} e^{-i\mathcal{H}_0\tau}.\end{aligned} \tag{19}$$

Here ρ is the initial approximation for coarse-grained statistical operator. In our further calculation we will assume that

$$\rho = w\left(Y(\mathbf{x}')\right) = \exp\left\{\Omega\left(Y(\mathbf{x}')\right) - \int d^3x' Y_A(\mathbf{x}')\hat{\zeta}_A(\mathbf{x}')\right\}, \tag{20}$$

where $Y_A(\mathbf{x}')$ are arbitrary functions and $\Omega\left(Y(\mathbf{x}')\right)$ is determined by normalization requirement $Spw\left(Y(\mathbf{x}')\right) = 1$. Such operator contains enough arbitrary functions and satisfies spatial correlation weakening principle.

Further we build perturbation theory for equations (19) and (20) over small interaction $\hat{V}$ and small spatial gradients of description parameters $\zeta_A(\mathbf{x})$. We use the symbol $D^{(n,m)}$ to label the terms of perturbation series of variable D. The term $D^{(n,m)}$ is derived in of the n-th order in magnitude of gradients of $\zeta_A(\mathbf{x})$ and of the m-th order in interaction $\hat{V}$. After some transformations and using symmetry properties we obtain the following results. Operator $w\left(Y(\mathbf{x}')\right)$ is expanded into series over small gradients:

$$\begin{aligned} w\left(Y(\mathbf{x})\right) &= w^{(0)}(\mathbf{x}) + w^{(1)}(\mathbf{x}) + ..., \\ w^{(0)}(\mathbf{x}) &= \exp\left\{\Omega\ (\mathbf{x}) - Y_A(\mathbf{x})\hat{\gamma}_A\right\}, \end{aligned} \tag{21}$$

Here variables $\hat{\gamma}_A$ are additive motion integrals $\hat{\gamma}_A = \int d^3x\hat{\zeta}_A(\mathbf{x})$, and thermodynamic potential $\Omega(\mathbf{x})$ and thermodynamic forces $Y_A(\mathbf{x})$ are derived from equations $Spw^{(0)}(\mathbf{x}) = 1$, $Spw^{(0)}(\mathbf{x})\hat{\zeta}_A(0) = \zeta_A(\mathbf{x})$. Further we use symbol $\langle...\rangle = Spw^{(0)}...$.

After some transformations (see [2,5]), we get

$$\begin{aligned} w^{(1)}(\mathbf{x}) = -\frac{\partial Y_A(\mathbf{x})}{\partial x_k} w^{(0)}(\mathbf{x}) \\ \times\int_0^1 d\lambda \int d^3x' x'_k \left(w^{(0)-\lambda}\hat{\zeta}_A(\mathbf{x}') w_p^{(0)\lambda} - \left\langle\hat{\zeta}_A\right\rangle\right). \end{aligned} \tag{22}$$

Now we find series expansion for coarse-grained statistical operator σ

$$\begin{aligned} \sigma(\mathbf{x}) &= w^{(0)}(\mathbf{x}) + \sigma^{(0,1)}(\mathbf{x}) + \sigma^{(1,0)}(\mathbf{x}) + ..., \\ \sigma^{(0,1)}(\mathbf{x}) &= -i\int_{-\infty}^{0} d\tau\, e^{i\mathcal{H}_0\tau}\left[\hat{V}, w^{(0)}(\mathbf{x})\right] e^{-i\mathcal{H}_0\tau}, \\ \sigma^{(1,0)}(\mathbf{x}) &= w^{(1)}(\mathbf{x}) + \frac{\partial Y_A(\mathbf{x})}{\partial x_k} w^{(0)}(\mathbf{x}) \int_{-\infty}^{0} d\tau \\ &\times \int_0^1 d\lambda \int d^3x' \left\{ e^{i\mathcal{H}_0\tau}\left(\hat{\zeta}'_{Ak}(\mathbf{x}',\lambda) - \left\langle\hat{\zeta}'_{Ak}\right\rangle\right) e^{-i\mathcal{H}_0\tau}\right\}. \end{aligned} \tag{23}$$

Motion equations for our system can de easily derived using (18) and (23)

$$\begin{aligned} \frac{\partial\zeta_A(\mathbf{x})}{\partial t} &= L_A^{(1,0)}(\mathbf{x}) + L_A^{(0,1)}(\mathbf{x}) \\ &+ L_A^{(1,1)}(\mathbf{x}) + L_A^{(0,2)}(\mathbf{x}) + L_A^{(2,0)}(\mathbf{x}), \\ L_A^{(1,0)}(\mathbf{x}) &= -\frac{\partial}{\partial x_k} Spw^{(0)}(\mathbf{x})\hat{\zeta}_{Ak}(0), \\ L_A^{(2,0)}(\mathbf{x}) &= -\frac{\partial}{\partial x_k} Sp\sigma^{(1,0)}(\mathbf{x})\hat{\zeta}_{Ak}(0), \\ L_A^{(0,1)}(\boldsymbol{x}) &= iSpw^{(0)}(\boldsymbol{x})\left[\hat{V},\hat{\zeta}_A(0)\right], \\ L_A^{(0,2)}(\mathbf{x}) &= iSp\sigma^{(0,1)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right], \\ L_A^{(1,1)}(\mathbf{x}) &= iSp\sigma^{(1,0)}(\mathbf{x})\left[\hat{V},\hat{\zeta}_A(0)\right] \\ &- \frac{\partial}{\partial x_k} Sp\sigma^{(0,1)}(\mathbf{x})\hat{\zeta}_{Ak}(0). \end{aligned} \tag{24}$$

5. RESULTS: EQUATIONS UNDER CONSIDERATION

After calculating traces in (24) we obtain a system of equations, describing the system. We assume that

$$\hat{V} = \sum_{\mathbf{p}_1,\mathbf{p}_2} \mathcal{I}(\mathbf{p}_1,\mathbf{p}_2) a^{+}_{\mathbf{p}_1} a_{\mathbf{p}_2}, \tag{25}$$

where operators $a^{+}_{\mathbf{p}_1}, a_{\mathbf{p}_2}$ are the creation and annihilation operators of medium. Relation $\mathcal{I}^{+}(\mathbf{p}_1,\mathbf{p}_2) = \mathcal{I}(\mathbf{p}_2,\mathbf{p}_1)$ follows from Hamiltonian hermicity.

The obtained equation system containes kinetic type equation for particles and hydrodynamic type equation for medium. In our calculations we assume that the medium particles are bosons and the particles interacting with the medium are fermions:

$$\begin{aligned}
&\frac{\partial f(\mathbf{p},\mathbf{x})}{\partial t}+\frac{p_k}{m}\frac{\partial f(\mathbf{p},\mathbf{x})}{\partial x_k}=L_{\mathbf{p}}(\mathbf{x}),\\
&\frac{\partial \zeta_a(\mathbf{x})}{\partial t}+\frac{\partial}{\partial x_k}\zeta^{(0)}_{\alpha k}(\mathbf{x})+\frac{\partial}{\partial x_k}\zeta^{(1)}_{\alpha k}(\mathbf{x})=L_\alpha(\mathbf{x}),\\
&L_{\mathbf{p}}(\mathbf{x})=2\pi\sum_{1,2}\delta_{2\mathbf{p}}\left\{I_{2,1}(\varepsilon_1-\varepsilon_2,\mathbf{x})f_1(\mathbf{x})\times\right.\\
&\quad\left.(1-f_2(\mathbf{x}))-I_{1,2}(\varepsilon_2-\varepsilon_1,\mathbf{x})f_2(\mathbf{x})(1-f_1(\mathbf{x}))\right\},\\
&L_\alpha(\mathbf{x})=-\sum_{\mathbf{p}}\chi_\alpha(\mathbf{p})L_{\mathbf{p}}(\mathbf{x}).
\end{aligned} \tag{26}$$

Expressions for $\zeta^{(0)}_{\alpha k}(\mathbf{x})$ and $\zeta^{(1)}_{\alpha k}(\mathbf{x})$ are the same as these in conventional hydrodynamics equations (see [2]).

Collision integral is determined by spectral function

$$\begin{aligned}
&I_{1,2}(\omega,\mathbf{x})=\frac{1}{2\pi}\\
&\times\int_{-\infty}^{\infty}d\tau\, Spw^{(0)}(\mathbf{x})e^{i\mathcal{H}_m\tau}\hat{\jmath}(1,2)e^{-i\mathcal{H}_m\tau}\hat{\jmath}(1,2)e^{i\omega\tau}.
\end{aligned} \tag{27}$$

It is easy to show, that $L_{\mathbf{p}}(\mathbf{x})=0$ when condition

$$f(\mathbf{p},\mathbf{x})=f_0(\mathbf{p},\mathbf{x})\equiv\left(\mathrm{e}^{Y_0(\mathbf{x})\varepsilon_{\mathbf{p}}+Y_i(\mathbf{x})p_i+c(\mathbf{x})}+1\right)^{-1} \tag{28}$$

takes place, i.e. when the particles distribution function is Fermi-like, and their mean velocity and temperature are equal to these of the medium.

6. HYDRODYNAMIC EQUATIONS

The last fact mentioned in the previous section allows us to assume that such an evolution stage exists, when the distribution function of particles is close to locally equilibrium Fermi distribution, though their density $n(\mathbf{x})=\sum_{\mathbf{p}} f(\mathbf{p},\mathbf{x})$ determined by parameter $c(\mathbf{x})$ in (22) is not constant. We use reduced description method formalism, assuming that distribution function $f(\mathbf{p},\mathbf{x})$ depends on reduced description parameters $\zeta_\alpha(\mathbf{x})$ and $n(\mathbf{x})$:

$$f(\mathbf{p},\mathbf{x})=f(\mathbf{p};n(\mathbf{x}),\zeta(\mathbf{x})). \tag{29}$$

We obtained our results assuming that particle density is small enough to use Maxwell's distribution function instead of Fermi: $f^{(0)}(\mathbf{p},\mathbf{x})=n(\mathbf{x})\vartheta\,\mathrm{e}^{-Y_ip_i-Y_0\varepsilon_{\mathbf{p}}}$. For more convenience we used the following set of reduced description variables: Medium density $\rho(\mathbf{x})$, temperature $T(\mathbf{x})=1/Y_0(\mathbf{x})$, local velocity $u_i(\mathbf{x})=\pi_i(\mathbf{x})/\rho(\mathbf{x})=-Y_i(\mathbf{x})/Y_0(\mathbf{x})$ and particles density $n(\mathbf{x})=\sum_{\mathbf{p}} f(\mathbf{p},\mathbf{x})$.

The following equations are obtained in the second order of perturbation theory

$$\frac{\partial n}{\partial t}+\nabla(n\mathbf{u})+\nabla\mathbf{\Phi}=0,\quad \frac{\partial\rho}{\partial t}+\nabla(n\rho)=0, \tag{30.h}$$

$$\begin{aligned}
&\frac{\partial T}{\partial t}+(\mathbf{u}\nabla)T+\Theta(\nabla\mathbf{u})\left(1-\frac{3n}{2\rho c_v}\right)+\frac{nT}{\rho c_v}(\nabla\mathbf{u})\\
&=\frac{1}{\rho c_v}\left(1-\frac{3n}{2\rho c_v}\right)\left\{\frac{\eta}{2}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right)^2\right.\\
&\left.+\left(\zeta-\frac{2\eta}{3}\right)\left(\frac{\partial u_l}{\partial x_l}\right)^2+\nabla(\kappa\nabla T)\right\}+\frac{1}{\rho c_v}\left\{\frac{m}{\rho}\mathbf{\Phi}\nabla p\right.\\
&\left.-\frac{3n\zeta_n}{2}\left(\frac{\Delta p}{\rho}-\frac{\nabla p\nabla\rho}{\rho^2}+\frac{\partial u_k}{\partial x_l}\frac{\partial u_l}{\partial x_k}\right)\right\}\\
&+\frac{1}{\rho c_v}\nabla\left\{\kappa_n\nabla n+n\kappa_T\nabla T+n\kappa_\rho\nabla\rho\right\}\\
&+\frac{3}{2}(\nabla\mathbf{u})^2\left\{\rho n\frac{\partial\zeta_n}{\partial\rho}+\Theta n\frac{\partial\zeta_n}{\partial T}-\frac{2}{3}n\zeta_n\right\},
\end{aligned} \tag{30.t}$$

$$\begin{aligned}
&\frac{\partial u_i}{\partial t}+(\mathbf{u}\nabla)u_i+\frac{\nabla_i p}{\rho}\left(1-\frac{mn}{\rho}\right)+\frac{\nabla_i(nT)}{\rho}\\
&=\frac{1}{\rho}\left(1-\frac{mn}{\rho}\right)\nabla_k(\eta u_{ik}+\zeta\delta_{ik}\nabla\mathbf{u})+\frac{1}{\rho}\nabla_k(m\eta_n u_{ik}\\
&+\zeta_n\delta_{ik}\nabla\mathbf{u})-\frac{m}{\rho}D\left\{\frac{\partial n}{\partial x_k}\frac{\partial u_k}{\partial x_i}+\nabla_i(n\nabla\mathbf{u})\right\}\\
&-\frac{mn}{\rho}D_\rho\left\{\frac{\partial\rho}{\partial x_k}\frac{\partial u_k}{\partial x_i}+\nabla_i(\rho\nabla\mathbf{u})\right\}\\
&-\frac{mn}{\rho}D_T\left\{\frac{\partial T}{\partial x_k}\frac{\partial u_k}{\partial x_i}+\nabla_i(\Theta\nabla\mathbf{u})\right\}\\
&+\frac{m}{\rho}\left\{\rho\frac{\partial\Phi_i}{\partial\rho}+n\frac{\partial\Phi_i}{\partial n}+\Theta\frac{\partial\Phi_i}{\partial T}+\Phi_i\right\}\nabla\mathbf{u}.
\end{aligned} \tag{30.v}$$

Here we introduce new values: particle diffusion flux vector $\mathbf{\Phi}=-D\nabla n-nD_T\nabla T-nD_\rho\nabla\rho$, medium velocity tensor $u_{ik}=\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}$, heat capacity $c_v=\frac{1}{\rho}\left(\frac{\partial\varepsilon_0}{\partial T}\right)_\rho$, and effective temperature $\Theta=\frac{T}{\rho c_v}\left(\frac{\partial p}{\partial T}\right)_\rho$. New constants appear in equations (30). We obtain particle diffusion constants

$$\begin{aligned}
&D_T=\frac{1}{3T}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2G(\tilde{p}),\\
&D_\rho=\frac{1}{3\rho}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2A(\tilde{p}),\; D=\frac{1}{3}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2A(\tilde{p}).
\end{aligned} \tag{31}$$

New kinetic coefficients appear due to transportation of heat and momentum with particles. They are thermodiffusivity:

$$\begin{aligned}
&\kappa_\rho=\frac{1}{3\rho}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2\left(\frac{\tilde{p}^2}{2m}-\frac{3T}{2}\right)A(\tilde{p}),\\
&\kappa_n=\frac{1}{3}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2\left(\frac{\tilde{p}^2}{2m}-\frac{3T}{2}\right)A(\tilde{p}),
\end{aligned} \tag{32}$$

specific particle thermoconductivity

$$\kappa_T=\frac{1}{3T}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2\left(\frac{\tilde{p}^2}{2m}-\frac{3T}{2}\right)G(\tilde{p}), \tag{33}$$

and specific particle viscosity coefficients

$$\eta_n = \frac{1}{15}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2 \frac{\tilde{p}^2}{T} B(\tilde{p}),$$
$$\zeta_n = \left(\frac{2}{3} - \frac{\Theta}{T}\right)\frac{m}{3}\sum_{\tilde{\mathbf{p}}}\left(\frac{\tilde{p}}{m}\right)^2 F(\tilde{p}). \quad (34)$$

Coefficients (32)-(34) can be derived from integral equations, containing linearized collision integral

$$\mathcal{L}_{\mathbf{p}}(\mathbf{x}) = 2\pi\sum_{1,2}\delta_{2\mathbf{p}}\left\{f_1(\mathbf{x})I_{2,1}(\varepsilon_1 - \varepsilon_2, \mathbf{x}) - f_2(\mathbf{x})I_{1,2}(\varepsilon_2 - \varepsilon_1, \mathbf{x})\right\}, \quad (35)$$

and function $\phi(\mathbf{p})$ relevant to Maxwell distribution:

$$\phi(\tilde{\mathbf{p}}) = (2\pi\hbar)^3 \mathcal{V}^{-1}(2\pi mT)^{\frac{3}{2}} e^{-\frac{\tilde{p}^2}{2mT}}. \quad (36)$$

The integral equations are:

$$\mathcal{L}_{\mathbf{p}}(A_k, \xi) = -\varphi(\tilde{\mathbf{p}})\tilde{p}_k, \quad (37)$$

$$\mathcal{L}_{\mathbf{p}}(F, \xi) = -\varphi(\tilde{\mathbf{p}})\left(\frac{\tilde{p}^2}{2mT} - \frac{3}{2}\right), \quad (38)$$

$$\mathcal{L}_{\mathbf{p}}(B_{ik}, \xi) = -\varphi(\tilde{\mathbf{p}})\left(\tilde{p}_i\tilde{p}_k - \frac{1}{3}\delta_{ik}\tilde{p}^2\right), \quad (39)$$

$$\mathcal{L}_{\mathbf{p}}(G_k, \xi) = -\varphi(\tilde{\mathbf{p}})\tilde{p}_k\left(\left(\frac{\tilde{p}^2}{2mT} - \frac{3}{2}\right) - \frac{m}{\rho}\left(\frac{\partial p}{\partial T}\right)_\rho\right), \quad (40)$$

$$\mathcal{L}_{\mathbf{p}}(R_k, \xi) = -\varphi(\tilde{\mathbf{p}})\left(-\frac{m\tilde{p}_k}{T}\left(\frac{\partial p}{\partial \rho}\right)_\rho\right). \quad (41)$$

Due to rotational symmetry, we have:

$$A_k(\tilde{\mathbf{p}}) = A(\tilde{p})\tilde{p}_k,\; F(\tilde{\mathbf{p}}) = F(\tilde{p}),$$
$$G_k(\tilde{\mathbf{p}}) = G(\tilde{p})\tilde{p}_k,\; R_k(\tilde{\mathbf{p}}) = R(\tilde{p})\tilde{p}_k, \quad (42)$$
$$B_{ik}(\tilde{\mathbf{p}}) = B(\tilde{p})\left(\tilde{p}_i\tilde{p}_k - \frac{1}{3}\tilde{p}^2\right).$$

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МЕТОД СОКРАЩЕННОГО ОПИСАНИЯ В ДИНАМИЧЕСКОЙ ТЕОРИИ ЧАСТИЦ, ВЗАИМОДЕЙСТВУЮЩИХ СО СРЕДОЙ

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Рассматриваются пространственно неоднородные состояния частиц, слабо взаимодействующих с гидродинамической средой в рамках метода сокращенного описания Боголюбова. Показано, что у такой системы существуют кинетический и гидродинамический этапы эволюции. На кинетическом этапе частицы описываются одночастичной функцией распределения, а среда описывается пятью гидродинамическими параметрами. Получена система связанных уравнений движения для параметров сокращенного описания. Рассмотрен переход от кинетического к гидродинамическому этапу эволюции системы. В качестве параметров сокращенного описания выбраны гидродинамические параметры среды и плотность частиц. Получены уравнения движения системы на гидродинамическом этапе эволюции. Эти уравнения могут, в частности, описывать нейтроны, распространяющиеся в среде без захвата и размножения.

МЕТОД СКОРОЧЕНОГО ОПИСУ В ДИНАМІЧНІЙ ТЕОРІЇ ЧАСТИНОК, ЩО ВЗАЄМОДІЮТЬ ІЗ СЕРЕДОВИЩЕМ

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Розглянуті просторово неоднорідні стани частинок, що слабко взаємодіють із гідродинамічним середовищем в рамках методу скороченого опису Боголюбова. Показано, що така система може знаходитися на кінетичному та гідродинамічному етапах еволюції. На кінетичному етапі частинки описуються одночастинковою функцією розподілу, а середовище описується п'ятьма гідродинамічними параметрами. Отримано систему зв'язаних рівнянь руху для параметрів скороченого опису. Розглянуто перехід від кінетичного до гідродинамічного етапу еволюції системи. Параметрами скороченого опису вибрані п'ять гідродинамічних параметрів середовища та густина частинок. Отримано рівняння руху системи на гідродинамічному етапі еволюції системи. Ці рівняння можуть, зокрема, описувати нейтрони, що поширюються у середовищі без розмноження та захвату.