ELECTROMAGNETIC FIELD CORRELATIONS AND SOUND WAVES

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Statistical operator of the many component plasma has been found on the basis of the Bogolyubov reduced description method and quasi-relativistic quantum electrodynamics. Calculations were carried out in the Hamilton gauge up to the second order of a perturbation theory in interaction. Closed system of equations for binary correlations of electromagnetic field and hydrodynamic variables of medium has been obtained and investigated near equilibrium. Classical Maxwell plasma approximation was studied. Coupled states of sound waves and waves of transversal correlation of the field were predicted. Waves of correlations of electromagnetic field can be excited by sound waves in plasma.

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1. INTRODUCTION

Recent theories of electromagnetic (EM) processes, which take into account fluctuations, are based on the Langevin equations and are semiphenomenological [1]. Besides, usual quasi-relativistic theories consider effective direct Coulomb interaction between charged particles [2] and use the Coulomb gauge of the vector potential $\operatorname{div}\vec{A}=0$. Our purpose is to build kinetics of EM field in hydrodynamic medium based on the Hamilton gauge $\varphi=0$ and quasi-relativistic quantum electrodynamics using. In the framework of this gauge one does not need to introduce the scalar potential φ and the Maxwell equations have the form of the Hamilton equations. The medium (plasma) consists of a few components of charged and neutral particles.

Description of nonequilibrium states of the system is based on the Bogolyubov reduced description method (RDM) [3]. For the first time this method was applied to the considered system in our paper [5]. In the present paper we pay the main attention to the study of the influence of binary correlations of the field on dynamics of the system. Mass densities of a neutral and charged components, mass speed and temperature of the plasma, electric field, vector potential and their binary correlations are chosen as variables that describe time evolution of the system (reduced description parameters). Therefore, in the considered model usual plasma waves (longitudinal EM waves) are absent because of equilibrium between particles of the components. Besides, following to [6] we restrict ourselves by consideration of the ideal liquid approximation. In the framework of the RDM statistical operator of the system is built using the Bogolyubov condition of the complete correlation weakening. Additional convenience in the consideration is possible because the field variables satisfy the Peletminsky-Yatsenko commutation condition [3].

As a small parameter of the theory λ ratio of the plasma frequency Ω and Cherenkov's frequency of absorption of the field $k\upsilon_T$ (υ_T is a characteristic equilibrium velocity) is chosen, that is equivalent to consideration of wave vectors bounded from the bottom by inverse Debye radius $k\geq k_{\min}$, $k_{\min}\sim r_D^{-1}$. On the other hand, in quasi-relativistic theory wave vectors of the field are bounded from the above too $k\leq k_{\max}$. For our purpose one can estimate $k_{\max}$ as reverse average interparticle distance $k_{\max}\sim r_0^{-1}$. As a result closed system of equations of hydrodynamics and fluctuation electrodynamics is built within the first order of the perturbation theory for statistical operator.

2. REDUCED DESCRIPTION OT THE SYSTEM

We will describe our system by electric field $E_n(x,t)$, vector potential $A_n(x,t)$, their binary correlations $(E_n(x)E_l(x'))_t$, $(E_n(x)A_l(x'))_t$, $(A_n(x)A_l(x'))_t$ (variables $\eta_i(t)$) and densities of mass $\sigma_a(x,t)$ (a is the component number), momentum $\pi_n(x,t)$ and energy $\varepsilon(x,t)$ of the medium (variables $\zeta_\mu(x,t)$). Exact definition of the correlations is given by the relation of the type $2(E_nA_l)_t=\overline{\{\hat{E}_n,\hat{A}_l\}}-2\overline{\hat{E}_n}\;\overline{\hat{A}_l}$. In the Hamilton gauge quasi-relativistic Hamilton operator of the system has the form

$$\hat{H}=\hat{H}_0+\hat{H}_{\text{int}}=(\hat{H}_m+\hat{H}_f)+(\hat{V}_1+\hat{V}_2),$$

$$\hat{H}_f=\frac{1}{8\pi}\int d^3x\{\hat{\vec{E}}^2(x)+\hat{\vec{H}}^2(x)\},$$

$$\hat{V}_1=-\frac{1}{c}\int d^3x\,\hat{A}_n(x)\hat{j}_{on}(x),\ \hat{V}_2=\frac{1}{2c^2}\int d^3x\,\hat{A}^2(x)\hat{\chi}(x);$$

$$\hat{\vec{H}}=\operatorname{rot}\hat{\vec{A}},\ \hat{\vec{E}}=-\frac{1}{c}\dot{\hat{\vec{A}}};\ \hat{\chi}(x)=\sum_a\frac{e_a^2}{m_a^2}\hat{\sigma}_a(x),$$

$$\hat{\vec{j}}_o(x)=\sum_a\frac{e_a}{m_a}\hat{\vec{\pi}}_{oa}(x),\qquad(1)$$

where $\hat{H}_m$ is the Hamilton operator of free medium particles. Formulae (1) contain a non-gauge invariant density of momentum $\hat{\vec{\pi}}_{oa}(x)$. Gauge invariant densities of momentum and energy can be introduced by usual way. It is easy to check that gauge invariant mass velocity $u_n(x,t)$ is given by the formula

$$u_n(x,t)=u_{on}(x,t)-\frac{1}{c}A_n(x,t)\rho(x,t)$$

$$(\rho=\sum_a\rho_a\ ,\ \rho_a=\frac{e_a}{m_a}\sigma_a\ ), \tag{2}$$

where $u_{on}(x,t)=\pi_{on}(x,t)/\sigma(x,t)$ is non-invariant velocity. This allows us to use the Galilei transformation for the medium and cast the results in a gauge invariant form.

Equations of motion for operators of reduced description parameters in the terms of the gauge invariant densities have usual form [5]

$$\partial_t\hat{E}_n=c\,\Delta_{nl}\hat{A}_l-4\pi\hat{j}_n\ ,\ \partial_t\hat{A}_n=-c\,\hat{E}_n$$
$$\partial_t\{\hat{E}_n,\hat{E}'_l\}=c\Delta_{nm}\{\hat{A}_m,\hat{E}'_l\}+c\Delta'_{lm}\{\hat{E}_n,\hat{A}'_m\}-$$
$$-4\pi(\{\hat{j}_n,\hat{E}'_l\}+\{\hat{E}_n,\hat{j}'_l\})$$
$$\partial_t\{\hat{E}_n,\hat{A}'_l\}=c\Delta_{nm}\{\hat{A}_m,\hat{A}'_l\}-c\{\hat{E}_n,\hat{E}'_l\}-4\pi\{\hat{j}_n,\hat{A}'_l\}\ ,$$
$$\partial_t\{\hat{A}_n,\hat{E}'_l\}=-c\{\hat{E}_n,\hat{E}'_l\}+c\Delta'_{lm}\{\hat{A}_n,\hat{A}'_m\}-4\pi\{\hat{A}_n,\hat{j}'_l\}\ ,$$
$$\partial_t\{\hat{A}_n,\hat{A}'_l\}=-c\{\hat{E}_n,\hat{A}'_l\}-c\{\hat{A}_n,\hat{E}'_l\}\ ;$$
$$\partial_t\hat{\sigma}_a=-\frac{\partial\hat{i}_{an}}{\partial x_n}\ ,\ \ \partial_t\hat{\varepsilon}=-\frac{\partial\hat{q}_n}{\partial x_n}+\frac{1}{2}\{\hat{j}_n,\hat{E}_n\}$$
$$\partial_t\hat{\pi}_l=-\frac{\partial\hat{t}_{ln}}{\partial x_n}+\hat{\rho}\hat{E}_l+\frac{1}{2c}\varepsilon_{lnm}\{\hat{j}_n,\mathrm{rot}_m\hat{\hat{A}}\}\ , \tag{3}$$

where

$$\Delta_{nl}=\partial_n\partial_l-\delta_{nl}\Delta\ ,\ \hat{j}_n(x)\equiv\hat{j}_{on}(x)-\frac{1}{c}\hat{A}_n(x)\hat{\chi}(x)\ ,$$

($\{\hat{E}_n,\hat{E}'_l\}\equiv\{\hat{E}_n(x),\hat{E}_l(x')\}$ and so on; $\hat{j}_n(x)$ is gauge invariant current; here ∂_t is the Schrödinger time derivative). Averaging equations (3) with nonequilibrium statistical operator (SO) of the system $\rho(\eta(t),\zeta_o(t))$ we obtain a closed system of equations for parameters describing its state (it is convenient to do this using non-gauge invariant medium variables $\zeta_{o\mu}(x,t)$).

To construct SO $\rho(\eta(t),\zeta_o(t))$ for this case one can use the RDM [3-5] starting from the Liouville equation

$$\partial_t\rho(\eta(t),\zeta_o(t))=-\frac{i}{\hbar}[\hat{H},\rho(\eta(t),\zeta_o(t))]\equiv$$
$$\equiv\mathbf{L}\rho(\eta(t),\zeta_o(t))\ . \tag{4}$$

This consideration is simplified by the remark that operators of the EM field and its correlations satisfy the Peletminsky-Yatsenko condition $\mathbf{L}_f\hat{\eta}_i=-i\sum_{i'}c_{ii'}\hat{\eta}_{i'}$. In this work we do not introduce direct interaction among charged particles and in the leading approximation the medium is described by the local equilibrium distribution of ideal gas

$$w(Y)=\exp\{F(Y)-\sum_a\int d^3xY_a(x)\hat{\zeta}_{o\mu}(x)\}\ .$$

Using boundary condition of the complete correlation weakening according to [3-5] we obtain the following integral equation for statistical operator $\rho(\eta,\zeta_o)$

$$\rho(\eta,\zeta_o)=\rho_q(\eta)w(Y(\zeta_o))+\int_0^{+\infty}d\tau e^{\tau\mathbf{L}_0}\{\mathbf{L}_{\mathrm{int}}\rho(\eta,\zeta_o)$$
$$+\rho_q(\eta)\mathbf{L}_m w(Y(\zeta_o))-\sum_i\frac{\partial\rho(\eta,\zeta_o)}{\partial\eta_i}\tilde{L}_i(\eta,\zeta_o)$$
$$-\sum_\mu\int dx\frac{\delta\rho(\eta,\zeta_o)}{\delta\zeta_{o\mu}(x)}M_\mu(x,\eta,\zeta_o)\}_{\eta\to e^{-ic\tau}\eta}\ ;$$
$$M_i(\eta,\zeta_o)\equiv-\mathrm{Sp}\,\rho(\eta,\zeta_o)\mathbf{L}_{\mathrm{int}}\hat{\eta}_i\ ,$$
$$M_\mu(x,\eta,\zeta_o)=-\mathrm{Sp}\,\rho(\eta,\zeta_o)(\mathbf{L}_m+\mathbf{L}_{\mathrm{int}})\hat{\zeta}_{o\mu}(x)\ ;$$
$$\mathrm{Sp}\,\rho(\eta,\zeta_o)\hat{\zeta}_{o\mu}(x)=\zeta_{o\mu}(x)\ . \tag{5}$$

Here $\rho_q(\eta)$ is statistical operator of the free field, which satisfy the Liouville equation

$$\sum_{ii'}\frac{\partial\rho_q(\eta)}{\partial\eta_i}ic_{ii'}\eta_{i'}=\mathbf{L}_f\rho_q(\eta)\ . \tag{6}$$

The last equation in (5) allows to find functions $Y_\mu(x,\zeta_o)$.

Equation (5) is solvable in perturbation theory in small interaction. In the Baryakhtar-Peletminsky picture in the frame of the local rest its solution has the form [3-5]

$$\rho^o(x,\eta,\zeta_o)=\rho_q(\eta)w^o(\zeta_o(x))-\frac{i}{c\hbar}\int_{-\infty}^0 d\tau\int dx'\sum_i\nu_{2nx',i}(\tau)$$
$$\times\{[\rho_q w^o(\zeta_o(x)),\hat{\eta}_i(\hat{j}_{on}(x'-x,\tau)+\hat{\rho}(x'-x,\tau)u_{on}(x))]$$
$$+w^o(\zeta_o(x))\rho(x)u_{on}(x)\sum_{i'}\frac{\partial\rho_q}{\partial\eta_{i'}}\mathrm{Sp}_f\,\rho_q[\hat{\eta}_{i'},\hat{\eta}_i]\}+O(\lambda^2)\ , \tag{7}$$

where

$$w^o(\zeta_o)=\exp\beta\{\Omega(\beta,\mu)-\hat{H}_m+\sum_a\mu_a\hat{M}_a\}$$

($\beta\equiv T^{-1}$). Here zero and first order contributions in interaction are given in zero approximation in gradients of hydrodynamic variables. The Dirac picture for operators $\hat{A}_n(x)$ and $\hat{j}_{on}(x)$ was introduced in (7) by usual formulae

$$\hat{A}_n(x,\tau)\equiv e^{-\tau\mathbf{L}_f}\hat{A}_n(x)=\sum_i\nu_{2nx,i}(\tau)\hat{\eta}_i\ , \tag{8}$$

$$\hat{j}_{on}(x,\tau)=e^{-\tau\mathbf{L}_m}\hat{j}_{on}(x)\ , \tag{9}$$

($\hat{A}_n(x)\equiv\hat{\eta}_{2nx}$, $\hat{E}_n(x)\equiv\hat{\eta}_{1nx}$) where entering first relation Fourier components of $\nu_{ii'}(\tau)$ are given by expressions

$$\mu_{nl}(k,\tau)=\tilde{k}_n\tilde{k}_l+\delta^t_{nl}\cos\omega_k\tau\ ,$$
$$\nu_{nl}(k,\tau)=-c\tau\tilde{k}_n\tilde{k}_l-\delta^t_{nl}\frac{\sin\omega_k\tau}{k}\ ,$$
$$(\nu_{2nx,1lx'}(\tau)=\nu_{nl}(x-x',\tau)\ ,\ \nu_{2nx,2lx'}(\tau)=\mu_{nl}(x-x',\tau)\ ;$$
$$\tilde{k}_n=k_n/k\ ,\ \delta^t_{nl}=\delta_{nl}-\tilde{k}_n\tilde{k}_l\ ). \tag{10}$$

3. EQUATIONS OF MOTION

Let us consider averaging of relations (3) with statistical operator (7). The first term in (7) can be transformed with the help of formula [1,7]

$$[\hat{j}_{on}(x,\tau),w^o]=-\beta\int_{-1}^{0}d\lambda[\hat{j}_{on}(x,\tau,\lambda),\hat{H}_m]w^o$$

$$=-i\hbar\beta\int_{-1}^{0}d\lambda\frac{\partial\hat{j}_{on}(x,\tau,\lambda)}{\partial\tau}w^o \quad (11)$$

($\hat{A}(\lambda)\equiv w^{o\lambda}\hat{A}w^{o-\lambda}$). Integrating by parts and taking into account, that lower limit vanishes due to the principle of correlation weakening, we obtain instead of (7)

$$\rho^o=\rho_q w^o+\frac{i}{c\hbar}\int_{-\infty}^{0}d\tau\int dx[\hat{A}_n(x,\tau),\rho_q]\hat{j}_{on}(x,\tau)w^o$$

$$+\beta\int_{-\infty}^{0}d\tau\int dx\int_{-1}^{0}d\lambda\hat{E}_n(x,\tau)\hat{j}_{on}(x,\tau,\lambda)\rho_q w^o$$

$$+\frac{\beta}{c}\int dx\int_{-1}^{0}d\lambda\,\hat{A}_n(x)\hat{j}_n(x,\lambda)\rho_q w^o$$

$$-\frac{i}{c\hbar}\rho u_{on}\int_{-\infty}^{0}d\tau\int dx'[\rho_q w^o,\hat{\rho}(x'-x,\tau)\hat{A}_n(x',\tau)]$$

$$+O(\lambda^2)\,. \quad (12)$$

Further we restrict ourselves by consideration of classical medium. Waves of correlations will be studied on the basis of equations of motion linearized near equilibrium, therefore, we will drop terms, which vanish after linearization, and only indicate them. Then formula (12) leads to the following expression for average gauge invariant current

$$j_n(x)\equiv\beta\int_{-\infty}^{0}d\tau\int dx' I^x_{nl}(x-x',-\tau)E_l(x',\tau)$$

$$+\frac{\beta}{c}\int dx' I^x_{nl}(x-x',\tau=0)A_l(x')-\frac{1}{c}A_n(x)\chi+O(u_o^2)\,,$$

$$I^{x'}_{nl}(x,\tau)\equiv\mathrm{Sp}_m\, w^o(\zeta_o(x'))\hat{j}_{on}(x,\tau)\hat{j}_{ol}(0)\,. \quad (13)$$

According (1), (9), (10) the Dirac picture for operator $\hat{E}_n(x)$ has the form

$$\hat{E}_n(x,\tau)=e^{-\tau\mathrm{L}_0}\hat{E}_n(x)=\int dx'\{\mu_{nl}(x-x',\tau)\hat{E}_l(x')$$

$$+\lambda_{nl}(x-x',\tau)\hat{A}_l(x')\}\,,$$

$$\lambda_{nl}(k,\tau)\equiv\delta^t_{nl}k\sin\omega_k\tau\,. \quad (14)$$

It the considered case correlation function $I_{nl}(x,\tau)$ in (13) corresponds to the Maxwell plasma and their Fourier transform can be written in the form

$$I^x_{nl}(k,\omega)=2\pi\sum_a e_a^2 n_a\int d^3\upsilon\upsilon_n\upsilon_l f_a(\upsilon)\delta(\omega-\vec{k}\vec{\upsilon})\,,$$

$$I^x_{nl}(x',\tau=0)=\frac{\chi}{\beta}\delta_{nl}\delta(x')\ \ (\int d^3\upsilon f_a(\upsilon)=1) \quad (15)$$

($f_a(\upsilon)$ is the Maxwell distribution; $\chi=\sum_a e_a^2 n_a/m_a$; here for simplicity we do not show dependence of n_a, β, χ on x). This leads to the following material equation

$$j_n(x)=\int dx'\{M^x_{nl}(x-x')E_l(x')+N^x_{nl}(x-x')A_l(x')\}$$

$$+O(u_o^2)\,, \quad (16)$$

with material coefficients

$$M^x_{nl}(k)=\beta\int_{-\infty}^{0}d\tau I^x_{nl}(k,\tau)\mu_{ml}(k,\tau)\,,$$

$$N^x_{nl}(k)=\beta\int_{-\infty}^{0}d\tau I^x_{nm}(k,\tau)\lambda_{ml}(k,\tau)\,. \quad (17)$$

Analogous to (16) calculation gives field-current correlations

$$(A^x_n j^{x'}_l)=\int dx''\{M^{x'}_{lm}(x'-x'')(A^x_n E^{x''}_m)$$

$$+N^{x'}_{lm}(x'-x'')(A^x_n A^{x''}_m)\}+S^{x'}_{nl}(x-x')+O(u_o^2)\,,$$

$$(j^x_n A^{x'}_l)=\int dx''\{M^x_{nm}(x-x'')(E^{x''}_m A^{x'}_l)+$$

$$+N^x_{nm}(x-x'')(A^{x''}_m A^{x'}_l)\}+S^x_{nl}(x-x')+O(u_o^2)\,,$$

$$(E^x_n j^{x'}_l)=\int dx''\{M^{x'}_{lm}(x'-x'')(E^x_n E^{x''}_m)$$

$$+N^{x'}_{lm}(x'-x'')(E^x_n A^{x''}_m)\}+T^{x'}_{nl}(x-x')+O(u_o^2)\,,$$

$$(j^x_n E^{x'}_l)=\int dx''\{M^x_{nm}(x-x'')(E^{x''}_m E^{x'}_l)$$

$$+N^x_{nm}(x-x'')(A^{x''}_m E^{x'}_l)\}+T^x_{nl}(x-x')+O(u_o^2)\,, \quad (18)$$

where the functions $S^{x'}_{nl}(x)$, $T^{x'}_{nl}(x)$ are also expressed through $M^{x'}_{nl}(x)$, $N^{x'}_{nl}(x)$

$$S^x_{nl}(k)=-\frac{8\pi T(x)}{k^2}N^x_{nl}(k)\,,\ T^x_{nl}(k)=-8\pi T(x)M^x_{nl}(k) \quad (19)$$

(in (22) and further notation of the type $(E_n(x)E_l(x'))_t=(E^x_n E^{x'}_l)=(E_n E'_l)$ are used for binary correlations). Relations (18) are additional material equations of the developed theory.

Material coefficients (17) determinate electromagnetic properties of the medium. In homogenous and isotropic media all second rank tensors can be presented as sum of longitudinal and transversal parts

$$C_{mn}(\vec{k})=\tilde{k}_n\tilde{k}_m C^l(k)+\left(\delta_{nm}-\tilde{k}_n\tilde{k}_m\right)C^{tr}(k)\,. \quad (20)$$

Using correlation function (15) and formulas (10), (14) we obtain the expressions for nonzero components of tensors $M_{nl}(k,\omega)$ and $N_{nl}(k,\omega)$

$$M^{x,l}(k)=0\,,\ M^{x,tr}(k)=-\sum_a\frac{n_a e_a^2}{m_a c}\mathrm{Im}J_+(\frac{c}{\upsilon_a})\,,$$

$$N^{x,tr}(k)=\sum_a\frac{n_a e_a^2}{m_a c}\mathrm{Re}J_+(\frac{c}{\upsilon_a})\,, \quad (21)$$

where the function $J_+(\frac{c}{\upsilon_a})$ is given by integral [7]

$$\int d^3\upsilon f_a(\upsilon)\frac{[\vec{k},\vec{\upsilon}]^2}{kc-\vec{k}\vec{\upsilon}+i0}=\frac{2kT}{m_a c}J_+(\frac{c}{\upsilon_a})\,,$$

$$(\upsilon_a\equiv\sqrt{3T/m_a});\ J_+(+\infty)=1\,. \quad (22)$$

In the considered here nonrelativistic approximation $c/\upsilon_a\gg 1$ relations (21), (22) give

$$M^{x,tr}(k)\simeq 0\,,\ N^{x,tr}(k)\simeq\Omega^2/4\pi c \quad (23)$$

($\Omega=\sqrt{4\pi\chi}$ is the plasma frequency). So, introduced material coefficients do not depend on wave vector and temperature.

Operator equations (3) after averaging over nonequlibrium statistical operator lead to the following set of electrodynamic and hydrodynamic equations [5]

$$\partial_t E_n=c\Delta_{nl}A_l-4\pi j_n\,,\ \partial_t A_n=-cE_n\,,$$

$$\partial_t(E_n A'_l)=c\Delta_{nm}(A_m A'_l)-c(E_n E'_l)-4\pi(j_n A'_l)\,,$$

$$\partial_t(A_n E'_l)=-c(E_n E'_l)+c\Delta'_{lm}(A_n A')_m-4\pi(A_n j'_l)\,,$$

$$\partial_t (A_n A_l') = -c(E_n A_l') - c(A_n E_l'),$$
$$\partial_t (E_n E_l') = c\Delta_{nm}(A_m E_l') + c\Delta_{lm}'(E_n A_m')$$
$$-4\pi\{(j_n E_l') + (E_n j_l')\};$$
$$\partial_t \pi_l = -\frac{\partial t_{ln}}{\partial x_n} + (\rho E_l) + \rho E_l + \frac{1}{c}\varepsilon_{lnm}\{(j_n B_m) + j_n B_m\},$$
$$\partial_t \varepsilon = -\frac{\partial q_n}{\partial x_n} + (j_n E_n) + j_n E_n, \ \partial_t \sigma_a = -\frac{\partial i_{an}}{\partial x_n}. \quad (24)$$

Material equations (16), (18) express the right hand sides of these equations through independent variables of the present theory by the relations

$$j_n^x = \frac{\Omega(x)^2}{4\pi c} A_n^{x,tr} + O(u_o^2),$$
$$(j_n^x E_l^{x'}) = \frac{\Omega(x)^2}{4\pi c}(A_n^{x,tr} E_l^{x'}) + O(u_o^2),$$
$$(E_n^x j_l^{x'}) = \frac{\Omega(x')^2}{4\pi c}(E_n^x A_l^{x',tr}) + O(u_o^2),$$
$$(j_n^x A_n^{x'}) = \frac{\Omega(x)^2}{4\pi c}(A_n^{x,tr} A_l^{x'}) + S_{nl}^x(x-x') + O(u_o^2),$$
$$(A_n^x j_l^{x'}) = \frac{\Omega(x')^2}{4\pi c}(A_n^x A_l^{x',tr}) + S_{nl}^{x'}(x-x') + O(u_o^2);$$
$$S_{nl}^x(k) = -\frac{2T(x)\Omega(x)^2}{ck^2}\tilde{\delta}_{nl}$$
$$(A_{nk} = A_{nk}^{tr} + A_{nk}^l, \ A_{nk}^{tr} \equiv A_{lk}\tilde{\delta}_{nl}, \ A_{nk} = A_{lk}\tilde{k}_n\tilde{k}_l). \quad (25)$$

4. SUBDYNAMICS OF ELECTROMAGNETIC CORRELATIONS OF ZERO RADIUS

In this section it will be demonstrated an influence of correlations of the electromagnetic field on dynamics of the system. It is convenient [6] to introduce auxiliary field $Z_n(x)$ by its Fourier transform $Z_n^k = \varepsilon_{nlm}\tilde{k}_l B_m^k = -ikA_n^{trk}$ and discuss correlations of the electromagnetic field in the terms of $(E_n E_l')$, $(E_n Z_l')$, $(Z_n Z_l')$. Equations (24), (25) must be linearized close to the equilibrium. We will restrict ourselves by a zero correlation length approximation, in which deviations of correlations have the structure

$$\delta(E_n^x E_l^{x'}) = \delta(E_n E_l)_x \delta(x-x'),$$
$$\delta(Z_n^x E_l^{x'}) = \delta(Z_n E_l)_x \delta(x-x'),$$
$$\delta(Z_n^x Z_l^{x'}) = \delta(Z_n Z_l)_x \delta(x-x') \quad (26)$$

(similar formulas are true for equilibrium correlations of the field). Simple calculation leads to following closed set of equations with full mass density as an independent variable

$$\partial_t \delta\sigma_k = -ik_n \sigma \delta u_{nk},$$
$$\partial_t \delta T_k = -ik_n w \delta u_{nk} + i\frac{2\Omega^2}{qc}\delta(E_n Z_n)_k,$$
$$\partial_t \delta u_{nk} = -ik_n(\alpha_\sigma \delta\sigma_k + \alpha_T \delta T_k) +$$
$$+i\frac{2\Omega^2}{rc^2}\tilde{k}_n\{\delta(Z_l Z_l)_k - 4\pi\delta T_k\};$$
$$\partial_t \delta(Z_n Z_l)_k = ick\delta(E_n Z_l)_k,$$
$$\partial_t \delta(E_n Z_l)_k = -ick\delta(Z_n Z_l)_k$$
$$-i\frac{\Omega^2}{ck}\{\delta(Z_n Z_l)_k - 2\pi\tilde{\delta}_{nl}\delta T_k\}, \quad (27)$$

where

$$q^{-1} = \int_{k_{\min}}^{k_{\max}} \frac{d^3k}{k\varepsilon_T}, \ \tilde{k}_n r^{-1} \equiv \int_{k_{\min}}^{k_{\max}} \frac{d^3k'\tilde{k}_n'}{4\pi|\vec{k}'-\vec{k}|\sigma};$$
$$\alpha_\sigma = \frac{T}{m\sigma}, \ \alpha_T = \frac{1}{m}, \ \varepsilon_T = \frac{3\sigma}{2m}, \ w = \frac{2T}{3}. \quad (28)$$

According to the mentioned above here $k_{\min} \sim r_D^{-1}$, $k_{\max} \sim r_0^{-1}$ (r_D is the Debye radius, r_0 is the average interparicle distance). Equations (27) are based on non-dissipative hydrodynamic fluxes and expressions for energy density and pressure $\varepsilon = 3\sigma T/2m$, $p = \sigma T/m$ in ideal gas approximation.

The Maxwell equations from (24), (25) are absent in (27) because in the considered approximation they are not connected with hydrodynamic variables and correlations of the field. They describe transverse electromagnetic waves with dispersion law $\omega_{f0}(k) = \sqrt{\Omega^2 + c^2k^2}$. Equation for $\delta(E_n E_l)_k$ from (24), (25) can be solved after consideration of the set (27) and does not lead to new branches of oscillations.

Hydrodynamic equations (27) without correlations of the field describe sound waves with dispersion law $\omega_{s0}(k) = ku$ (sound velocity $u = \sqrt{5T/3m}$). Equations for correlations (27) contain only transversal $\delta(ZZ)_k^{tr}$, $\delta(EZ)_k^{tr}$ parts of the field correlations. In the case of equilibrium medium these equations describe waves with dispersion law $\omega_{f0}(k)$.

5. CONNECTED SOUND AND CORRELATION OF THE FIELD WAVES

Equations (27) in the terms of variables $\delta\sigma_k$, $\delta u_k^l \equiv \tilde{k}_n \delta u_{nk}$, δT_k, $\delta(ZZ)_k^{tr}$, $\delta(EZ)_k^{tr}$ give

$$\partial_t \delta\sigma_k = -ik\sigma\delta u_k^l,$$
$$\partial_t \delta T_k = -ikw\delta u_k^l + i\frac{2\Omega^2}{qc}\delta(EZ)_k^{tr},$$
$$\partial_t \delta u_k^l = -ik(\alpha_\sigma\delta\sigma_k + \alpha_T\delta T_k) + i\frac{2\Omega^2}{rc^2}\{\delta(ZZ)_k^{tr} - 2\pi\delta T_k\},$$
$$\partial_t \delta(EZ)_k^{tr} = -\frac{i}{ck}\{c^2k^2 + \Omega^2\}\delta(ZZ)_k^{tr} + i\frac{2\pi\Omega^2}{ck}\delta T_k,$$
$$\partial_t \delta(ZZ)_k^{tr} = ick\delta(EZ)_k^{tr}. \quad (29)$$

The matrix of coefficients of this set of linear equations for $\delta\sigma_k$, $\delta u_k^l \equiv \tilde{k}_n \delta u_{nk}$, δT_k, $2\,\delta(EZ)_k^{tr}$, $2\,\delta(ZZ)_k^{tr}$ has the form

$$\begin{pmatrix} 0 & -ik\sigma & 0 & 0 & 0 \\ -ik\alpha_T & 0 & -ik\alpha_\sigma - i4\pi\frac{\Omega^2}{rc^2} & 0 & i\frac{\Omega^2}{rc^2} \\ 0 & -ikw & 0 & i\frac{\Omega^2}{qc} & 0 \\ 0 & 0 & i\frac{4\pi\Omega^2}{kc} & 0 & -i\frac{\Omega^2}{kc} - ikc \\ 0 & 0 & 0 & -ikc & 0 \end{pmatrix} \quad (30)$$

Nonzero eigenvalues of this matrix, which correspond to frequencies of own oscillations in the system, can be written as it follows

$$\lambda = \pm\frac{i}{\sqrt{2}}\{\frac{1}{c^2kqr}(c^4k^3qr + c^2kqr(k^2u^2 + \Omega^2)$$
$$+4\pi\Omega^2(k^2qw + r\Omega^2) \pm \{(c^2k^3qr(c^2 + u^2)$$
$$+kq(c^2r + 4k\pi w)\Omega^2 + 4\pi r\Omega^4)^2$$
$$-4c^2k^3qr(c^4k^3qru^2 + c^2kq(ru^2 + 4k\pi w)\Omega^2$$
$$+4\pi\alpha_T r\Omega^4)\}^{1/2})\}^{1/2}. \quad (31)$$

In general solution of equations (30) all combinations of the signs must be used. In the leading order in c^{-1} dispersion laws is given by formula

$$\omega(k) = 2^{-1/2}\{k^2(c^2 + u^2) + \Omega^2 \pm$$
$$\pm((k^2(c^2 - u^2) + \Omega^2)^2 - 16\pi w\Omega^2k^3/r)^{1/2}\}^{1/2}. \quad (32)$$

In the small interaction limit field correlation and sound frequencies (32) take the form

$$\omega_{\mathrm{f}}(k) = ck + \frac{\Omega^2}{2ck}$$
$$+\frac{c^2(16\pi k - q)r + qru^2 + 16\pi qwk - 16\pi\alpha_T r\sigma k}{8c^3qr(c^2 - u^2)k^3}\Omega^4 + O(\Omega^5),$$
$$\omega_{\mathrm{s}}(k) = uk + \frac{2\pi w\Omega^2}{c^2ru}$$
$$+2\pi\frac{\pi qu^2w^2k - c^2\{qru^2w + \pi qw^2k + r^2(u^4 - \alpha_T u^2\sigma)\}}{c^4qr^2u^3(c^2 - u^2)k^2}\Omega^4$$
$$+O(\Omega^5). \quad (33)$$

In the leading approximation in c^{-1} these expressions can be written as

$$\omega_{\mathrm{f}}(k) \simeq ck + \frac{\Omega^2}{2ck} + \frac{(16\pi k - q)}{8c^3qk^3}\Omega^4 + O(\Omega^5),$$
$$\omega_{\mathrm{s}}(k) \simeq uk + \frac{2\pi w\Omega^2}{c^2ru} \quad (34)$$
$$-2\pi\frac{qru^2w + \pi qw^2k + r^2(u^4 - \alpha_T u^2\sigma^0)}{c^4qr^2u^3k^2}\Omega^4 + O(\Omega^5).$$

6. CONCLUSION

Obtained on the base of the Bogolyubov reduced description method results [5], were applied to the system of the electromagnetic field in hydrodynamic plasma considered as an ideal gas [6,7]. A closed system of equations (24), (25) for density, mass velocity and temperature of plasma and field correlations was built, which linearized near the equilibrium. The subdynamics of correlations with zero radius was considered. New modes of oscillations in this system were studied (31), which correspond to coupled due to electromagnetic interaction sound and transversal correlation modes. The case of small interaction (33) was considered.

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КОРРЕЛЯЦИИ ЭЛЕКТРОМАГНИТНОГО ПОЛЯ И ЗВУКОВЫЕ ВОЛНЫ

А.И. Соколовский, А.А. Ступка

Статистический оператор многокомпонентной плазмы найден на основе метода сокращенного описания Боголюбова и квазирелятивистской квантовой электродинамики. Вычисления проведены в калибровке Гамильтона с точностью до второго порядка теории возмущений по взаимодействию. Получена замкнутая система уравнений для бинарных корреляций электромагнитного поля и гидродинамических переменных среды и исследована около равновесия. Изучено приближение классической максвелловской плазмы. Предсказаны связанные состояния звуковых волн и волн поперечных корреляций поля. Волны корреляций электромагнитного поля могут быть возбуждены звуковыми волнами в плазме.

КОРЕЛЯЦІЇ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ ТА ЗВУКОВІ ХВИЛІ

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Статистичний оператор багатокомпонентної плазми знайдено на основі метода скороченого опису Боголюбова та квазірелятивістської квантової електродинаміки. Обчислення проведено у калібровці Гамільтона з точністю до другого порядку теорії збурень за взаємодією. Одержано замкнену систему рівнянь для бінарних кореляцій поля та гідродинамічних змінних середовища і досліджено біля рівноваги. Вивчено наближення класичної максвеллівської плазми. Передбачено зв'язані стани звукових хвиль та хвиль поперечних кореляцій поля. Хвилі кореляцій електромагнітного поля можуть бути збуджені звуковими хвилями у плазмі.