CONSTRUCTION OF PROBABILISTIC SOLUTIONS OF THE BOLTZMANN EQUATION

Yu.P. Virchenko and T.V. Karabutova

Belgorod State University, Belgorod, Russia;
e-mail: virch@bsu.edu.ru

The proving method of the Cauchy problem solvability of the Boltzmann kinetic equation with spatially uniform initial data in the case of particle scattering cross-section finiteness is proposed. It is based on the construction of the auxiliary vector-valued random process such that the particle velocity distribution function satisfying the Boltzmann equation is the first order marginal probability distribution of this random process.

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1. INTRODUCTION

The Boltzmann equation has got the origin of the physical kinetics. Building of the equation and its physical predictions have played the great role during the development of representations concerning evolution irreversibility that are taken place in nature. Later, this equation has got also the practical mean for calculation of kinetic coefficients of real gently dense gases and also for the study of the motion of solids in the dilute gas environment. In this connection, the mathematical correct results concerning some solution properties of the Boltzmann equation have gained special importance. In particular, the Cauchy problem solvability of the equation is of interest. First results in this direction have been obtained by D.Hilbert [1] and T.Carleman [2]. All stationary solutions of the Boltzmann equation have been found and the theorem of the final behavior of solutions at the unbounded increasing of time has been proved. Besides, the spectrum of corresponding linearized equation has been investigated. Further, in connection with the development of the approximate methods of the equation solving, the Chepmen-Enskog asymptotical analysis has been created [3]. The modern state of the mathematical physics area which is connected with the Boltzmann equation see in [4]. In the present work, we propose the new approach to study the Boltzmann equation solutions which is based on the random process theory. This method permits to approach in a new fashion to the Cauchy problem solving of the Boltzmann equation and to the construction of approximations of corresponding solutions with the controlled accuracy.

2. THE BOLTZMANN EQUATION

For formulation of the Boltzmann equation in the form that is convenient for our aim, it is required the following presentation of scattering data in the classical mechanics for the pair of identical particles. Let $\mathbf{v}$ and $\mathbf{v}'$ be two particle velocities before the collision when they are on such a distance where one may consider them as the noninteracting ones. Further, let $\mathbf{V}$ and $\mathbf{V}'$ be corresponding velocities after the collision when the particles have gone away so far that they become noninteracting again. These velocities are represented by the definite functions of velocities $\mathbf{v}$, $\mathbf{v}'$ and the vector $\mathbf{r}$ which is on the plane being orthogonal to vectors $\langle \mathbf{v}, \mathbf{v}' \rangle$, i.e. $\mathbf{V} = (\mathbf{v}, \mathbf{v}', \mathbf{r}), \mathbf{V}' = (\mathbf{v}, \mathbf{v}', \mathbf{r})$. On the definition, the vector $\mathbf{r}$ begins at the straight line on which the first particle arrives from the infinity and it is finished at the analogous straight line of the second particle. The vector $\mathbf{r}$ has the minimal length among all vectors possessing the properties pointed out. The functions $\mathbf{V}$, $\mathbf{V}'$ are defined by the interaction potential $\Phi(\mathbf{r}_1 - \mathbf{r}_2)$ between two particles, where $\mathbf{r}_1, \mathbf{r}_2$ are space position vectors of particles.

For the determination of these functions, it is necessary to solve the mechanical scattering problem of two particles interacting by means of the potential Φ. Further, we consider only the spatially homogeneous gas of particles. In this case, the Boltzmann equation is formulated for the distribution function $f(\mathbf{v},t)$ that depends on the velocity $\mathbf{v}$ of the mentioned particle and the time t. It has the following form (see, for example, [5])

$$\dot{f}(\mathbf{v},t) = \rho \int |\mathbf{v} - \mathbf{v}'| (f(\mathbf{V},t) f(\mathbf{V}',t) - f(\mathbf{v},t) f(\mathbf{v}',t)) d\mathbf{v}' d\sigma \tag{1}$$

where the dot means the differentiation on t, the parameter $\rho > 0$ represents the density of particles and the integral on $d\sigma$ points out the integration on all values of the vector $\mathbf{r}$.

The function $f(\mathbf{v},t)$ is the probability density function, i.e. $f(\mathbf{v},t) \geq 0$ and the equality

$$\int_{\mathbf{R}^3} f(\mathbf{v},t) d\mathbf{v} = 1 \tag{2}$$

takes place. The functions $\mathbf{V} = (\mathbf{v}, \mathbf{v}', \mathbf{r}), \mathbf{V}' = (\mathbf{v}, \mathbf{v}', \mathbf{r})$ have the properties

$$\mathbf{V} + \mathbf{V}' = \mathbf{v} + \mathbf{v}', \quad \mathbf{V}^2 + \mathbf{V}'^2 = \mathbf{v}^2 + \mathbf{v}'^2 \tag{3}$$

which express the conservation laws of momentum and energy at collisions. From Eq. (3), the equality of relative velocities follows,

$$|\mathbf{V} - \mathbf{V}'| = |\mathbf{v} - \mathbf{v}'|. \tag{4}$$

Besides, the symmetry property

$$\mathbf{V} = (\mathbf{v}', \mathbf{v}, \mathbf{r}) = \mathbf{V}' = (\mathbf{v}, \mathbf{v}', \mathbf{r}) \tag{5}$$

takes place. It corresponds to the particle identity. Let us introduce the vector function $\mathbf{n} = \mathbf{n}(\mathbf{v}, \mathbf{v}', \mathbf{r}), |\mathbf{n}| = 1$, satisfying to the equality

$$\mathbf{V}=\frac{1}{2}[(\mathbf{v}+\mathbf{v}')+\mathbf{n}\,|\,\mathbf{v}-\mathbf{v}'|],$$

$$\mathbf{V}'=\frac{1}{2}[(\mathbf{v}+\mathbf{v}')-\mathbf{n}\,|\,\mathbf{v}-\mathbf{v}'|]. \qquad (6)$$

Further, we notice that for the complete characterization of the scattering of two particles, it is necessary to introduce, in addition to the functions $\mathbf{V}$, $\mathbf{V}'$, the vector function $\mathbf{R}(\mathbf{v},\mathbf{v}',\mathbf{r})$ also. Its values lay in the plane being orthogonal to vectors $\mathbf{V}$, $\mathbf{V}'$. The vector $\mathbf{R}$ is defined similarly to the vector $\mathbf{r}$ but it is done relative to the straight trajectories of particles going away with the velocities $\mathbf{V}$, $\mathbf{V}'$. From the energy and momentum conservation, the equality $|\mathbf{R}|=|\mathbf{r}|=r$ follows. The reversibility of mechanical motion leads to the fact that the functions $\mathbf{V}=(\mathbf{v},\mathbf{v}',\mathbf{r}), \mathbf{V}'=(\mathbf{v},\mathbf{v}',\mathbf{r})$ satisfy to identities

$$\mathbf{v}=-\mathbf{V}(-\mathbf{V},-\mathbf{V}',\mathbf{R}),\ \mathbf{v}'=-\mathbf{V}'(-\mathbf{V},-\mathbf{V}',\mathbf{R}) \qquad (7)$$

and, besides, $\mathbf{r}=\mathbf{R}(-\mathbf{V},-\mathbf{V}',\mathbf{R})$.

In the case when the potential is spherically symmetric, all vectors $\mathbf{v},\mathbf{v}',\mathbf{r},\mathbf{V},\mathbf{V}',\mathbf{R}$ lay in the common plane.

Let us transform the right-hand side of the equation (1) to another form which is more suitable for our construction. With this aim, introducing the additional integration by means of δ-functions, we write down

$$\dot f(\mathbf{v},t)=\int w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')(f(\mathbf{u},t)f(\mathbf{u}',t) - f(\mathbf{v},t)f(\mathbf{v}',t))d\mathbf{v}'\,d\mathbf{u}d\mathbf{u}'. \qquad (8)$$

The non-negative function w is named the intensity of scattering $\langle\mathbf{v},\mathbf{v}'\rangle\Rightarrow\langle\mathbf{u},\mathbf{u}'\rangle$ process. It is determined by the formula

$$w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')= =\rho\,|\,\mathbf{v}-\mathbf{v}'|\int\delta(\mathbf{V}-\mathbf{u})\delta(\mathbf{V}'-\mathbf{u}')d\sigma. \qquad (9)$$

According to Eqs. (5,7), it satisfies to the following identities

$$w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')=w(\mathbf{v}',\mathbf{v};\mathbf{u}',\mathbf{u}), \qquad (10)$$

$$w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')=w(-\mathbf{u},-\mathbf{u}';-\mathbf{v},-\mathbf{v}'). \qquad (11)$$

The function w contains the δ-functional singularities which are connected with conservation laws. Indeed, changing the argument of the second δ-function in Eq. (9) according to the first identity of Eq. (3) and also using the condition presented by the second δ-function, we obtain

$$w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')= =\rho\,|\,\mathbf{v}-\mathbf{v}'|\,\delta(\mathbf{v}+\mathbf{v}'-\mathbf{u}-\mathbf{u}')\int\delta(\mathbf{V}-\mathbf{u})d\sigma. \qquad (12)$$

We change the function $\mathbf{V}$ in the integrand expression according to Eq. (6) and we use the first identity of Eq. (3),

$$\int\delta(\mathbf{V}-\mathbf{u})d\sigma=2\int\delta(\mathbf{u}-\mathbf{u}'-\mathbf{n}\,|\,\mathbf{v}-\mathbf{v}'|)d\sigma.$$

Decomposing three-dimensional δ-function depending on the vector argument $(\mathbf{u}-\mathbf{u}')$ on the product

$$|\,\mathbf{v}-\mathbf{v}'|^{-2}\delta(|\,\mathbf{v}-\mathbf{v}'|-|\,\mathbf{u}-\mathbf{u}'|) \times\delta((\mathbf{u}-\mathbf{u}')/|\,\mathbf{v}-\mathbf{v}'|-\mathbf{n})$$

and after that transforming the first δ-function with the account of equality $\mathbf{v}+\mathbf{v}'=\mathbf{u}+\mathbf{u}'$ which is carried out in the integration domain, we find

$$\int\delta(\mathbf{V}-\mathbf{u})d\sigma=2\,|\,\mathbf{v}-\mathbf{v}'|^{-1} \times\delta(\mathbf{v}^2+\mathbf{v}'^2-\mathbf{u}^2-\mathbf{u}'^2) \times\int\delta((\mathbf{u}-\mathbf{u}')/|\,\mathbf{u}-\mathbf{u}'|-\mathbf{n})d\sigma. \qquad (13)$$

Since the function $\mathbf{n}(\mathbf{v},\mathbf{v}',\mathbf{r})$ realizes one-to-one correspondence between the plane of the vector $\mathbf{r}$ changing and the unit sphere, the last integral is equal to the Jacobean of corresponding map

$$\int\delta((\mathbf{u}-\mathbf{u}')/|\,\mathbf{u}-\mathbf{u}'|-\mathbf{n})d\sigma =|\,(D(\mathbf{n})/D(\mathbf{r}))_{\mathbf{n}=\mathbf{m}}\,|^{-1}$$

where $\mathbf{m}=(\mathbf{u}-\mathbf{u}')/|\,\mathbf{u}-\mathbf{u}'|$. Together with Eq. (12) and Eq. (13), it gives the final formula

$$w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')=4\rho\delta(\mathbf{v}+\mathbf{v}'-\mathbf{u}-\mathbf{u}') \times\delta(\mathbf{v}^2+\mathbf{v}'^2-\mathbf{u}^2-\mathbf{u}'^2)\,|\,(D(\mathbf{n})/D(\mathbf{r}))_{\mathbf{n}=\mathbf{m}}\,|^{-1}.$$

3. M.KAC PROBLEM

Solutions which possess the following properties:

a) $f(\mathbf{v},t)\ge 0$ at all $t\ge 0$, if $f(\mathbf{v},0)\ge 0$;

b) $\int\limits_{\mathbf{R}^3} f(\mathbf{v},t)d\mathbf{v}$ does not depend from t,

we name as *the probabilistic ones*.

In connection with the existence of probabilistic solutions of the equation (1), M. Kac has set the problem [6] of the construction of such a random process $\langle\tilde{\mathbf{v}}(t);t\ge 0\rangle$ for which the function $f(\mathbf{v},t)$ is the first order marginal distribution density,

$$f(\mathbf{v},t)=\frac{d}{d\omega(\mathbf{v})}\Pr\{\tilde{\mathbf{v}}(t)\in\omega(\mathbf{v})\}. \qquad (14)$$

This function should completely define the probability distribution of the process in its sample space $(\mathbf{R}^3)^{\mathbf{R}_+}, t\in\mathbf{R}_+$, i.e. the function should define all set of marginal distribution densities $f_n(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n)$ of order $n\in\mathbf{N}$, which are consistent, i.e.

$$\int f_n(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n)d\mathbf{v}_n = = f_{n-1}(\mathbf{v}_1,t_1;...;\mathbf{v}_{n-1},t_{n-1}).$$

They generate the process probability distribution according to the Kolmogorov theorem. It means that $f(\mathbf{v},t)=f_1(\mathbf{v},t)$ and all densities of higher order are some functionals $f_n(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n)=f_n[f(\mathbf{v},t)]$.

Constructive building of such a random process on $[0,\infty)$ for which the probability distribution of initial values satisfies $d\Pr\{\tilde{\mathbf{v}}(0)\in\omega(\mathbf{v})\}/d\omega(\mathbf{v})=f(\mathbf{v},0)$ solves the Cauchy problem on $\mathbf{R}_+$ for the equation (1) with initial data $f(\mathbf{v},0)$. Below, we offer a building of this process in the case of finiteness of cross-section of particles scattering.

4. THE WEAK SOLVABILITY

We construct the process $\langle\tilde{\mathbf{v}}(t);t\ge 0\rangle$ as the weak limit in the space $(\mathbf{R}^3)^{\mathbf{R}_+}$ of the sequence $\langle\tilde{\mathbf{v}}^{(N)}(t);t\ge 0\rangle, N=1,2...$ of random processes. It means that all marginal densities $f_n^{(N)}(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n), n\in\mathbf{N}$ converge weakly at $N\to\infty$ to the infinite set of densities $f_n(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n), n\in\mathbf{N}$ consistent with each other, i.e. sequences of corresponding characteristic functions converge

$$\int \exp\left[i\sum_{j=1}^{n}(\mathbf{k}_j,\mathbf{v}_j)\right] f_n^{(N)}(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n)d\mathbf{v}_1...d\mathbf{v}_n$$
$$\to$$
$$\int \exp\left[i\sum_{j=1}^{n}(\mathbf{k}_j,\mathbf{v}_j)\right] f_n(\mathbf{v}_1,t_1;...;\mathbf{v}_n,t_n)d\mathbf{v}_1...d\mathbf{v}_n .$$

In particular, the weak convergence $f^{(N)}(\mathbf{v},t)\to f(\mathbf{v},t)$ to the Boltzmann equation solution takes place.

We use the Prokhorov criterion of weak compactness measure sequence for the proof of the weak convergence of random processes sequence. It is formulated as follows.

If the sequence of measures $\mu^{(N)}, N=1,2...$ *on a metrical separable space* $\mathbf{L}$ *(being not necessarily complete) such that for this sequence there exist a compact function* Ψ *on space* $\mathbf{L}$ *having uniformly bounded average values on measures* $\mu^{(N)}$, $\int_{\mathbf{L}}\Psi(x)d\mu^{(N)}(x)<const$, *so this sequence is weakly compact.*

We choose the space of all piecewise constant functions $\tilde{\mathbf{v}}(t), t\in[0,T]$ with vector values as the sample space $\mathbf{L}$ of all processes.

Each random trajectory $\tilde{\mathbf{v}}(t)$ is characterized by the sequence of pairs $\left\langle\left\langle\tilde{\mathbf{a}}_n,\tilde{t}_n\right\rangle; n\in\mathbf{N}\right\rangle$, where $\tilde{\mathbf{a}}_n=\tilde{\mathbf{v}}_n-\tilde{\mathbf{v}}_{n-1}, n=1,2...$ are random function jumps which occur in random time points $\tilde{t}_n, n=1,2...$. We put

$$p_1[\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)]=|\tilde{\mathbf{v}}'(0)-\tilde{\mathbf{v}}''(0)|+ \\ +\sum_n|\tilde{\mathbf{a}}_n'-\tilde{\mathbf{a}}_n''| \tag{15}$$

where the summation is carried out on all jumps of both functions being in the one-to-one correspondence according to their order on $[0,T]$. If one of the functions, for example, $\tilde{\mathbf{v}}''(t)$ has greater number of jumps in comparison with the other function, so it is necessary formally to put that the last has zero jumps in those points $\tilde{t}_n''$ where jumps $\tilde{\mathbf{a}}_n''$ do not have corresponding jumps $\tilde{\mathbf{a}}_n'$.

The functional $p_1[\cdot,\cdot]$ is the *deviation* on $\mathbf{L}$ between two functions $\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)$. The functional $p_2[\cdot,\cdot]$ defined by the formula

$$p_2[\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)]=\sum_n|\tilde{t}_n'-\tilde{t}_n''| \tag{16}$$

is the deviation too. Here, points $\tilde{t}_n',\tilde{t}_n''$ are set in one-to-one correspondence according to the one-to-one correspondence between jumps of the functions $\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)$. It is easily to verify that the functional

$$\mathrm{dist}[\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)]=p_1[\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)] \\ +p_2[\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)] \tag{17}$$

is the distance in the space $\mathbf{L}$ between functions $\tilde{\mathbf{v}}'(t),\tilde{\mathbf{v}}''(t)$. The space $\mathbf{L}$ is separable relative to this distance. Besides, it is established that those function sets for which the jump number does not surpass anything number $m\in\mathbf{N}$, are precompact relative to the topology connected with the distance $\mathrm{dist}[\cdot,\cdot]$. Therefore, for the fixed interval $[0,T]$, we construct the function $\Psi[\cdot]$ on $\mathbf{L}$ putting that its values for each function $\tilde{\mathbf{v}}\in\mathbf{L}$ are equal $|\langle\tilde{t}_n;n\in\mathbf{N}\rangle\cap[0,T)|$ to the random number of its jumps on $[0,T]$. Then, we get that sets $\{\tilde{\mathbf{v}}\in\mathbf{L}:\Psi[\tilde{\mathbf{v}}]\le M\}$ are compact for any $M>0$ and, hence, the function $\Psi[\cdot]$ is compact.

5. THE BOLTZMANN RANDOM PROCESS

We construct the random process $\left\langle\tilde{\mathbf{v}}(t);t\in\mathbf{R}_+\right\rangle$ which solves the Kac problem. It is named the Boltzmann process. At the assumption of the cross-section σ finiteness, we introduce the functional

$$q[\mathbf{v};f]=\int w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')f(\mathbf{v}',t)d\mathbf{u}d\mathbf{u}'d\mathbf{v}' \\ =\rho\sigma\int|\mathbf{v}-\mathbf{v}'|f(\mathbf{v}')d\mathbf{v}' \tag{18}$$

and an the kernel

$$Q(\mathbf{v};\mathbf{u},\mathbf{u}')=\int_{\mathbf{R}^3}w(\mathbf{v},\mathbf{v}';\mathbf{u},\mathbf{u}')d\mathbf{v}' . \tag{19}$$

Due to Eq. (11), it takes place

$$\int_{\mathbf{R}^3}Q(\mathbf{v};\mathbf{u},\mathbf{u}')d\mathbf{v}=\rho\sigma|\mathbf{u}-\mathbf{u}'| . \tag{20}$$

In terms of introduced values, equation (8) is represented in the form of

$$\dot{f}(\mathbf{v},t)=\int Q(\mathbf{v};\mathbf{u},\mathbf{u}')f(\mathbf{u},t)f(\mathbf{u}',t)d\mathbf{u}d\mathbf{u}' \\ -q[\mathbf{v};f]f(\mathbf{v},t). \tag{21}$$

Let us require that the first order marginal densities $f^{(N)}(\mathbf{v},t), N=1,2...$ of the processes $\left\langle\tilde{\mathbf{v}}^{(N)}(t);t\ge 0\right\rangle, N=1,2...$, satisfy to the identity

$$f^{(N)}(\mathbf{v},t+\Delta)=f^{(N)}(\mathbf{v},t)\exp(-q[\mathbf{v};f^{(N)}]\Delta) \\ +(1-\exp(-q[\mathbf{v};f^{(N)}]\Delta))\times \\ \times\int Q(\mathbf{v};\mathbf{u},\mathbf{u}')f^{(N)}(\mathbf{u},t)f^{(N)}(\mathbf{u}',t)d\mathbf{u}d\mathbf{u}' \tag{22}$$

where $\Delta=T/N$ and $N\to\infty$.

To achieve this aim, we construct for any $N\in\mathbf{N}$ the random process $\left\langle\tilde{\mathbf{v}}^{(N)}(t);t\in\mathbf{R}_+\right\rangle$. Trajectories of the process at $N\in\mathbf{N}$ are defined by the formula

$$\tilde{\mathbf{v}}^{(N)}(t)=\{\tilde{\mathbf{v}}_l^{(N)};t\in[(l-1)T/N,lT/N),l\in\mathbf{N}\}$$

where the random sequence $\left\langle\tilde{\mathbf{v}}_l^{(N)};l\in\mathbf{N}\right\rangle$ is the *nonlinear Markov chain* with the state space $\mathbf{R}^3$ which is defined by the distribution density

$$f_l(\mathbf{v})\equiv\frac{d}{d\omega(\mathbf{v})}\Pr\{\tilde{\mathbf{v}}_l\in\omega(\mathbf{v})\}$$

of random variable $\tilde{\mathbf{v}}_l^{(N)}$ at the moment l. This density is changed during one evolution step by the following way

$$f_{l+1}^{(N)}(\mathbf{v}) = f_l^{(N)}(\mathbf{v})\exp(-q[\mathbf{v}; f_l^{(N)}]\Delta) + (1-\exp(-q[\mathbf{v}; f_l^{(N)}]\Delta))q^{-1}[\mathbf{v}; f_l^{(N)}]\times \times\int Q(\mathbf{v};\mathbf{u},\mathbf{u}')f_l^{(N)}(\mathbf{u})f_l^{(N)}(\mathbf{u}')d\mathbf{u}d\mathbf{u}'. \tag{23}$$

The changing points of the process $\left\langle\tilde{\mathbf{v}}^{(N)}(t); t\in\mathbf{R}_+\right\rangle$ belong to the set $\{lT/N; l\in\mathbf{N}\}$.

Let $\Psi[\tilde{\mathbf{v}}(t)]$ be the number of the changing points of the function in $\mathbf{L}$. The average number of changing points of the chain trajectory $\tilde{\mathbf{v}}_l, l = 0,1,...,n$ is equal to

$$\sum_{l=1}^{n-1}(1-\exp(-q[\mathbf{v}; f_l^{(N)}]))q^{-1}[\mathbf{v}; f_l^{(N)}]\times \times\int\left(\int Q(\mathbf{v};\mathbf{u},\mathbf{u}')f_n^{(N)}(\mathbf{u}')d\mathbf{u}'\right)f_l^{(N)}(\mathbf{u})d\mathbf{u}d\mathbf{v}.$$

Since all random points of the chain are in one-to-one correspondence with changing points of the process $\left\langle\tilde{\mathbf{v}}^{(N)}(t); t\in\mathbf{R}_+\right\rangle$, the average value $\left\langle\Psi[\tilde{\mathbf{v}}^{(N)}(t)]\right\rangle$ is defined by above expression too. Then, at $\Delta\to 0$, we have

$$\left\langle\Psi[\tilde{\mathbf{v}}^{(N)}(t)]\right\rangle \propto \int_0^t\left(\int Q(\mathbf{v};\mathbf{u},\mathbf{u}')f^{(N)}(\mathbf{u}',t)f^{(N)}(\mathbf{u},s)d\mathbf{u}'d\mathbf{u}\right)d\mathbf{v}ds.$$

Consequently, this average value is bounded,

$$\left\langle\Psi[\tilde{\mathbf{v}}^{(N)}(t)]\right\rangle \propto \rho\sigma\times \int_0^t\left(\int|\mathbf{u}-\mathbf{u}'|f^{(N)}(\mathbf{u}',t)f^{(N)}(\mathbf{u},s)d\mathbf{u}'d\mathbf{u}\right)ds \le, \le 2\rho\sigma t\langle\mathbf{u}^2\rangle^{1/2} \tag{24}$$

where the squared average $\langle\mathbf{u}^2\rangle^{1/2}$ of the velocity $\tilde{\mathbf{v}}^{(N)}(t)$ does not depend on t, since the Boltzmann equation conserves the kinetic energy. Thus, due to the inequality (24), one may apply the Prokhorov criterion for measures connected with processes $\left\langle\tilde{\mathbf{v}}^{(N)}(t); t\in\mathbf{R}_+\right\rangle$. On the basis of this criterion, these processes weakly converge to a process $\langle\tilde{\mathbf{v}}(t); t\in\mathbf{R}_+\rangle$. In this case, all marginal distributions of the processes $\left\langle\tilde{\mathbf{v}}^{(N)}(t); t\in\mathbf{R}_+\right\rangle$, weakly converge to corresponding marginal distributions of the process $\langle\tilde{\mathbf{v}}(t); t\in\mathbf{R}_+\rangle$ at $\Delta\to 0$. Then, on the basis of Eq. (22),

$$\frac{f^{(N)}(\mathbf{v},t+\Delta)-f^{(N)}(\mathbf{v},t)}{1-e^{-q[\mathbf{v};f^{(N)}(t)]\Delta}} = q^{-1}[\mathbf{v}; f^{(N)}(t)] \times\int Q(\mathbf{v};\mathbf{v}_1,\mathbf{v}_2)f^{(N)}(\mathbf{v}_1,t)f^{(N)}(\mathbf{v}_2,t)d\mathbf{v}_1d\mathbf{v}_2 - f^{(N)}(\mathbf{v},t).$$

Therefore, the one-point probability density $f_1(\mathbf{v},t)$ of the limit process $\langle\tilde{\mathbf{v}}(t); t\in\mathbf{R}_+\rangle$ satisfy to Eq. (1), i.e. the existence of the random process $\langle\tilde{\mathbf{v}}(t); t\in\mathbf{R}_+\rangle$ leads to the existence of the weak solution of the equation (1).

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ПОСТРОЕНИЕ ВЕРОЯТНОСТНЫХ РЕШЕНИЙ УРАВНЕНИЯ БОЛЬЦМАНА

Ю.П. Вирченко, Т.В. Карабутова

Предлагается метод доказательства разрешимости задачи Коши для кинетического уравнения Больцмана с пространственно однородными начальными данными в случае конечности сечения рассеяния сталкивающихся частиц. Метод основан на построении вспомогательного векторнозначного случайного процесса, такого, что функция распределения по скоростям частиц, удовлетворяющая уравнению Больцмана, является его частным распределением вероятностей первого порядка.

ПОБУДОВА ЙМОВІРНОСТНИХ РОЗВ'ЯЗКІВ РІВНЯННЯ БОЛЬЦМАНА

Ю.П. Вірченко, Т.В. Карабутова

Пропонується метод доведення розв'язання проблеми Коші для кінетичного рівняння Больцмана з просторово однорідною початковою функцією у випадку скінченності перерізу розсіяння частинок, що зіткаються. Метод оснований на побудові векторнозначного випадкового процесу такого, що функція розподілу за швидкостями частинок, яка задовольняє рівнянню Больцмана, є його частинним розподілом ймовірностей першого порядку.