NONLINEAR TRANSITION RADIATION OF THE MODULATED ELECTRON BEAM FROM THE SHARP VACUUM-PLASMA BORDER

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The transition radiation of the radially unbounded modulated electron beam on the sharp vacuum-isotropic plasma boundary is studied up to quadratic nonlinearity approximation. Solution for the second harmonic radioemission and stationary forced fields is considered in details.

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1. INTRODUCTION

It has already known that the effectiveness of the modulated electron beam transition radiation increases with the beam current [1]. But large currents result to appearance of the nonlinear effects. Their threshold might be extremely low in plasmas. The "slow" nonlinearity was studied in [2-3]. It takes place in the weakly inhomogeneous plasma due to the electrons' pressing-out from the local plasma resonance region by the high-frequency electric field of the modulated electron beam. In this article the "fast" nonlinear effects are studied for the simplest model including modulated electron beam of the infinite radius passing through the sharp vacuum-isotropic plasma border.

2. MODEL DESCRIPTION, INITIAL EQUATIONS AND METHOD OF SOLUTION

The charge compensated and obliquely modulated electron beam of the infinite radius produces the current wave

$$j_{\exp} = j_m e_z \cos(\omega t - \chi r),\ \chi = \{0; \chi_\perp; \chi_\parallel\},\ \ \chi_\parallel = \frac{\omega}{v_0},$$

where v_0 is the beam velocity. This beam normally passes through the plain border between vacuum and cold homogeneous isotropic plasma.

Approximation of the given beam current is used. Plasma perturbations caused by the beam are described by the Maxwell's equations and the motion equation for the plasma electrons:

$$\begin{cases} rotrot\vec{A} = \dfrac{4\pi}{c}\vec{j}_{\exp} - \dfrac{4\pi e}{c} n\vec{v} - \dfrac{1}{c^2}\dfrac{\partial^2 A}{\partial t^2}; \\ \dfrac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} = \dfrac{e}{mc}\dfrac{\partial \vec{A}}{\partial t}; \\ div\left(-\dfrac{1}{c}\dfrac{\partial A}{\partial t}\right) = -4\pi e(n - n_0), \end{cases} \quad (1)$$

where vector-potential is used to describe the electromagnetic field (calibration condition is taken in the form $\varphi=0$). Non-linearity is caused by the quadratic terms in the equations of motion and continuity for the plasma electrons.

Magnitude of the beam current is considered to be a small parameter. Than one can find the vector-potential in plasma as a sum of series on the beam current magnitude:

$$A = A_1 + A_2 + \ldots \quad (2)$$

(here index indicates the order of smallness).

The boundary conditions for finding out the transition radiation magnitude should be written taking into account the surface charges on the vacuum-plasma bound.

3. OBTAINING OF THE RECURRENT EQUATIONS FOR ELECTROMAGNETIC FIELD

By substituting (2) into (1) and picking out the terms of the same order of smallness one can obtain the set of recurrent equations. The first order perturbations are described by the set:

$$\begin{cases} n_1 = \dfrac{1}{4\pi ec}\dfrac{\partial}{\partial t} div\vec{A}_1; \\ \vec{v}_1 = \dfrac{eA_1}{mc}; \\ rotrot\vec{A}_1 + \dfrac{4\pi e^2 n_0}{mc^2}\vec{A}_1 + \dfrac{1}{c^2}\dfrac{\partial^2 \vec{A}_1}{\partial t^2} = \dfrac{4\pi}{c}\vec{j}_{\exp}. \end{cases} \quad (3)$$

The next order perturbations are described by the equations:

$$\begin{cases} n_i = \dfrac{1}{4\pi ec}\dfrac{\partial}{\partial t} div\vec{A}_i \\ \vec{v}_i = \dfrac{eA_i}{mc} - \sum\limits_{k=1}^{i-1}\int dt(\vec{v}_k\nabla)\vec{v}_{i-k} \\ rotrot\vec{A}_i + \kappa_p^2\vec{A}_i + \dfrac{1}{c^2}\dfrac{\partial^2 \vec{A}_i}{\partial t^2} = \\ = -\dfrac{4\pi e}{c}\sum\limits_{k=1}^{i-1} n_k\vec{v}_{i-k} + \dfrac{4\pi e n_0}{c}\sum\limits_{k=1}^{i-1}\int dt(\vec{v}_k\nabla)\vec{v}_{i-k} \end{cases} \quad (4)$$

Here the first equation includes a beam current in the right side. Others have in the right sides the nonlinear terms caused by the magnitudes of the lower orders of smallness.

To obtain the transition radiation it is necessary to take into account both the existing solutions of the equations (3)-(4) (that correspond to the eigen field of the modulated electron beam) and the solutions of the appropriate homogeneous equation (that describe the waves moving from the border into plasma). In vacuum the inhomogeneous wave equation remains linear.

Boundary conditions at the vacuum-plasma border can be presented in the form:

$H_1=H_2$; $E_{\tau 1}=E_{\tau 2}$; $E_{n1}-E_{n2}=4\pi\sigma$,

where σ is the surface charge density.

4. THE FIRST APPROXIMATION SOLUTION

Magnitudes of the transition radiation into plasma and vacuum calculated in the first approximation after the beam current magnitude are in good agreement with the well-known results [4]:

$$A_p = -\frac{4\pi\chi_\perp j_m k_p^2 k_{\parallel p}[(\chi_\parallel + k_{\parallel v})(k_p^2 - k_0^2) + (\chi^2 - k_0^2)\chi_\parallel]}{c(\chi^2 + k_p^2 - k_0^2)(k_p^2 - k_0^2)(\chi^2 - k_0^2)[(k_{\parallel p} + k_{\parallel v})k_0^2 - k_p^2 k_{\parallel v}]}$$

$$A_v = -\frac{4\pi\chi_\perp j_m k_p^2 k_{\parallel v}[(k_{\parallel p} - \chi_\parallel)k_0^2 + (\chi^2 + k_p^2 - k_0^2)\chi_\parallel]}{c(\chi^2 + k_p^2 - k_0^2)(k_p^2 - k_0^2)(\chi^2 - k_0^2)[(k_{\parallel p} + k_{\parallel v})k_0^2 - k_p^2 k_{\parallel v}]},$$

(5)

(5)

where $k_0=\omega/c$, $k_p=\omega_p/c=(e/c)(4\pi n_0/m)^{1/2}$; n_0 – average concentration of plasma electrons, $\chi^2=\chi_{//}^2+\chi_\perp^2$, $k_{//p}=(k_0^2-k_p^2-\chi_\perp^2)^{1/2}$ and $k_{//v}=(k_0^2-\chi_\perp^2)^{1/2}$ – longitudinal wave numbers for the radiation into plasma and vacuum correspondingly.

5. THE SECOND APPROXIMATION SOLUTION

Let us find out transition radioemission in the second approximation. Using the relation

$$\vec{E} = -\frac{1}{c}\frac{\partial A}{\partial t},$$

the third equation of the set (4) can be transformed into:

$$rot\,rot\,\vec{E}_2 + \frac{1}{c^2}\frac{\partial^2 E_2}{\partial t^2} + k_p^2\vec{E}_2 = \frac{4\pi e}{c^2}\frac{\partial}{\partial t}(n_1\vec{v}_1) - \frac{4\pi e n_0}{c^2}(\vec{v}_1\nabla)\vec{v}_1. \qquad (6)$$

There is a quadratic non-linearity in the right side of equation (6) that results to appearance of the second harmonic of the modulation frequency and the stationary field.

Components with the temporal dependence exp(i2ω t) correspond to the second harmonic of the eigen field of the beam and the sum mixed harmonic. Electromagnetic radiation of the frequency 2ω is emitted both into plasma and into vacuum.

The quadratic non-linearity produces also the spatially inhomogeneous stationary field corresponding to the difference mixed wave. It results to the appearance of the constant component of the surface charge density.

Due to the different time dependence electromagnetic fields of the second and zero harmonics can be considered separately.

6. WAVES AT THE SECOND HARMONIC

According to the boundary conditions one can write down the set of equations to obtain the magnitude of the second harmonic transition radiation into plasma and vacuum in the second approximation after the beam current magnitude:

$$\frac{k_0}{k_{\parallel v}}E'_v = -\frac{4k_0^2 - k_p^2}{4k_0 k'_{\parallel p}}E'_p - \frac{4\pi^2 e j_m^2\chi_\parallel}{c^4 m(k_p^2 - k_0^2)k_0}\times$$
$$\times\left[\frac{2\chi_\perp(k_p^2 + 2k_0^2)}{(4\chi^2 + k_p^2 - 4k_0^2)(\chi^2 + k_p^2 - k_0^2)} - \right.$$
$$-\frac{A_p^0[2k_0^2(k_0^2 + \chi_\perp^2)\chi_\parallel - k_p^2(k_p^2 + k_0^2)\chi_\parallel]}{k_{\parallel p}(\chi^2 + 2(\chi_\parallel k_{\parallel p} + \chi_\perp^2) - 3k_0^2)\chi_\parallel} -$$
$$\left. - \frac{A_p^0[k_{\parallel p}(k_p^2(k_0^2 - k_p^2) + 2k_0^2\chi_\parallel^2)]}{k_{\parallel p}(\chi^2 + 2(\chi_\parallel k_{\parallel p} + \chi_\perp^2) - 3k_0^2)\chi_\parallel}\right]; \qquad (7)$$

(7)

$$E'_v = E'_p - \frac{8\pi^2 e j_m^2\chi_\parallel}{c^4 m(k_p^2 - k_0^2)}\times$$
$$\times\left[\frac{\chi_\parallel\chi_\perp(k_p^2 + 2k_0^2)(4\chi^2 + 5k_p^2 - 8k_0^2)}{(4\chi^2 + k_p^2 - 4k_0^2)(k_p^2 - k_0^2)(\chi^2 + k_p^2 - k_0^2)(k_p^2 - 4k_0^2)}\right.$$
$$-\frac{A_p^0}{k_{\parallel p}(\chi^2 + 2(\chi_\parallel k_{\parallel p} + \chi_\perp^2) - 3k_0^2)}\times$$
$$\times\left(-2k_0^2 k_{\parallel p} + \frac{2\chi_\perp^2(2k_0^2(\chi_\parallel - k_{\parallel p}) + k_p^2(\chi_\parallel + k_{\parallel p}))}{k_p^2 - 4k_0^2} - \right.$$
$$-\frac{k_p^2(k_0^2 - k_p^2)(4\chi_\perp^2 + k_p^2 - 4k_0^2)}{(k_p^2 - 4k_0^2)\chi_\parallel} -$$
$$\left.\left. - \frac{2\chi_\perp^2(k_{\parallel p} - \chi_\parallel)k_p^2(4\chi_\perp^2 + k_p^2 - 4k_0^2)}{(\chi^2 + k_p^2 - k_0^2)(k_p^2 - 4k_0^2)}\right)\right] \qquad (8)$$

where

$$k'_{\parallel p} = \sqrt{k_0^2 - k_p^2/4 - \chi_\perp^2}; \qquad A_p^0 = cA_p/4\pi j_m.$$

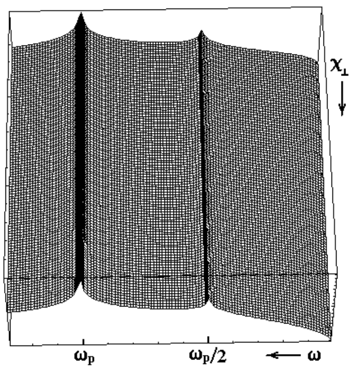
Solution of the set (7)-(8) is too bulky so it is not presented here.

Dependencies of the forced waves' magnitudes excited in plasma due to the non-linearity on frequency and transversal wave number are shown on Fig. 1.

Maximums at $\omega=\omega_p$ and $\omega=\omega_p/2$ correspond to the plasma resonance at the first and second harmonics of the modulation frequency (the last maximum exists only in plasma). Another maximums correspond to the vanishing of the denominators

$4k_0^2(k'_{//p} + k_{//v}) - k_p^2 k_{//v}$ and $k_0^2(k_{//p} + k_{//v}) - k_p^2 k_{//v}$

in the expression for the transition radiation magnitudes.



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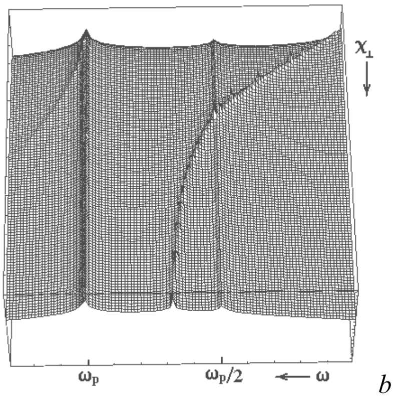
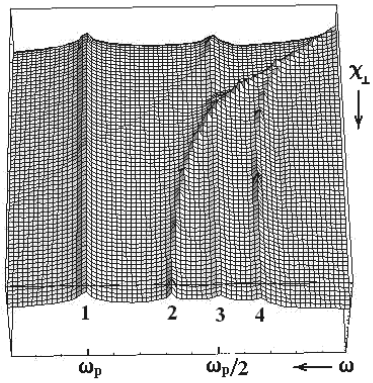


Fig. 1. Magnitude of the forced waves in plasma at frequency 2ω (a – second harmonic of the beam field; b – sum mixed harmonic of beam field and radiation at frequency ω) in dependence on frequency and transversal wave number

The dependencies of transition radiation magnitudes in plasma and vacuum on frequency and transversal wave number are shown on Fig.2.



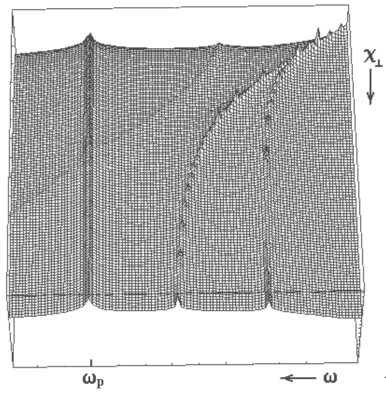


Fig. 2. Magnitude of the z-component of transition radiation into plasma (a) and into vacuum (b) at frequency 2ω dependent on the modulation frequency and transversal wave number

From comparison of fig.2 and fig.3 one can see that maximums 1-3 are caused by the resonant growth of the stimulated field of the beam. Maximum 4 corresponds to the resonant excitation of some quasi-eigen mode (see, e.g., [1]) of the vacuum-plasma boundary.

7. STATIONARY FIELD GENERATION

It was already noticed that the spatially inhomogeneous stationary field corresponding to the difference mixed wave appears in the system due to the quadratic non-linearity. The magnitude of this field can be presented in the form

$$E_z = \frac{2\pi e j_m \chi_\perp A_p(\chi_\parallel \chi_\perp^2 + (2k_{\parallel p} - \chi_\parallel)(\chi_\parallel^2 + k_p^2 - k_0^2))}{c^3 m(k_p^2 - k_0^2)k_{\parallel p}(\chi^2 + k_p^2 - k_0^2)}$$

Dependence of this magnitude upon the modulation frequency and transversal wave number is presented on fig. 3. Maximums of fig.3 are similar to the ones on fig. 2.

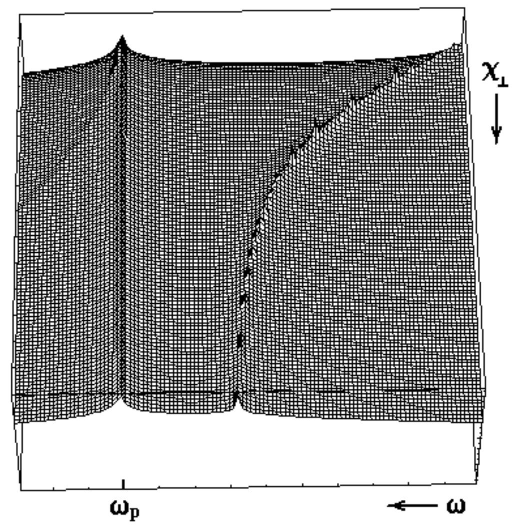


Fig. 3. Magnitude of the difference mixed harmonic (stationary in time field)

8. CONCLUSION

Nonlinear transition radiation from the obliquely modulated electron beam of the infinite radius is calculated in the second approximation after the current wave magnitude. It results to the appearance of the second harmonic and stationary field of the difference wave number (between the current wave and the electromagnetic wave).

The next approximations result to the appearance the corresponding harmonics in the radiation spectrum and some modification of the main harmonic magnitude.

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