THE WORLD OF CHAOS

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The present paper is devoted to discussion of origins of the modern paradigm of deterministic chaos: the transition from the older philosophy understanding chaos as a result of uncontrolled external effects to the theory based on internal mechanism of the chaos appearance in nonlinear systems. The report briefly presents the main steps of research in framework of new approach. Success of the new approach is demonstrated on two important dynamical systems – billiards and atomic nuclei.

PACS: 05.45.Gg

Currently one can consider as a rigorously established fact an existence of such dynamical systems with a small number of degrees of freedom ($N \geq 2$) for which under certain conditions classical motion could not be distinguished from random one [1,2]. Typical features of these systems are nonlinearity and absence of any external source of randomness. Thus, using such synonyms for the term "random" as "chaotic", "stochastic", "irregular", one can state that there are nonlinear deterministic systems, for which these notions express adequately internal fundamental properties that compromise and represent an important and interesting subject for investigation. Examples of chaotic motion for the last 30-40 years have been detected in every field of natural sciences.

What are mechanisms of generation of chaos in rigorous deterministic systems? To answer this question let us consider perhaps the most important example of dynamical laws, Newton laws. Newton's laws are deterministic because for any system the same initial conditions will always produce identically the same outcome. Term "the same initial conditions" requires more precise definition. One of the fundamental physics principles is that no real measurement is infinitely precise: uncertainty which is present in any real measurement arises from the fact that any imaginable measuring device — even if designed and used perfectly — can record its measurement only with a finite precision. This uncertainty can never be eliminated completely, even as a theoretical idea. The presence of uncertainty in any real measurement means that the initial conditions cannot be specified to infinite accuracy. The uncertainty present in the initial conditions of a system yields a corresponding uncertainty in the range of the prediction for any later time. It should be particularly emphasized that the uncertainty in the dynamical outcome does not arise from any randomness in the equations of motion — since they are completely deterministic — but rather from the lack of the infinite accuracy in the initial conditions.

Throughout most of the modern history of physics, it has been assumed that it is possible to shrink the uncertainty in the final dynamical prediction by measuring the initial conditions to greater and greater accuracy. However, this procedure loses every sense for the system, possessing property of dynamical instability. Dynamical instability refers to a special kind of behavior of dynamical systems which was discovered around the year 1900 by H. Poincaré. He noticed that certain astronomical systems did not seem to obey the rule that shrinking the initial conditions always shrank the final prediction in a corresponding way. For these types of systems, Poincaré showed that a very tiny imprecision in the initial conditions would grow in time at an enormous rate. Thus two nearly-indistinguishable sets of initial conditions for the same system would result in two final predictions which differed vastly from each other. In other words, Poincaré proved for these systems the only way to obtain predictions with any degree of accuracy lies in the fact that to entail the initial conditions to absolutely infinite precision. But it is impossible. The extreme sensitivity to initial conditions received the name dynamical instability, or simply chaos.

Many decades would pass before the dynamical chaos ideology was realized by the science community as a whole. According to the old ideology:

- Chaos is an attribute of the compound system.
- In any compound system it is possible to find out the elements of chaos.
- The useful information is contained in those few places in which chaos is absent.
- Physicist must look for nonchaos.

New ideology in principle changed the situation:

- Chaos is a universal inalienable property of the simple deterministic systems.
- Chaotic dynamics is most general way of evolution of arbitrary nonlinear system
- New interesting information is contained exactly in those regions of natural sciences, where chaos is present.
- Chaos is the major object of study.
- Physicist must look for chaos!

The Newtonian model is often depicted as a billiard game, in which the outcome unfolds mathematically from the initial conditions in predetermined fashion, like a movie that can be run forwards or backwards in time. The billiard game is a useful analogy, because on the microscopic level, the motion of molecules can be compared to the collisions of the balls on the billiard table, with the same dynamical laws invoked in both cases.

Billiards conception is of great importance in physics. A lot of physical processes are described in billiard terms. Billiards also play an important part in the forming of statistical physics basic ideas. So let us pay detailed attention at this conception. From the physical point of view, billiard conception, as a matter of fact, has dual nature or origin. This duality is very deep and is connected with two most important physical objects — particle and wave. Really, according to one conception, billiard can be regarded as a region limited by a boundary. Inside the region, the particle moves freely and is elastically reflected from the boundary. In this conception boundary coincides with singular potential energy, the potential barrier of which is situated on the billiard boundary. The other conception is based on beams spreading in the region with reflection from the boundaries; according to the law, falling angle equals to reflection angle. At first sight, both conceptions look like absolutely equal. The dual nature of the billiards, described above, is reflected in the approaches to billiard dynamics description and, in particular, in case of phase space introduction.

In case of regarding the billiard as a particle inside a singular barrier, the traditional idea of billiard condition arises. For example, the particle state is defined by its position $\vec{x}$ and momentum $\vec{p}$. Now let us discuss phase space, which naturally follows from beams conception. The beam state, which uniquely defines its evolution, is, in its turn, defined by the beam's segment with the direction indicated. In case of boundary parameterization by the parameter $0 \le s \le 1$, the coordinates of segment are $\left(s_1, s_2\right)$ — where s_1 is the beginning of the segment on the boundary, and s_2 is segment's second end coordinate on the billiard boundary. Let us notice that both coordinates are equal by their geometric meaning. This important and deep difference is apparent in the structure of phase space. Such approach to billiards is proposed in the works [3–5]. In a way, these approaches correlate as Hamiltonian and Lagrangean formalisms in classic mechanics.

In the geometric approach some problems look more natural. For example, it is there where universal type of billiard reflections with interesting features appears [4], a close connection with projective geometry is found [3], new characteristics appear, such as invariant billiard boundary attendance distribution function [5], which satisfies the non-linear analogue of Frobenius–Perron equation. In such approach, an important progress in theory of normal forms in billiard periodic orbits [7], as well as in topological billiards classification is achieved. Besides, other criteria types of chaos arising in billiards appear [6]. When developing this approach, an interesting class of chaotic billiards, different from Sinai billiards [8] and Bunimovich ones [9] was found — polymorphous billiards [10]. Such billiards can prove useful models for studying strongly deformed states of compact drops. One can mention new spectrometric uniformities of light-gathering in scintillation detectors as an important example of physical application for billiard, theoretically found and experimentally proved [11].

Another example of billiards, genetically connected with wave conception, can be composite billiard [12]. Among the examples of systems leading to such billiard type, we can mention scintillation detector, consisting of two materials with different refraction coefficients. Such a physical system, naturally, leads to a generalized billiard model, in which two new elements appear. They are beams "multiplication" at reflection and refraction of a beam on the media boundary and the law of beams refraction, which is complementary to the mirror reflection law. Both these elements are connected with the appearance of transparent media boundaries, ordinary billiard boundaries preserved. So, by composite billiard we shall mean the billiard inside which there are transparent boundaries of media.

Such billiards have comparatively complicated phase spaces and dynamics, compared to ordinary billiards. In chaotization mechanisms in them, proceeding from the effective boundary conception, different from ordinary billiards chaotization variants are admitted [12]. In a sense, in such billiards even a different type of deterministic chaos arises, connected not with exponential sensitivity to initial condition, but with the appearance of deterministically chaotic beams laws of motion.

Conception of dynamic chaos is universal and may be realized on all spatial scales, from cosmological to subnuclear. By reason of the richness of experimental data and sufficient precision of the theory, the nuclear dynamics provides useful realistic model for studying classical chaos and quantum manifestations of the classical stochasticity.

Conception of chaos has been introduced in the nuclear theory within the last twenty years. This conception brought birth to the new notion in the nuclear structure, nuclear reactions, could resolve a sequence of the very old contradictions in the nuclear theory. A radically new universal approach to the problem of statistical properties of the energy spectra was developed on the basis of the general nonlinear theory of dynamical systems. Considerable advances have been made in the area of concrete nuclear effects. Finally, straightforward observations of the chaotic regimes in the course of simulations of the heavy-ions reactions verify in favor of the general considerations.

In our researches of collective nuclear dynamics description of surface quadruple oscillations held a central position [13]. This investigations include a complete description of classical dynamics generated by the Hamiltonian of quadruple oscillations along with identifications of those peculiarities of quantum dynamics which can be interpreted as quantum manifestations of classical stochasticity. We have pointed up an intimate connection between dynamical features and geometry of the potential energy surface. Interpretation of negative curvature of the potential energy surface as the source of the local instability allows to correctly predict the critical energy of the transition to chaos for one-well potentials.

Particular attention has been given to the investigation of classical dynamics in the parameter region corresponding to the potentials with a few local minima. Researches of these potentials would be treated as one of the indispensable steps on the way to transition from description of the model systems to direct consideration of much more realistic systems.

As was shown, one of the main peculiarities of the many-well Hamiltonians is the existence of the mixed state: realization of diverse dynamical regimes (regular or chaotic) at one and the same energy in different local minima [14]. We proposed a new approach to investigation of quantum manifestations of classical stochasticity in wave functions structure, which can be realized in potentials with two and more local minima [15]. The main advantage of the proposed approach is the possibility to detect quantum manifestations of classical stochasticity in comparison not different wave functions, but different parts of the same wave function. Efficiency of the approach is demonstrated for two potentials: surface quadruple oscillations and lower umbilic catastrophe D_5 [16].

We proved that the type of classical motion is correlated with the structure of the stationary wave functions of highly excited states in the regularity-chaos-regularity transition. Correlations were found both in the coordinate space (the lattice of nodal curves and the distribution of the probability density) and in the Hilbert space associated with the integrable part of Hamiltonian (the distribution of the wave functions in the oscillator basis and the entropy of individual eigenstates). Calculations with the scaled Planck constant, that make it possible to obtain wave functions with equal quantum numbers and energies corresponding to different types of classical motion, enabled us to separate unambiguously correlation effects in the structure of wave functions.

The Hamiltonian of quadruple oscillations was used as an example to study the shell structure destruction induced by the increase of nonintegrable perturbation which models residual nucleon-nucleon interaction [17]. In the vicinity of the classical critical energy there were observed multiple quasicrossings of the energy levels, violation of the quasiperiodical energy dependence of the entropy, and increase of the average value fluctuations of the operators used to classify the eigenstates of the integrable problem.

Recently Zaslavsky and Edelman [18] considered a model of a billiard-type system, which consists of two chambers connected through a hole. One chamber has a circle-shaped scatterer inside, and the other one has a Cassini oval with a concave border. As was shown, the corresponding distribution function does not reach equilibrium even during the anomalously large time. We want to note, that the mixed state, at energy a little exceeding the saddle, can serve as a more realistic model for study of anomalous kinetics.

The so-called complex systems have attracted considerable attention at the last time. This is a wide class of systems that even includes some biological objects. The very different systems can belong to this class, if they exhibit the following common features: (i) a complex system is composed of several interacting components; (ii) its phase space contains regions of regular and chaotic dynamics; (iii) it exhibits a multiscale spatiotemporal behavior. Because of the presence of different component, it is expected that even a weak perturbation induces transition between them. In this case one can formulate a problem of control of the dynamics of the complex system.

This problem can be solved by application of standard methods of control of the chaotic systems. The general idea of these methods is to optimize the dynamics and to obtain the desired behavior by applying an intentional small perturbation to the system. As a result, chaotic oscillations are transformed into periodic ones. It is assumed, that the perturbation, being weak, does not change the topology of the phase space.

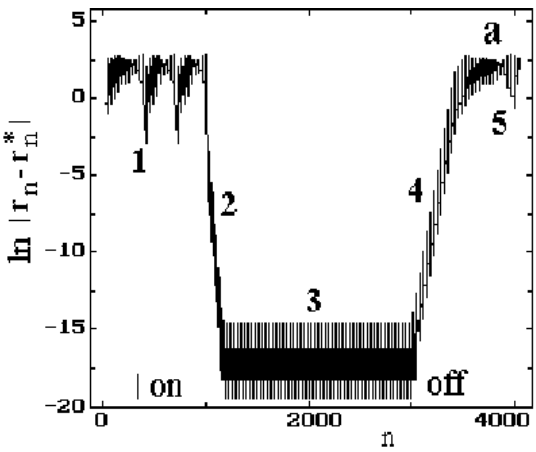
The above mentioned complexity is not necessarily an attribute of high-dimensional systems only. It may also be found in low-dimensional systems. In particular, the low-dimensional dynamics is realized in so-called reversible systems which exhibit typical complex behavior. The phase space of such systems usually contains elements of Hamiltonian systems (stability islands and resonances) as well as elements of dissipative systems (attractors and invariant attractive sets). The interplay between these elements gives rise to rather complicated dynamics as compared with the dynamics of either pure Hamiltonian or pure dissipative systems.

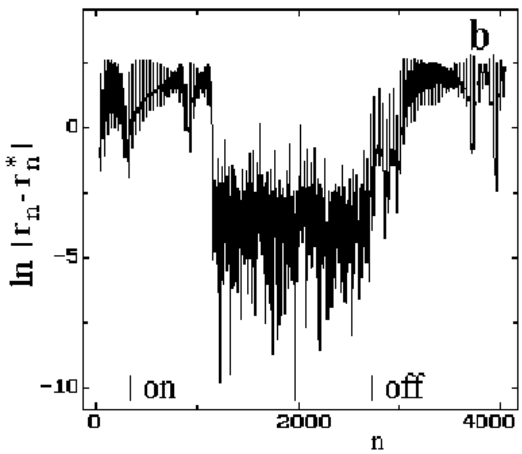
We studied [19,20] the possibility of controlling a high-period unstable orbit in two-dimensional reversible map

$$\mathbf{r}_{n+1} = \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} x_n + y_{n+1} \pmod 2 \\ y_n - \varepsilon(a - y_n)x_n \end{pmatrix}.$$

Dynamics of the complex system described by this map is essentially affected by the interaction between the attractor and the stability islands. This interaction gives rise to strong spatial and temporal inhomogeneity — a typical trajectory consists of regular parts (close to the attractor) and chaotic one far away from it. In the chaotic region the trajectory exhibits intermittency, i.e., a diffusive motion along y axis is suddenly interrupted by long jumps. When $y \to \pm\infty$ the diffusion becomes anomalously fast: the root-mean-square displacement grows exponentially with time.

Because of all these peculiarities, which are typical for any complex system, a direct application of the standard method to control high-period unstable orbit fails. Effective at every step of iteration, it nevertheless requires hard computational efforts to calculate stable and unstable directions at each point of the orbit. Alternative option — to apply control perturbation only on each period — is much easier from the point of view of calculations, but is unstable with respect to external noise. Our version of the standard method of control [20] is free from these difficulties. It was developed especially for systems with strongly nonhomogeneous phase space and is based on conception of local and global control. This conception is useful in situations where there are "dangerous" points on the target orbit, i.e., the points where the probability of breakdown of control is high. As a result, the dangerous points turn out to be much more sensitive to external noise than other points on the orbit. And only the dangerous points determine how effective the control is.





Stabilization of the coordinate $\mathbf{r}_n$ *when control is switched on: (a) without noise; (b) with Gaussian noise*

In Figure we show the behavior of the deviation $\mathbf{r}_n - \mathbf{r}_n^*$ ($\mathbf{r}_n^*$ is the target period — 34 orbit for the considered map) without noise (a) and with Gaussian noise (b). The concept of local and global control turns out to be sufficiently powerful and effective and makes it essential element of modern control methods.

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МИР ХАОСА

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Работа посвящена обсуждению природы современной парадигмы детерминированного хаоса: перехода от старой философии понимания хаоса как результата неконтролируемых воздействий на систему к теории внутреннего механизма появления хаоса в нелинейных системах. Доклад кратко знакомит с основными этапами исследований в рамках нового подхода. Достижения этого нового подхода демонстрируются на двух важных системах – бильярдах и атомных ядрах.

ВСЕСВІТ ХАОСУ

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Робота присвячена обговоренню природи сучасної парадигми детермінованого хаосу: переходу від старої філософії розуміння хаосу, як результату неконтрольованих впливів на систему до теорії внутрішнього механізму появи хаосу в нелінійних системах. Доповідь стисло знайомить з головними етапами досліджень у рамках нового підходу. Досягнення цього нового підходу демонструються на двох важливих системах – більярдах та атомних ядрах.