RADIATION REACTION AND RENORMALIZATION FOR A PHOTON-LIKE CHARGED PARTICLE

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A renormalization scheme which relies on energy-momentum and angular momentum balance equations is applied to the derivation of effective equation of motion for a massless point-like charge. Unlike the massive case, the rates of radiated energy-momentum and angular momentum tend to infinity whenever the massless source is accelerated. The external electromagnetic fields which do not change the velocity of the particle admit only its presence within the interaction area. The effective equation of motion is the equation on eigenvalues and eigenvectors of the electromagnetic tensor. The same solution arises in Rylov's model of magnetosphere of a rapidly rotating neutron star (pulsar).

PACS: 03.50.De, 11.10.Gh, 97.60.Gb

1. INTRODUCTION

In the paper [1] massless charged particles of spin one or larger are excluded in quantum electrodynamics by the argument that masslessness, Lorentz invariance, and electromagnetic coupling, are mutually incompatible. Roughly speaking, the interaction with an external electromagnetic field drastically changes incoming massless particle state, so that outgoing state does not describe a particle without rest mass. Further in Ref.[2] the existence of massless charges is forbidden in general by the condition that the energy of such particles in the electromagnetic field has no lower bound. In the present paper we consider the problem of reality of a massless charge within the realm of classical field theory.

Since the concept of a "zero-mass interacting particle" is quite different in quantum and classical theories, it would be more appropriate to obtain the equation of motion as a limiting case of the well-known Lorentz-Dirac equation [3]. (It defines the motion of point-like charge with rest mass m under the influence of an external force as well as its own electromagnetic field, for a modern review see [4,5].) In Ref.[6] the motion of massive charged particles in a very strong electromagnetic field is studied. The guiding center approximation [7] is used in the Lorentz-Dirac equation. In this approximation the particle motion is described as a combination of forward and oscillatory motions (the field changes are small during a gyration period). If the gradient of the field potential is much larger than the rest mass of the particle, the strong radiation damping suppresses the particle gyration. It is shown [6] that the particle velocity is directed along one of the eigenvectors of the (external) electromagnetic tensor if $m \to 0$ in the rewritten Lorentz-Dirac equation. The equation on eigenvalues and eigenvectors of the electromagnetic tensor governs the motion of charges in the massless approximation.

According to Ref.[6], the effective equation of motion for this charge does not contain derivatives higher than 1. This conclusion is in contradiction with that of [8] where the 5-th order differential equation determines the evolution of photon-like charge.

In the present paper we apply the regularization procedure based on Noether conservation laws to the problem of radiation reaction for a massless charge in response to the electromagnetic field. The conservation laws are an immovable fulcrum about which tips the balance of truth regarding renormalization and radiation reaction.

2. GENERAL SETTING

We consider a massless point-like particle which carries an electric charge q and moves on a lightlike world line γ: $\mathrm{I\!R} \to \mathrm{I\!M}_4$ described by functions $z^\mu(\tau)$, in which τ is an arbitrary parameter. A tangent vector to each point $z^\mu(\tau) \in \gamma$ lies on the future light cone with vertex at this point:

$$\dot{z}^2 = 0\,. \tag{1}$$

(We use an overdot on z to indicate differentiation with respect to the evolution parameter τ.) We let $u^\alpha(\tau)=\mathrm{d}z^\alpha/\mathrm{d}\tau$ the 4-velocity, and $a^\alpha(\tau)=\mathrm{d}u^\alpha/\mathrm{d}\tau$ is the 4-acceleration. Initially we take the world line to be arbitrary; our main goal is to find the particle's equation of motion.

Following [8], we deal with an obvious generalization of the standard variational principle for massive charge

$$I = I_{particle} + I_{\rm int} + I_{field}\,, \tag{2}$$

with

$$I_{field} = -\frac{1}{16\pi}\int \mathrm{d}^4 x f^{\alpha\beta} f_{\alpha\beta}\,, I_{\rm int} = \int \mathrm{d}^4 x A_\mu j^\mu\,. \tag{3}$$

The particle part of variational principle should be consistent with the field and the interaction terms. So, if we require that the renormalized mass be zero, a nonzero bare mass is necessary to absorb a divergent self-energy. Hence the world line of the bare particle should be assumed time-like rather than lightlike. We may also require that the world line be lightlike before renormalization as well as after this procedure. To solve the dilemma we establish the structure of the bound and radiative terms of energy-momentum and angular momentum carried by electromagnetic field of the photon-like charge.

Having variated (3) with respect to potential A_μ we obtain the Maxwell field equations [4]

$$\Box A_\mu(x) = -4\pi j_\mu(x)\,, \tag{4}$$

where current density is zero everywhere, except at the particle's position where it is infinite

$$j_\mu(x) = q\int d\tau u_\mu(\tau)\delta[x - z(\tau)]\,, \tag{5}$$

and $\Box=\eta^{\alpha\beta}\partial_\alpha\partial_\beta$ is the wave operator.

The components of the momentum 4-vector carried by the electromagnetic field are [4,5]

$$p^\nu_{em}(\tau) = \int_\Sigma d\sigma_\mu T^{\mu\nu}\,, \tag{6}$$

where $d\sigma_\mu$ is the outward-directed surface element on an arbitrary space-like hypersurface Σ. The angular momentum tensor of the electromagnetic field is written as

$$M^{\mu\nu}_{em}(\tau) = \int_\Sigma d\sigma_\alpha \left(x^\mu T^{\alpha\nu} - x^\nu T^{\alpha\mu}\right) \tag{7}$$

where

$$T^{\mu\nu} = \frac{1}{4\pi}\left(f^{\mu\alpha} f^\nu_\alpha - 1/4\eta^{\mu\nu} f^{\alpha\beta} f_{\alpha\beta}\right) \tag{8}$$

is the electromagnetic field's stress-energy tensor.

3. ELECTROMAGNETIC FIELD OF A PHOTON-LIKE CHARGE

Let the past light cone with vertex at an observation point x is punctured by the particle's world line γ at point $z(s)$. The retarded Green function associated with the d'Alembert operator $\Box$ and the charge-current density (5) is valuable only. The components of the Liénard-Wiechert potential $\hat{A}=A_\alpha dx^\alpha$ are

$$A_\alpha = q\frac{u_\alpha(s)}{r}\,, \tag{9}$$

where $r = -(R\cdot u)$ is the retarded distance [4,5]; $R^\mu=x^\mu-z^\mu(s)$ is the null vector pointing from $z(s)$ to x. The 4-potential is not defined at points on the ray in the direction of momentary 4-velocity $u(s)$ by reason of the isotropy condition (1).

Straightforward computation reveals that $\Box A=0$ everywhere, except at the particle's position.

Unlike the massive case, the photon-like charge generates the far electromagnetic field $\hat{f} = d\hat{A}$:

$$\hat{f} = q\frac{a\wedge k + (a\cdot k)u\wedge k}{r}\,, \tag{10}$$

where the symbol $\wedge$ denotes the wedge product. Because of isotropy condition the retarded distance vanishes on the ray in the direction of particle's 4-velocity taken at the instant of emission. The field diverges at all the points of this ray with vertex at the point of emission.

To calculate the stress-energy tensor of the electromagnetic field we substitute the components (10) into expression (8). Contrary to the massive case [5, eqs.(5.3)-(5.5)], the "photon-like" Maxwell energy-momentum density contains the radiative component only:

$$4\pi T^{\alpha\beta} = q^2 a^2 \frac{k^\alpha k^\beta}{r^2}\,. \tag{11}$$

Hence the divergent self-energy which is due to volume integration of the bound part of the electromagnetic field's stress-energy tensor [9] does not arise. Unlike the massive case, the photon-like charge does not possess an electromagnetic "cloud" permanently attached to it.

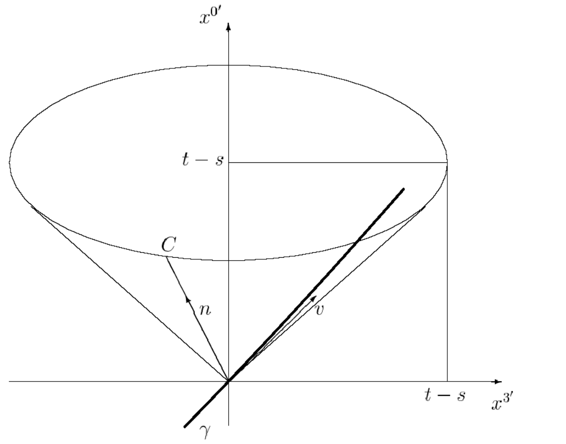

Fig. 1. *In the particle's momentarily comoving frame the massless charge is placed at the coordinate origin; its 4-velocity is (1,0,0,1). The point C is linked to the coordinate origin by a null ray characterized by the angles (φ,θ). (The null vector n=(1,**n**) defines this direction.) For a given point C with coordinates $x^{\alpha'}=(t-s)n^{\alpha'}$ the retarded distance is $x^{0'}-x^{3'}=(t-s)(1-\cos\theta)$*

4. ENERGY-MOMENTUM AND ANGULAR MOMENTUM CARRIED BY THE ELECTROMAGNETIC FIELD

Now we calculate the electromagnetic field momentum (6) where an integration hypersurface $\Sigma_t=\{x\in M_4: x^0=t\}$ is a surface of constant t. Volume integration of the radiative energy-momentum density (11) over a hyperplane Σ_t gives the amount of radiated energy-momentum at fixed instant t. An appropriate coordinate system is a very important for the integration. We introduce the set of curvilinear coordinates for flat space-time M_4 involving the observation time t and the retarded time s:

$$x^\alpha = z^\alpha(s) + (t-s)\Omega^\alpha_{\alpha'} n^{\alpha'}\,. \tag{12}$$

The null vector n=(1,**n**) has the components (1,cosφsinθ, sinφsinθ,cosθ); θ and φ are two polar angles. Matrix space-time components are $\Omega^0{}_\mu= \Omega^\mu{}_0=\delta^0{}_\mu$; its space components $\Omega^i{}_j$ constitute the orthogonal matrix which rotates space axes of the laboratory Lorentz frame until new z-axis is directed along three-vector **v**. (Particle's 4-velocity has the form $(1,v^i)$, $|\mathbf{v}|=1$, if parametrization of the world line γ is provided by a disjoint union of hyperplanes Σ_t.) In terms of curvilinear coordinates (t,s,θ,φ) the retarded distance is as follows:

$$r = (t-s)(1-\cos\theta)\,. \tag{13}$$

The situation is pictured in Fig. 1.

The surface element is given by $d\sigma_0=(-g)^{1/2}dsd\theta d\varphi$ where

$$\sqrt{-g}=(t-s)^2\sin\theta\,(1-\cos\theta) \tag{14}$$

is the determinant of metric tensor of Minkowski space viewed in curvilinear coordinates (12). In these coordinates the components of the electromagnetic field's stress-energy tensor (11) have the form:

$$4\pi T^{00}=q^2\frac{a^2(s)}{(t-s)^2(1-\cos\theta)^4};$$

$$4\pi T^{0i}=q^2\frac{a^2(s)\Omega^i_{i'}n^{i'}}{(t-s)^2(1-\cos\theta)^4}. \tag{15}$$

The angular integration results the radiated energy-momentum:

$$p^0_{em}(t)=\tfrac{1}{2}q^2J_0\int_{-\infty}^{t}\mathrm{d}s\,a^2(s)$$

$$p^i_{em}(t)=\tfrac{1}{2}q^2J_1\int_{-\infty}^{t}\mathrm{d}s\,a^2(s)\mathrm{v}^i(s), \tag{16}$$

where factors J_0 and J_1 diverge:

$$J_0=-\frac{1}{8}+\lim_{\theta\to 0}\frac{1}{2(1-\cos\theta)^2};$$

$$J_1=\frac{3}{8}-\lim_{\theta\to 0}\left[\frac{1}{1-\cos\theta}-\frac{1}{2(1-\cos\theta)^2}\right].$$

Similarly, the computation of the electromagnetic field angular momentum (7) which flows across the hyperplane Σ_t gives rise to the divergent quantities:

$$M^{0i}_{em}(t)=\tfrac{1}{2}q^2\begin{bmatrix} J_1\int_{-\infty}^{t}\mathrm{d}s\,a^2(\mathrm{s})\mathrm{s}\mathrm{v}^i(s) \\ -J_0\int_{-\infty}^{t}\mathrm{d}s\,a^2(\mathrm{s})\mathrm{z}^i(s)\end{bmatrix};$$

$$M^{ij}_{em}(t)=\tfrac{1}{2}q^2J_1\int_{-\infty}^{t}\mathrm{d}s\,a^2(s)\left[z^i\mathrm{v}^j-z^j\mathrm{v}^i\right]. \tag{17}$$

The energy-momentum (16) and the angular momentum (17) of electromagnetic field generated by the accelerated photon-like charge tend to infinity in the direction of particle's velocity at the instant of emission. The divergent terms are not bound terms which should be absorbed by corresponding particle characteristics within the renormalization procedure. Indeed, they do not depend on the distance from the particle's world line. Secondly, the energy-momentum and the angular momentum accumulate with time at the observation hyperplane Σ_t (see Fig.2). Hence the divergent Noether quantities cannot be referred to an electromagnetic "cloud" which is permanently attached to the charge and is carried along with it.

As a consequence, the Brink-Di Vecchia-Howe action term [10,eq.(2)]:

$$I_{particle}=\tfrac{1}{2}\int\mathrm{d}\tau e(\tau)\dot{z}^2 \tag{18}$$

is consistent with the field and interaction terms (3). Variation of (18) with respect to Lagrange multiplier $e(\tau)\neq 0$ yields the isotropy condition (1). The particle part (18) of the total action (2) describes already renormalized massless charge.

Further in this paper we shall use a disjoint union of hyperplanes Σ_t for parametrization of the particle world line γ. We define $\mathrm{v}^\alpha(t)=\mathrm{d}z^\alpha(t)/\mathrm{d}t$ as the 4-velocity; 4-acceleration $a^\alpha(t)=\mathrm{dv}^\alpha(t)/\mathrm{d}t$ looks as $(0,\dot{\mathrm{v}}^i)$ in this para-metrization.

Changes in energy-momentum and angular momentum radiated by accelerated charge should be balanced by changes in already renormalized 4-momentum and angular momentum of the particle. But the accelerated photon-like charge emits infinite amounts of radiation (see Fig. 2). To change the velocity of the massless charge the energy is necessary which is too large to be observed. Threfore, the effective equation of motion should be supplemented with the condition of absence of radiative damping.

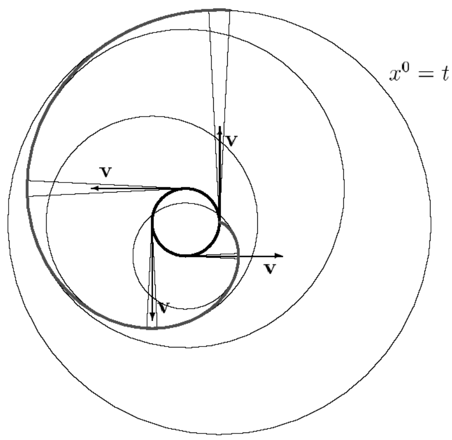

Fig. 2. *The bold circle pictures the trajectory of a photon-like charge. The others are spherical wave fronts viewed in the observation hyperplane Σ_t. The circling photon-like charge radiates infinite rates of energy-momentum and angular momentum in the direction of its velocity* **v** *at the instant of emission. The energy-momentum and angular momentum carried by electromagnetic field of accelerated charge tend to infinity on the spiral curve*

According to expression (10), non-accelerated photon-like charge itself does not generate the electromagnetic field. The evolution of the particle beyond an interaction area is determined by the Brink-Di Vecchia-Howe Lagrangian. The particle's 4-momentum $p_{part}=e(t)\dot{z}$ does not change with time:

$$\dot{e}(t)\dot{z}^\mu+e(t)\ddot{z}^\mu=0. \tag{19}$$

Since γ is degenerate (see eq.(1)), the 4-acceleration vanishes in adapted parametrization. Since $\dot{z}\neq 0$, the Lagrange multiplier $e=e_0$ does not depend on time. We deal with a photon-like particle moving in the **v**-direction with frequency $\omega_0=e_0$, such that its energy-momentum 4-vector can be written $p^\mu_{part}=(\omega_0,\omega_0\mathrm{v}^i)$.

5. MASSLESS CHARGE WITHIN AN INTERACTION AREA

When considering the system under the influence of an external device the change in particle's 4-momentum should be balanced by an external force F_{ext}:

$$\dot{p}^{\mu}_{part} = F^{\mu}_{ext} . \qquad (20)$$

This effective equation of motion is supplemented with the condition of absence of radiative damping. In other words, the external device admits a massless charge if and only if the components of null vector of 4-velocity do not change with time despite the influence of the external field. The conclusion is similar to that of [1,2].

When the photon-like charged particle moves in the external electromagnetic field $\hat{F}$, the Lorentz force balances the change in its 4-momentum:

$$\dot{e}\mathrm{v}^{\mu} = qF^{\mu}_{\nu}\mathrm{v}^{\nu} . \qquad (21)$$

It is convenient to decompose $\hat{F}$ into an electric field E and a magnetic field B. Equation (21) is then rewritten as

$$\dot{e} = q(E\cdot \mathrm{v}) , \; \dot{e}\mathrm{v} = qE + q[\mathrm{v}\times B] . \qquad (22)$$

We have the following 4-th order algebraic equation on eigenvalues $\dot{e}$:

$$\dot{e}^4 + \dot{e}^2 q^2 (B^2 - E^2) - q^4 (B\cdot E)^2 = 0 . \qquad (23)$$

In general, it possesses two real solutions [6,7]:

$$\dot{e}_{\pm} = \pm q\sqrt{\left(E^2 - B^2 + \mu\right)/2} , \qquad (24)$$

where

$$\mu = \sqrt{\left(B^2 - E^2\right)^2 + 4(B\cdot E)^2} . \qquad (25)$$

The field admits a photon-like charge if and only if corresponding eigenvectors

$$\mathrm{v}_{\pm} = \frac{[E\times B] \pm (\lambda E + \kappa\nu B)}{\sigma} , \; \kappa = \mathrm{sgn}(B\cdot E) \qquad (26)$$

are of constant values. Here

$$\lambda = \sqrt{(E^2 - B^2 + \mu)/2} , \; \nu = \sqrt{(B^2 - E^2 + \mu)/2} ,$$

$$\sigma = (E^2 + B^2 + \mu)/2 . \qquad (27)$$

The expression (26) is obtained in [11,eq.(2.3)] where the model of magnetosphere of a rapidly rotating neutron star (pulsar) is elaborated. It defines the velocity of the massless charged particles which constitute the so-called "dynamical phase" of the gas of ultrarelativistic electrons and positrons moving in a very strong electromagnetic field of the pulsar. In Rylov's model [11] the massless charges as a limiting case of massive ones are considered. The reason is that the gradient of star's potential is much larger than the particle's rest energy $m_e c^2$.

6. CONCLUSIONS

Our consideration is founded on the Maxwell equations with point-like source which governs the propagation of the electromagnetic field produced by a photon-like charge. Unlike the massive case, it generates the **far** electromagnetic field which does not yield to divergent Coulomb-like self-energy. Hence the world line is null before renormalization as well as after this procedure. We choose Brink-Di Vecchia-Howe action [10] for a bare particle moving on the world line which is proclaimed then to be lightlike.

A surprising feature of the study of the radiation back reaction in dynamics of the photon-like charge is that the Larmor term (15) diverges whenever the charge is accelerated. Since the emitted radiation detaches itself from the charge and leads an independent existence, it cannot be absorbed within a renormalization procedure.

Inspection of the energy-momentum and angular momentum carried by the electromagnetic field of a photon-like charge reveals the reason why it is not yet detected (if it exists). Noninteracting massless charges do manifest themselves in no way. Any external electromagnetic field (including that generated by a detecting device) will attempt to change the velocity of the charged particle. Whenever the effort will be successful, the radiation reaction will increase extremely. In general, this circumstance forbids the presence of the photon-like charges within the interaction area.

Nevertheless, there exists the electromagnetic fields which do not change the velocities of the massless charged particles. For example, superposition of plane waves propagating along some base line admits the massless charges moving analogously (see Appendix). (But any disturbance annuls such a "loyalty".) It is worth noting that the quantum mechanical results [1,2] are in favour the conception that the external field distinguishes the directions of non-accelerating motions of photon-like charges (if they exist).

To survive photon-like charges need an extremely strong field of specific configuration, as that of the rotating neutron star (pulsar). In [11] the model of the pulsar magnetosphere is elaborated. It involves the so-called **dynamical phase** which consists of the massless charged particles moving with speed of light along some base line determined by the electromagnetic field of the star. (The massless approximation is meant where the gradient of star's potential is much larger than electron's rest energy.) It is worth noting that the expression for the particles' velocity [11,eq.(2.2)] coincides with the solution (26) of the "massless" equations of motion derived in the present paper.

Equation (21) on eigenvalues and eigenvectors of the electromagnetic tensor governs the motion of charges in zero-mass approximation. This conclusion is in contradiction with that of [8] where the 5-th order differential equation determines the evolution of photon-like charge. The reason is that regularization approach to the radiation back reaction (smoothing the behaviour of the Lorentz force in the immediate vicinity of the particle's world line), employed by Kazinski and Sharapov, is not valid in the case of the photon-like charged particle and its field. Indeed, the field diverges not only at point of world line but at all points of the ray in the direction of particle's 4-velocity taken at the instant of emission (see Fig. 2).

The ray singularity is stronger that δ-like singularity of Green's function involved in [8] in the self-force expression. Hence integration over world line does not yield a finite part of the self-force.

APPENDIX: MOTION OF MASSLESS CHARGES IN A PLANE WAVE

In case of a plane wave with front moving in the positive z-direction, the electric and magnetic fields are related to each other as follows:

$$E_x = B_y\,,\ E_y = -B_x\,,\ E_z = B_z = 0\,. \qquad \text{(A1)}$$

Since $B^2 - E^2$ as well as the scalar product (B·E) vanish, the eigenvalues' equation (23) get simplified: $\dot{e}^4 = 0$. The eigenvector corresponding to the fourthly degenerate eigenvalue $\dot{e} = 0$ is defined by [6]

$$\mathrm{v} = \frac{[E \times B]}{B^2} = n_z\,. \qquad \text{(A2)}$$

Hence the plane wave admits massless charges moving along z-line in the positive direction. Their frequencies do not change with time.

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ПЕРЕНОРМИРОВАНИЕ И РЕАКЦИЯ ИЗЛУЧЕНИЯ ФОТОНОПОДОБНОГО ЗАРЯДА

Ю.Г. Яремко

Найдены энергия-импульс и момент количества движения электромагнитного поля безмассового точечного заряда. В отличие от точечного источника с ненулевой массой покоя, излученные интегралы движения поля ускоренного безмассового заряда неограниченно возрастают. Вследствие этого фотоноподобный заряд может существовать лишь в таком внешнем поле, которое не изменяет его скорости. Эффективным уравнением движения является уравнение на собственные векторы и собственные значения тензора напряжения внешнего электромагнитного поля. Такое же уравнение появляется в предложенной Рыловым модели магнитосферы пульсара.

ПЕРЕНОРМУВАННЯ ТА РЕАКЦІЯ ВИПРОМІНЮВАННЯ ФОТОНОПОДІБНОГО ЗАРЯДУ

Ю.Г. Яремко

Пораховані енергія-імпульс та момент кількості руху електромагнітного поля безмасового точкового заряду. На відміну від точкового джерела з ненульовою масою спокою, інтеграли руху, що переносяться полем прискореного безмасового заряду, необмежено зростають. Тому фотоноподібний заряд може існувати лише в такому зовнішньому полі, яке не змінює його швидкості. Ефективним рівнянням руху є рівняння на власні вектори та власні значення тензора напруженості зовнішнього електромагнітного поля. Таке ж рівняння виникає в запропонованій Риловим моделі магнітосфери пульсара.