# FEMTOSECOND SCANNING, CHOPPING (SLICING) AND MEASUREMENT OF LENGTH OF ELECTRON BUNCHES BY LASER PULSES: PRINCIPLES OF FEMTOSECOND OSCILLOSCOPES 

E.D. Gazazian ${ }^{1}$, V. Guidi ${ }^{2}$, K.A. Ispirian ${ }^{1}$, M.K. Ispirian ${ }^{1}$, and D.K. Kalantaryan ${ }^{3}$<br>${ }^{l}$ Yerevan Physics Institue, Yerevan, Armenia; e-mail: karo@mail.yerphi.am;<br>${ }^{2}$ Department of Physics and INFN, Ferrara, Italy; e-mail: guidi@smtp.fe.infn.it;<br>${ }^{3}$ CANDLE, Acharyan 31, 375040, Yerevan, Armenia;<br>e-mail:kalantaryan@asls.candle.am


#### Abstract

The equations of motion of electrons in a plane monochromatic wave with linear, circular and elliptical polarizations are solved taking into account the initial conditions. The cases when the interaction of the electrons with field of linearly and circularly polarized laser photon beams takes place during a finite time, while the electrons are detected after a long field-free region being scanned with period of the order of femtosecond, are suitable for femtoslicing and measurement of length of femtosecond electron pulses. The principles and limitations of slicing and of construction of laser femtosecond oscilloscopes are considered, and a scheme of experiment is proposed.


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## 1. INTRODUCTION

Following optical femtosecond (fs) pulses the fs Xray pulses found wide application in time resolved experiments on the study of fast processes of atomic and molecular investigations of physics, biology and other sciences [1]. After production of optical and low energy electron attosecond (as) pulses (see, for instance, $[2,3]$ ), the human fancy allows to discuss [4] the possibility of production of nuclear-time-scale zeptosecond (zs) bursts of duration $10^{-21}-10^{-22} \mathrm{~s}$. Bunches of electrons $[5,6]$ and X-rays $[7,8]$ with lengths down to a few tens of fs have been obtained. There are some projects [ 9,10$]$ for production of high energy electron and then X-ray pulses of a few fs and a few hundreds as.

The measurement of the length and electron distribution in fs bunches is necessary for FELs, linear colliders, advanced acceleration methods and for many other fields of science and technology. However, it has been experimentally shown that the contemporary methods of time and frequency domains for fs measurements (see [11]) using RF fields, electro-optic (EO) modulation and various types of coherent radiations are applicable for measurement of down to a few tens of fs only.

In the time domain the old $[12,13]$ methods based on the transverse deflection of electron beams by the fields in RF cavities has been modified in many works $[14,15]$, and the best results at present are $72 \mathrm{fs} / \mathrm{mm}$ sensitivity and 15 fs resolution [16]. Various modifications of the EO techniques based on the refraction index variation of some none linear materials by Coulomb field of particles have allowed to measure electron pulse length close to the possible limit, $\sim 100 \mathrm{fs}$ [17-20].

In the frequency domain one measures the spectral characteristics of coherent synchrotron (CSR), transition (CTR), diffraction (CDR) and Smith-Purcell
(CSPR) radiation, nevertheless, the methods of this domain are connected with assumptions and complicated time-frequency conversion. Though it has been shown [21] that, for instance, the X-ray CTR allows the measuring of time intervals down to a few as, nevertheless, the shortest pulse length measurement resolution achieved at present is $\sim 100 \mathrm{fs}$ [11].

The method of bunch slicing $[7,8]$ for the production of fs electron and hence X-ray bunches is and will be applied at various synchrotron radiation sources [22]. The fs slicing is the result of transversal deflection of a small part of electron bunches due to their interaction with co propagating intense fs laser bunches in undulators, and the best results show that only the $10^{-6}$ part of the electron bunches is sliced with FWHM length $\sim 100$ fs [22].

As it is well known (see [23]) it is difficult to accelerate the charged particles by laser beams, especially in vacuum, because the fields are perpendicular to the direction of propagation of photons. Despite the fact [23] that there are very strong electric fields $E$ in laser beams $\quad\left(E(V / \mathrm{cm}) \approx 20\left[I\left(W / \mathrm{cm}^{2}\right)\right]^{1 / 2}\right.$ equal to $E \approx 2.10^{10} \mathrm{~V} / \mathrm{cm}$ for an available laser beam intensity $I \approx 10^{18} \mathrm{~W} / \mathrm{cm}^{2}$ ) and optimistic acceleration rates theoretically predicted for various methods of advanced acceleration methods, the achieved record acceleration rates are low and only for short distances, and the progress in this field is very slow.

In [24-27] it has been shown that the interaction of an electron beam with polarized electromagnetic wave of laser photons propagating in the same direction in a finite interaction region results in significant transversal deflection of the electrons even when this interaction takes place not in magnetic undulators. In the case of linear polarization one can explain such a phenomenon
by the fact that despite to the difference of the electron and laser velocities the electron undergoes almost the same strong electric field on a significant interaction length $\sim \lambda \gamma^{2}$, where $\lambda$ is the laser photon wavelength and $\gamma=E / m c^{2}=1 / \sqrt{1-(v / c)^{2}}$. It has been shown that such deflections can be used for chopping (slicing), for production of fs electron and synchrotron radiation beams, construction of sub-femtosecond oscilloscope and other purposes. It has been shown that the processes are similar to those taking place in the RF deflection devices by replacing the RF fields inside cavities by intense laser field.

In this work the motion of electrons in laser beams in a finite length interaction region and then in a fieldfree drift length is investigated in details. Let us note that there is no contradiction with the LawsonWoodward theorem [28] since the interaction takes place in finite space-time interval. The results obtained by solving the Maxwell equations by Hamilton-Jacobi method are in agreement with the well known results [29, 30]. The principles of fs slicing and construction of femtosecond oscilloscopes are discussed.

## 2. THE MOTION OF ELECTRONS IN FINITE LENGTH, PLANE, MONOCHROMATIC ELECTROMAGNETIC FIELD OF A LASER BEAM

The equations of the electron motion in the electric $\vec{E}$ and magnetic field $\vec{H}$ of a wave have the form

$$
\begin{align*}
& \frac{d \vec{r}}{d t}=c \frac{\vec{p}}{\sqrt{m^{2} c^{2}+\vec{p}^{2}}}  \tag{1}\\
& \frac{d \vec{p}}{d t}=e\left[\vec{E}+\frac{[\vec{p} \vec{H}]}{\sqrt{m^{2} c^{2}+\vec{p}^{2}}}\right] \tag{2}
\end{align*}
$$

where $\vec{r}$ is the radius vector of the point where the electron is at the moment $t$, and $\vec{p}$ is its momentum.
Let the initial conditions when $t=0$ are:

$$
\begin{align*}
& \vec{r}(t=0)=\vec{r}_{0}=\left\{x_{0}, y_{0}, z_{0}\right\} \\
& \vec{p}(t=0)=\vec{p}_{0}=\left\{p_{x 0}, p_{y 0}, p_{z 0}\right\} \tag{3}
\end{align*}
$$

For the case when the particle and the plane monochromatic wave of elliptic polarization are propagating along the Cartesian axis OX

$$
\begin{align*}
& \vec{E}=\vec{E}\left(0, E_{y}, E_{z}\right), \quad \vec{H}=\vec{H}\left(0, H_{y}, H_{z}\right)  \tag{4a}\\
& E_{y, z}=E_{y, z 0} \cos \left(\omega t-k x+\varphi_{y, z 0}\right)  \tag{4b}\\
& H_{z, y}= \pm E_{y, z 0} \cos \left(\omega t-k x+\varphi_{y 0}\right) \tag{4c}
\end{align*}
$$

where $k=k_{x}=\omega / c, \varphi_{y 0}$ and $\varphi_{z 0}$ are the initial phases. Going from the variable $t$ to $\eta=\omega t-k x$, one can find the following final solutions

$$
\begin{aligned}
& x=C_{1}+\frac{2 B}{k}\left\{p_{x 0}\right. \\
& \left.+B\left[C_{y}^{2}+C_{z}^{2}-p_{y 0}^{2}-p_{z 0}^{2}+\frac{e^{2}}{2 \omega}\left(E_{y 0}^{2}+E_{z 0}^{2}\right)\right]\right\} \eta
\end{aligned}
$$

$$
\begin{align*}
& -\frac{4 e B^{2}}{k \omega}\left[C_{y} E_{y 0} \cos \left(\eta+\varphi_{y 0}\right)+C_{z} E_{z 0} \cos \left(\eta+\varphi_{z 0}\right)\right] \\
& -\frac{e^{2} B^{2}}{2 k \omega^{2}}\left[E_{y 0}^{2} \sin \left(\eta+\varphi_{y 0}\right)+E_{z 0}^{2} \sin 2\left(\eta+\varphi_{z 0}\right)\right] \\
& y=C_{2}+\frac{2 B C_{y}}{k} \eta-\frac{2 B e E_{y 0}}{k \omega} \cos \left(\eta+\varphi_{y 0}\right) \\
& z=C_{3}+\frac{2 B C z}{k} \eta-\frac{2 B e E_{z 0}}{k \omega} \cos \left(\eta+\varphi_{z 0}\right) \\
& p_{x}=p_{x 0}+B\left[\left(p_{y}^{2}-p^{2} y 0\right)+\left(p_{z}^{2}-p_{z 0}^{2}\right)\right] \\
& p_{y}=p_{y 0}+\frac{e E_{y 0}}{\omega}\left[\sin \left(\eta+\varphi_{y 0}\right)-\sin \left(-k x_{0}+\varphi_{y 0}\right)\right] \\
& p_{z}=p_{z 0}+\frac{e E_{z 0}}{\omega}\left[\sin \left(\eta+\varphi_{z 0}\right)-\sin \left(-k x_{0}+\varphi_{z 0}\right)\right] \tag{5}
\end{align*}
$$

Here the constants

$$
\begin{equation*}
C_{y, z}=p_{y 0, z 0}-\frac{e E_{y 0, z 0}}{\omega} \sin \left(-k x_{0}+\varphi_{y 0, z 0}\right) \tag{6}
\end{equation*}
$$

$C_{1}, C_{2}, C_{3}$, and the constant $B$ are determined by (3),

$$
\begin{equation*}
B=\frac{\sqrt{m^{2} c^{2}+p_{x 0}^{2}+p_{y 0}^{2}+p_{z 0}^{2}}+p_{x 0}}{2\left(m^{2} c^{2}+p_{y 0}^{2}+p_{z 0}^{2}\right)} \tag{7}
\end{equation*}
$$

In the simplest case of linear polarization when

$$
\begin{equation*}
E_{z}=H_{y}=E_{z 0}=0 \tag{8}
\end{equation*}
$$

the solutions (5) give

$$
\begin{align*}
& x=C_{1}+\frac{2 B}{k}\left\{p_{x 0}+B\left[C_{y}^{2}-p_{y 0}^{2}+\frac{e^{2} E_{0}^{2}}{2 \omega^{2}}\right]\right\} \\
& -\frac{4 e B^{2} C_{y} E_{0}}{k \omega} \cos \left(\eta+\varphi_{0}\right)-\frac{e^{2} B^{2} E_{0}^{2}}{2 k \omega^{2}} \sin 2\left(\eta+\varphi_{0}\right) \\
& y=C_{2}+\frac{2 B C_{y}}{k} \eta-\frac{2 B e E_{0}}{k \omega} \cos \left(\eta+\varphi_{0}\right) \\
& z=z_{0}+\frac{2 \cdot B \cdot p_{z 0}}{k} \cdot \eta \\
& p_{x}=p_{x 0}+B\left(p_{y}^{2}-p_{y 0}^{2}\right) \\
& p_{y}=p_{y 0}+\frac{e E_{0}}{\omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \left(-k x_{0}+\varphi_{0}\right)\right] \\
& p_{z}=p_{z 0} . \tag{9}
\end{align*}
$$

The obtained results are in agreement with the well known results [29, 30] obtained in a frame where in the average the electron is in rest. Indeed, using (9) one can show that the electrons have 8-type trajectories in the plane $z=z_{0}$ with a certain center.

In the case of circular polarization

$$
\begin{equation*}
E_{y 0}=E_{z 0}, \varphi_{y 0}=\varphi_{0}, \varphi_{z 0}=\varphi_{0}+\pi / 2 \tag{10}
\end{equation*}
$$

the expressions (5) give

$$
\begin{aligned}
& x=C_{1}+\frac{2 B}{k}\left\{p_{x 0}+\left\{\frac { 2 B e E _ { 0 } } { \omega } \left[\frac{e E_{0}}{\omega}\right.\right.\right. \\
& \left.\left.\left.-p_{y 0} \sin \left(-k x_{0}+\varphi_{0}\right)-p_{z 0} \cos \left(-k x_{0}+\varphi_{0}\right)\right]\right\}\right\} \eta \\
& -\frac{4 B^{2} e E_{0}}{k \omega}\left[\frac{e E_{0}}{\omega} \sin \left(\eta+k x_{0}\right)\right. \\
& \left.+p_{y 0} \cos \left(\eta+\varphi_{0}\right)-p_{z 0} \sin \left(\eta+\varphi_{0}\right)\right]
\end{aligned}
$$

$$
\begin{align*}
& y=C_{2}+\frac{2 B C_{y}}{k} \eta-\frac{2 B e E_{0}}{k \omega} \cos \left(\eta+\varphi_{0}\right) ; \\
& z=C_{3}+\frac{2 B C_{z}}{k} \eta+\frac{2 B e E_{0}}{k \omega} \sin \left(\eta+\varphi_{0}\right) ; \\
& p_{x}=p_{x 0}+B\left\{\frac{2 e^{2} E_{0}^{2}}{\omega^{2}}\left[1-\cos \left(\eta+k x_{0}\right)\right]\right. \\
& +\frac{4 e E_{0}}{\omega}\left[p_{y 0} \cos \left(\frac{\eta-k x_{0}}{2}+\varphi_{0}\right)\right. \\
& \left.\left.-p_{z 0} \sin \left(\frac{\eta-k x_{0}}{2}+\varphi_{0}\right)\right] \cdot \sin \frac{\eta+k x_{0}}{2}\right\} ; \\
& p_{y}=p_{y 0}+\frac{e E_{0}}{\omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \left(-k x_{0}+\varphi_{0}\right)\right] ; \\
& p_{z}=p_{z 0}+\frac{e E_{0}}{\omega}\left[\cos \left(\eta+\varphi_{0}\right)-\cos \left(-k x_{0}+\varphi_{0}\right)\right] . \tag{11}
\end{align*}
$$

As it follows from formulae (5), (9) and (11) the particle energy is a periodic function with zero average increase in agreement with the Lawson-Woodward theorem [28].

To compare (11) with the ones in $[29,30]$ consider the case when in the average the electrons are in rest, and, in particular, have momenta parallel to magnetic field

$$
\begin{align*}
& \varphi_{0}=0, x_{0}=0, y_{0}=-\frac{2 B e E_{0}}{k \omega}, z_{0}=0, \\
& p_{x 0}=0, p_{y 0}=0, p_{z 0}=\frac{e E_{0}}{\omega} . \tag{12}
\end{align*}
$$

Using (11) and (12) one can show that in agreement with [29, 30] the velocity of the particle remains always parallel to the magnetic field, and they make circular motion with the frequency of the field and radius

$$
\begin{equation*}
R=\frac{e E_{0}}{k \omega \sqrt{m^{2} c^{2}+\frac{e^{2} E_{0}^{2}}{\omega^{2}}}} . \tag{13}
\end{equation*}
$$

Let us note that in all the cases when the electron is in rest the maximal transversal deflections of the electrons are very small. It is not difficult to show that even when the fields are equal to critical fields $E_{c r}=m^{2} c^{3} / e \hbar=1.32 .10^{16} \mathrm{~V} / \mathrm{cm}, H_{c r}=10^{13} \mathrm{Gs}$, and the intensity parameter $\xi=e E_{y o} \lambda / 2 \pi m c^{2} \gg 1$, they are of the order of $\lambda$. This is explained by the fact that, though the transversal velocities may become relativistic, nevertheless, due to small time of quarter period $T=2 \pi / \omega$ the electron has no time to acquire large transversal deflection.

## 3. LINEAR AND CIRCULAR SCANNING OF THE ELECTRON BEAM. PRINCIPLES OF FEMTOSECOND OSCILLOSCOPE

Consider the motion of electrons in the arrangement (see Fig. 1) proposed in [24-27]. After the interaction region $L_{\text {int }}$ the electrons fly a field free-region L and are detected by, for instance, CCD pixel detectors.


Fig. 1. The scheme of the proposed fs oscilloscope

For the initial conditions

$$
\begin{align*}
& \varphi_{0} \neq 0, x_{0}=y_{0}=z_{0}=0, p_{x 0} \neq 0 ; \\
& p_{y 0}=p_{z 0}=0, \tag{14}
\end{align*}
$$

in the case of linear and circular polarization from (9) and (11) one obtains, respectively

$$
\begin{align*}
& \left.x=\frac{2 B}{k}\left\{p_{x 0}+\frac{B e^{2} E_{0}^{2}}{2 \omega^{2}}\left[1+2 \sin ^{2} \varphi_{0}\right]\right]\right\} \eta+\frac{B^{2} e^{2} E_{0}^{2}}{2 k \omega^{2}} \\
& \times\left[8 \sin \varphi_{0} \cos \left(\eta+\varphi_{0}\right)-3 \sin 2 \varphi_{0}-\sin 2\left(\eta+\varphi_{0}\right)\right] ; \\
& y=-\frac{2 B e E_{0} \sin \varphi_{0}}{k \omega} \eta-\frac{2 B e E_{0}}{k \omega}\left[\cos \left(\eta+\varphi_{0}\right)-\cos \varphi_{0}\right] ; \\
& p_{x}=p_{x 0}+\frac{B e^{2} E_{0}^{2}}{\omega^{2}}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \varphi_{0}\right]^{2} ; \\
& p_{y}=\frac{e E_{0}}{\omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \varphi_{0}\right], \tag{9'}
\end{align*}
$$

and

$$
\begin{aligned}
& x=\frac{2 B}{k}\left(p_{x 0}+\frac{2 B e^{2} E_{0}^{2}}{\omega^{2}}\right) \cdot \eta-\frac{4 B^{2} e^{2} E_{0}^{2}}{k \omega^{2}} \cdot \sin \eta \\
& y=-\frac{2 B e E_{0} \sin \varphi_{0}}{k \omega} \eta-\frac{2 B e E_{0}}{k \omega}\left[\cos \left(\eta+\varphi_{0}\right)-\cos \varphi_{0}\right]
\end{aligned}
$$

$$
;
$$

$$
\begin{align*}
& z=-\frac{2 B e E_{0} \cos \varphi_{0}}{k \omega} \eta+\frac{2 B e E_{0}}{k \omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \varphi_{0}\right] \\
& p_{x}=p_{x 0}+\frac{2 B e^{2} E_{0}^{2}}{\omega^{2}}(1-\cos \eta) \\
& p_{y}=\frac{e E_{0}}{\omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \varphi_{0}\right] \\
& p_{z}=\frac{e E_{0}}{\omega}\left[\cos \left(\eta+\varphi_{0}\right)-\cos \varphi_{0}\right] .
\end{align*}
$$

The trajectories of 50 keV electrons with $\varphi_{0}=0, \pi / 4, \pi / 2,3 \pi / 4, \pi, 5 \pi / 4,3 \pi / 2,7 \pi / 4$, in the interaction region calculated with the help of $\left(9^{\prime}\right)$ and (11') for $\mathrm{CO}_{2}$ laser $(\lambda=10 \mu) E_{0}=2.10^{8} \mathrm{~V} / \mathrm{cm}$ are
shown in the upper and bottom figures of Fig. 2 ( curves $1,2 \ldots 8$, respectively).


Fig. 2. Trajectories of electrons in the interaction region. The coordinates in the bottom figure are given in $\mu m$

As it is seen the maximal deflections, $y_{1}$ (and $z_{1}$ ) from the x-axis at the end of $L_{\text {int }}$ are always very small, while the deflection angles, $d y / d x$ (and $d z / d x$ ), with respect to OX can be significant large for certain $L_{\mathrm{int}}$. The coordinates, $y_{2}, z_{2}$, of the electrons on the detectors placed on the distance $L$ are equal to

$$
\begin{align*}
& y_{2}=y_{1}+\left.\left.L \frac{d y}{d x}\right|_{x=L_{\mathrm{int}}} \approx L \frac{d y}{d x}\right|_{x=L_{\mathrm{int}}} \\
& \left(z_{2}=z_{1}+\left.\left.L \frac{d z}{d x}\right|_{x=L_{\mathrm{int}}} \approx L \frac{d z}{d x}\right|_{x=L_{\mathrm{int}}}\right) \tag{15}
\end{align*}
$$

Using the above expressions one may write for linear polarization with linear scanning of electrons

$$
\begin{equation*}
\left.\frac{d y}{d x}\right|_{x=L_{\mathrm{int}}}=\frac{A}{\sqrt{\gamma^{2}-1}+\frac{A^{2}}{2}\left[\gamma+\sqrt{\gamma^{2}-1}\right]} \tag{16}
\end{equation*}
$$

where

$$
\begin{align*}
& A=\frac{e E_{0}}{m_{0} c \omega}\left[\sin \left(\Delta \eta+\varphi_{0}\right)-\sin \varphi_{0}\right]  \tag{17}\\
& \Delta \eta=\omega t_{1}-k L_{\mathrm{int}}
\end{align*}
$$

$t_{1}$ is time of electron fly through $L_{\text {int }}$.
For circular polarization with circular scanning $x=\frac{2 B}{k}\left(p_{x 0}+\frac{2 B e^{2} E_{0}^{2}}{\omega^{2}}\right) \cdot \eta-\frac{4 B^{2} e^{2} E_{0}^{2}}{k \omega^{2}} \cdot \sin \eta ;$

$$
\begin{align*}
& y=-\frac{2 B e E_{0} \sin \varphi_{0}}{k \omega} \eta-\frac{2 B e E_{0}}{k \omega}\left[\cos \left(\eta+\varphi_{0}\right)-\cos \varphi_{0}\right] \\
& z=-\frac{2 B e E_{0} \cos \varphi_{0}}{k \omega} \eta+\frac{2 B e E_{0}}{k \omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \varphi_{0}\right] \\
& p_{x}=p_{x 0}+\frac{2 B e^{2} E_{0}^{2}}{\omega^{2}}(1-\cos \eta) \\
& p_{y}=\frac{e E_{0}}{\omega}\left[\sin \left(\eta+\varphi_{0}\right)-\sin \varphi_{0}\right] ; \\
& p_{z}=\frac{e E_{0}}{\omega}\left[\cos \left(\eta+\varphi_{0}\right)-\cos \varphi_{0}\right] . \tag{18}
\end{align*}
$$

Taking $x=L_{\text {int }}$ equation (18) gives the relation
$L_{\text {int. }}=\frac{2 B}{k}\left(p_{x 0}+\frac{2 B e^{2} E_{0}^{2}}{\omega^{2}}\right) \cdot \Delta \eta$
$-\frac{4 B^{2} e^{2} E_{0}^{2}}{k \omega^{2}} \cdot \sin \Delta \eta$,
which for $\beta \rightarrow 1$ takes the form $L_{\mathrm{int}}=\lambda \gamma^{2} \frac{\Delta \eta}{\pi}$.
After $\tau_{\mathrm{int}} \approx L_{\mathrm{int}} / c$ the electrons make straight-line motion and hit the detectors along a circle with radius

$$
\begin{align*}
& R=\sqrt{y_{2}^{2}+z_{2}^{2}}=\frac{2 e E_{0}}{k \omega}\left[B^{2} \Delta \eta^{2}+\left(4 B^{2}+\frac{L^{2} k^{2}}{p_{x 1}^{2}}\right.\right.  \tag{20}\\
& \left.\left.+\frac{2 L B k \Delta \eta}{p_{x 1}}\right) \cdot \sin ^{2} \frac{\Delta \eta}{2}-2 B^{2} \Delta \eta \sin \Delta \eta\right]^{1 / 2}
\end{align*}
$$

where now

$$
\begin{align*}
& p_{x 1}=p_{x 0}+\frac{2 B e^{2} E_{0}^{2}}{\omega^{2}}(1-\cos \Delta \eta) ;  \tag{21}\\
& B=\frac{p_{x 0}+\sqrt{m^{2} c^{2}+p_{x 0}^{2}}}{2 m^{2} c^{2}} . \tag{22}
\end{align*}
$$

Thus, in the cases of linear and circular polarizations the electrons make oscillatory and circular motion on the pixel detectors with period $\mathrm{T}=2 \pi / \omega$. If the length $\tau_{b}$ of the electron bunch is less than $T$ then only a part of the circle with length $L_{\tau}$ will be detected. With the help of such an oscilloscope one can measure longitudinal electron distribution and length of fs pulses

$$
\begin{equation*}
\tau_{b}=\frac{L_{\tau} T}{2 \pi R} \tag{23}
\end{equation*}
$$

Various dependences upon the electron and laser parameters have been studied numerically in [24-27].

## 4. BUNCH CHOPPING (SLICING) FOR PRODUCTION OF FEMTOSECOND ELECTRON AND X-RAY PULSES

The method of bunch fs slicing [7,8] is based on the interaction of electron and laser beams in an undulator due to inverse FEL process. However, using the above results one can achieve [22] the same goals without undulator using linearly polarized laser beams (Fig. 3). Indeed, taking electron energy equal to $1.5 \mathrm{GeV}, L_{\text {int }}$ ~ $1 \mathrm{~cm}, L=300 \mathrm{~cm}$ and $\mathrm{CO}_{2}$ laser pulses with wave period
$T_{L}=\lambda / c=33.33 \mathrm{fs}, E_{0}=2 \times 10^{8} \mathrm{~V} / \mathrm{cm}$, with the help of above formulae one obtains $K=2 \times 10^{-5}$, $y_{2}=60 \mu \mathrm{~m}$ which is greater than the ALS e-bunch vertical size in the dipole magnets, $\sigma_{V}=12 \mu \mathrm{~m}$.


Fig. 3. The time structure of the electron, laser and train of $f$ s pulses

If the laser length $\tau_{L}$ is greater than the electron bunch length $\tau_{e}=30 \mathrm{ps}$ then after separating the slices with length $0.1 y_{2}=0.012 \mathrm{~cm}$ one obtains trains of fs pulses consisting of $N_{f s}=\tau_{e} / T_{L}=30 \mathrm{ps} / 33.3 \mathrm{fs}=900$ pulses with length equal to $T_{f s} \approx 0.2 T_{L}=6.66 \mathrm{fs}$ (see Fig. 3). Using Nd:Glass lasers with wavelength 1.06 micron one obtains X-ray pulses with length 0.66 fs . The expected intensities in the case of the proposed method, of course, are much higher than in the case of the method $[7,8]$. The advantage of the method is in the fact that it does not require wigglers, provides fs pulses. However it is necessary to confirm these results by simulations and experimental study.

## 5. DISCUSSION, CONCLUSION, FUTURE WORKS

It has been assumed that the electron beam has no transversal size and angular spread. The former is not essential if the beam cross section is less than the beam transversal deflection and the detector granularity, i.e. the sizes of detector pixels. The latter can be important if the deflection angles are not larger than the beam angular spread. The influences of both these factors as well as the effect of the electron energy spread will be considered in future analytical and simulation works.

In the above derivations it has been assumed that the laser field is sharply limited by the mirrors M1, M2 which besides the field distortions give multiple scattering. As it has been mentioned above it is reasonable to use interaction regions formed by the methods which are under intense investigation in connection with $\gamma \gamma$-colliders and X-ray Compton scattering sources [31, 32].

There are many other problems which can be solved only experimentally. For instance, at present the synchronization between the electron and laser pulses can be carried out with an accuracy of a few tens of fs, using accelerator RF signals, while for the realization of the above methods fs accuracies are required.

As a first step to find the solution of some problems we propose to perform the following experiment (see Fig. 4) that is the laser analog of the RF experiment [15]
with 50 keV electrons. The $\sim 600$ fs pulses from the $\mathrm{CO}_{2}$ (or Nd:Glass) laser are splitted into 3 pulses. After necessary delays the first splitted laser pulses are sent to the photo-cathode of the 50 keV RF electron gun + accelerator providing $\sim 600 \mathrm{fs}$ long electron pulses.


Fig. 4. The scheme of the proposed experiment
The second splitted laser pulses are sent to the "pulse slicing" interaction region (IR) after which one obtains trains of $\sim 18$ (180) deflected e-pulses of 6 (0.6) fs length using slits and apertures and long laser-free region for separating these trains from the long $\sim 600 \mathrm{fs}$ pulses. The third laser pulses are given to the IR "pulse length measurement" which scans the $6(0.6)$ fs e-pulses on the fine granularity detectors. Since the period of scanning is equal to 33.3 (3.33) fs, arcs with length equal only to one fifth of circle will be detected. One can calculate the parameters of the experimental set up as well as of the electron and laser beams using the above results. Thus the concept of the pulse fs slicing and length measurement can be tested.

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# ФЕМТОСЕКУНДНОЕ СКАНИРОВАНИЕ, ЧОППИРОВАНИЕ (ВЫРЕЗАНИЕ) И ИЗМЕРЕНИЕ ДЛИНЫ ЭЛЕКТРОННЫХ БУНЧЕЙ ЛАЗЕРНЫМИ ИМПУЛЬСАМИ И ПРИНЦИПЫ ФЕМТОСЕКУНДНЫХ ОСЦИЛЛОГРАФОВ 

Э.Д. Газазян, В. Гуиди, К.А. Испирян, М.К. Испирян, Д.К. Калантарян

С учетом начальных условий решаются уравнения движения электронов в плоской монохроматической волне с линейной, циркулярной и эллиптической поляризацией. Случаи, когда взаимодействие электронов с полем линейно и циркулярно поляризованными лазерными фотонными пучками происходит в течение конечного времени и электроны детектируются после длинного свободного пространства, будучи сканированными с периодом порядка фемтосекунд, удобны для "фемтослайсинга" и измерения длины фемтосекундных импульсов. Рассматриваются принципы и ограничения построения лазерного фемтосекундного осциллографа и предлагается схема эксперимента.

# ФЕМТОСЕКУНДНЕ СКАНУВАННЯ, ЧОПІРОВАННЯ (ВИРІЗАННЯ) I ВИМІРЮВАННЯ ДОВЖИНИ ЕЛЕКТРОННИХ БУНЧЕЙ ЛАЗЕРНИМИ ІМПУЛЬСАМИ Й ПРИНЦИПИ ФЕМТОСЕКУНДНИХ ОСЦИЛОГРАФІВ 

## Е.Д. Газазян, В. Гуіді, К.А. Іспірян, М.К. Іспірян, Д.К. Калантарян

З урахуванням початкових умов вирішуються рівняння руху електронів у плоскій монохроматичній хвилі $з$ лінійною, циркулярною й еліптичною поляризацією. Випадки, коли взаємодія електронів з полем лінійно й циркулярно поляризованими лазерними фотонними пучками відбувається в перебігу скінченного часу й електрони детектуются після довгого вільного простору, будучі сканованими з періодом порядку фемтосекунд, зручні для "фемтослайсингу" і виміру довжини фемтосекундних імпульсів. Розглядаються принципи й обмеження побудови лазерного фемтосекундного осцилографа й пропонується схема експерименту.

