SELF-SUSTAINED REGIME OF NUCLEAR BURNING WAVE IN U–Pu FAST REACTOR WITH Pb–Bi COOLANT

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The regime of the nuclear burning wave in a fast reactor is described using the non-stationary diffusion equation for neutron transport and equations of the fuel component burn-up and of nuclear kinetics for precursor nuclei of delayed neutrons. A critical two-zone fast reactor of cylindrical form with metal fuel of the U–Pu cycle with Pb–Bi coolant is considered. The neutron leakage in transverse direction was taken into account using the concept of radial buckling. The calculation results of the space-time evolution of neutron flux in this system using the effective multi-group approximation are presented.

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1. INTRODUCTION

The new conception of the safe fast reactor (FR) with the inner safety [1–13] has got further development in the present work. This FR is a new type of long-life operation critical reactor which can solve the problems of radioactive waste transmutation and nuclear proliferation constraints. The main advantage of this FR is that it does not require a reactivity control and therefore the initial fuel composition of the reactor will evolve according to nuclear processes without external intervention during the full FR lifetime without any refueling or fuel shuffling. In this regime the FR is automatically sustained in a state close to the critical one due to the mechanism of reactivity feedback (see [10]). The lifetime of the reactor could be very long without human access inside the FR core during its operation time and so can be placed underground.

The operation of FR is based on the non-linear self-organizing regime of the nuclear burning wave (NBW) that arises owing to a high conversion ratio from fertile to fissile materials in the FR. Feoktistov [1, 2] was the first to show up this regime in a schematic model of FR with the U–Pu fuel cycle. Later Teller et al. [3, 4] proposed a concept of long-life FR with fuel of the Th–U cycle. The corresponding complicated non-stationary problem was solved with help of the Monte-Carlo simulation. Further the concept of NBW (named CANDLE) was developed in the framework of multigroup diffusion approximation [5, 6] using the self-similar solution approach. In [7, 8] authors showed by means of mathematical modeling the possibility of formation of non-linear self-organizing regime in the FR (called as self-adjusting neutron-nuclide regime) that however did not go over into the NBW regime in the non-stationary scheme considered.

The possibility of creating the NBW regime in a linear FR was also confirmed in [9] by calculations carried out for the simple model [1] both in the self-similar solution approach and in the corresponding non-stationary one.

In our previous works [10–12], the space-time evolution of a self-organizing regime in the form of NBW was studied for a plane one-dimensional model of critical FR on the basis of one-group and effective multi-group diffusion approaches. The mathematical formulation of the problem included the solution of the non-stationary diffusion equation for neutron transport together with the burn-up equations for fuel components and the equations of nuclear kinetics for precursor nuclei of delayed neutrons. The questions of stability of the NBW regime relative to distortions of the neutron flux and of the mechanism of reactivity feedback were investigated. The calculations performed in [11, 12] showed that the allowance for the space-time alteration of the neutron spectrum during burn-up process is very important for the numerical simulation of the regime of the propagating NBW. In [13] the approach [11, 12] was further developed for simulating the initiation and propagation of the NBW in a cylindrical FR with U–Pu fuel and Na coolant taking into account the neutron leakage in the transversal direction by using the concept of radial buckling. This made it possible to study the influence of finite transversal size of FR upon rise and evolution of the NBW regime and to find certain variants of parameters of the FR under consideration in which the nuclear burning rate, the energy production intensity and the neutron flux in NBW take values acceptable from the practical point of view.

Since the commonly used in fast neutrons reactors Na coolant seems to lead to rather strong safety constraints on the construction and operation of the reactors, in this work we present the results of calculations in which the Pb-Bi eutectic has been considered as coolant. These investigation is the next step of development of the deterministic approach [13] for description of the space-time evolution of the burn-up process in cylindrical FR with taking into account of the transversal neutron leakage using the radial buckling concept.

2. THE DETAILS OF CALCULATION

We study the space-time evolution of the nuclear burning process in a critical FR by solving the set of partial differential equations that includes the non-stationary neutron diffusion equation, equations of fuel burn-up and equations of nuclear kinetics for precursors of delayed neutrons. The calculations are carried out in the effective multi-group approach [11, 12] in which the effective one-group cross sections are determined with the help of averaging the group cross-sections over the local neutron spectra found from solving the corresponding critical multi-group problem for the given time moment. These effective cross section are used for solving the non-stationary problem under consideration in the one-group approximation.

The one-group non-stationary neutron diffusion equation with taking into account delayed neutrons can be written for the cylindrical model of FR in the form

$$\frac{1}{v}\frac{\partial\Phi}{\partial t}-\frac{\partial}{\partial z}D\frac{\partial\Phi}{\partial z}-\frac{1}{r}\frac{\partial}{\partial r}rD\frac{\partial\Phi}{\partial r}+\Sigma_a\Phi-(1-\overline{\beta})(\nu_f\Sigma_f)\Phi = \sum\nolimits_l\sum\nolimits_i\lambda_l^iC_l^i . \qquad (1)$$

Here Φ is the scalar neutron flux, $\Sigma_\alpha=\Sigma_j\sigma_\alpha^jN_j$ is the macroscopic cross section of the neutron reaction (the index α corresponds to the reactions of neutron absorption (a) and fission (f)), N_j is the concentration of j'th nuclide; σ_α^j is the corresponding effective one-group microscopic cross section; $\nu_f\Sigma_f=\Sigma_j\nu_f^j\sigma_f^jN_j$, ν_f^j is the mean number of neutrons produced at a single nuclear fission event for the j'th fissile nuclide; $\overline{\beta}=\Sigma_j\beta_j(\nu_f\Sigma_f)_j/\nu_f\Sigma_f$ is the effective fraction of delayed neutrons, $\beta_j=\Sigma_i\beta_j^i$, and β_j^i, C_j^i and λ_j^i are the portion of delayed neutrons, the concentration and decay constant of the precursor nuclei in the i'th group of the j'th fissile nuclide, correspondingly; $D=1/(3\Sigma_{tr})$ is the diffusion coefficient, Σ_{tr} is the macroscopic transport cross-section, v is the one-group neutron velocity.

The boundary conditions for the neutron flux Φ are written in the form

$$\left.\left(\Phi-2D\frac{\partial\Phi}{\partial z}\right)\right|_{z=0}=2j_{ex}, \quad \left.\left(\Phi+2D\frac{\partial\Phi}{\partial z}\right)\right|_{z=L}=0. \qquad (2)$$

In (2) j_{ex} is the external neutron flux falling onto the left boundary of FR ($z=0$) while the right boundary ($z=L$) is free.

The conditions at the central axis and at the lateral surface of FR are:

$$\frac{\partial\Phi}{\partial r}=0 \text{ at } r=0\,; \quad \Phi=0 \text{ at } r=R+\delta_{extr}, \qquad (3)$$

where R is the FR radius and $\delta_{extr}=0.71/\Sigma_{tr}$ is the extrapolation length.

These conditions are valid for any moment of time within the considered time interval $0\le t\le T$. Besides, the scalar neutron flux in the corresponding critical assembly Φ_0 is chosen as an initial condition for Φ at the moment $t=0$.

We consider the nuclear burning wave propagation in the axial direction and reduce the problem to a one-dimensional case using the concept of radial buckling that takes an approximate account of the neutron leakage in the transversal direction. Then the equation (1) takes the form

$$\frac{1}{v}\frac{\partial\Phi}{\partial t}+\frac{\partial V}{\partial z}+DB_r^2\Phi+\Sigma_a\Phi-(1-\overline{\beta})(\nu_f\Sigma_f)\Phi = \sum\nolimits_l\sum\nolimits_i\lambda_l^iC_l^i, \quad V=-D\frac{\partial\Phi}{\partial z}, \qquad (4)$$

where $B_r=2.405/(R+\delta_{extr})$ is the radial buckling parameter.

The boundary conditions (2) can be rewritten as

$$\left.(\Phi+2V)\right|_{z=0}=2j_{ex}, \quad \left.(\Phi-2V)\right|_{z=L}=2j_{ex}. \qquad (5)$$

The burn-up equations describe the time evolution of the fuel composition according to the chain of nuclear transformations. In the case of FR with the U–Pu fuel cycle we consider the chain including only 10 nuclides, whose numeration is presented in Table 1:

$$\frac{\partial N_l}{\partial t}=-\left(\sigma_{al}\Phi+\Lambda_l\right)N_l +\left(\sigma_{cl}\Phi+\Lambda_{(l-1)}\right)N_{(l-1)}, \quad l=1\ldots8, \qquad (6)$$

$$\frac{\partial N_9}{\partial t}=\Lambda_6N_6, \qquad (7)$$

$$\frac{\partial N_{10}}{\partial t}=\sum_{l=1,4,5,6,7}\sigma_{fl}N_l\Phi. \qquad (8)$$

Here $\sigma_{al}=\sigma_{cl}+\sigma_{fl}$, σ_{cl} is the microscopic cross section of radiation neutron capture by the l'th nuclide, $\Lambda_l=ln2/T^l_{1/2}$ and $T^l_{1/2}$ are the decay constant and half-life of the l'th nuclide.

It should be noted that using the radial buckling conception we neglect the effects of the fuel nuclide burn-up non-uniformity in the radial direction.

***Table 1**. The numeration of nuclei in the ^{238}U–^{239}Pu transformation chain*

№	1	2	3	4	5
Nucleus	^{238}U	^{239}U	^{239}Np	^{239}Pu	^{240}Pu
№	6	7	8	9	10
Nucleus	^{241}Pu	^{242}Pu	^{243}Am	^{241}Am	FP

In Table 1 FP denotes the fission product nuclide that effectively represents any pair of fission fragments whose properties are averaged for all possible fission events.

The initial values of nuclide concentrations are

$$N_l(z,t=0)=N_l^0(z). \qquad (9)$$

The burn-up of nuclei ^{239}U, ^{239}Np, ^{241}Am, ^{243}Am is neglected ($\sigma_{a2}=\sigma_{a3}=\sigma_{a8}=\sigma_{a9}=0$) since it is small as. The changes of the fission fragments due to the neutron absorption also were not considered.

An important characteristics of nuclear processes in FR is equilibrium concentration of plutonium (see, for example, [7]) which in a stationary state can be written as

$$N_{eq}=\frac{\sigma_c^1N_1}{\sigma_f^4+\sigma_c^4+\sigma_{(n,2n)}^4}, \qquad (10)$$

where $\sigma^4_{(n,2n)}$ is the cross section of the (n,2n) reaction for ^{239}Pu.

At the initial stage of the nuclear burning inside the FR the following expression for N_{eq} is more correct

$$\overline{N_{eq}}=\frac{\Lambda_3N_3}{(\sigma_f^4+\sigma_c^4+\sigma_{(n,2n)}^4)\Phi}. \qquad (11)$$

Since the decay time of the precursor nuclei emitting delayed neutrons is much less than the characteristic time of variation of the scalar neutron flux Φ in the NBW regime (see, for example, [10]), we can use the approximation of one equivalent group of delayed neutrons for the nuclear kinetic equations

$$\frac{\partial C_l}{\partial t} = -\lambda_l C_l + \beta_l (\nu_f \Sigma_f)_l \Phi\,, \tag{12}$$

$$C_l(z, t=0) = C_l^0(z)\,, \tag{13}$$

where $\lambda_l = \beta_l / \sum_i \beta_l^i / \lambda_l^i$.

If the reactivity is less than the effective fraction of delayed neutrons $\overline{\beta}$, the behaviour of FR is completely controlled by delayed neutrons [14]. For this reason, in the considered case of small deviations from the critical state delayed neutrons play an important role in the initiation and stability of the nuclear burning process.

The complete formulation of the non-stationary problem under consideration includes the set of 16 nonlinear partial differential equations and the corresponding initial and boundary conditions for them. For numerical solution of the diffusion equation (4) we use the conservative finite-difference method (see, for example, [15]) and the implicit Crank-Nicolson difference scheme [16] with variable time step. This symmetric-in-time scheme has the approximation of the second order of accuracy in space and time steps and shows an unconditional stability at any relation between them.

The solution of the burn-up equations (4)-(6) and equations of nuclear kinetics (10) has been simplified assuming that the effective one-group cross sections and the neutron flux Φ are constant during the time step τ. The assumption about the constancy of cross-sections is well fulfilled for the FR conditions because of the weak sensitivity of the effective cross-sections to changes in the fast neutron spectrum. The assumption for Φ can be easily satisfied by choosing sufficiently small time intervalsτ. This fact allowed us to obtain an approximate analytical solution of Eqs. (5)-(7) and (10) for the concentrations of the corresponding nuclides (see for details [10]).

Because of non-linearity of the used implicit finite-difference scheme, the neutron flux at a new time layer has been found by means of successive iterations [10].

To calculate the effective one-group microscopic cross sections we use the group neutron fluxes Φ^g (g is the number of neutron energy group) found from solving the stationary multigroup problem. The calculations were performed in the 26-group approximation using the group neutron constants from [17] and [18]. We have used the method of averaging the group cross-sections that takes into account the requirement of conservation of rates of corresponding reactions during this procedure (see, for example, [14]). The scheme of passing from the group microscopic cross sections to the effective one-group cross sections is described in [11,12].

3. RESULTS AND DISCUSSION

The approach described above has been employed to carry out a series of calculations that simulate the initiation and propagation of the NBW in the critical FR under consideration. According to the boundary conditions (5) the process of nuclear burning in the FR is initiated by the external neutron flux j_{ex} coming onto the left end of the system.

Fig.1 presents a schematic layout of the two-zone homogeneous cylindrical FR under consideration. In the reactor a metal U–Pu fuel of porosity p=0.8 is used. Near the left end of the reactor there is an ignition zone in which the uranium fuel is enriched with plutonium with concentration 10%. This value has been chosen so as to be less than the equilibrium plutonium concentration N_{eq} (10). The plutonium in the fuel has the standard isotope composition: ^{239}Pu : ^{240}Pu : ^{241}Pu : ^{242}Pu = 0.70 : 0.22 : 0.05 : 0.03. In the breeding zone that is adjacent to the ignition one and occupies the main part of the FR volume the fuel consists of the fertile ^{238}U (we neglect the presence of ^{235}U in the present FR model). Both zones also include the constructional material Fe and the Pb-Bi eutectic coolant.

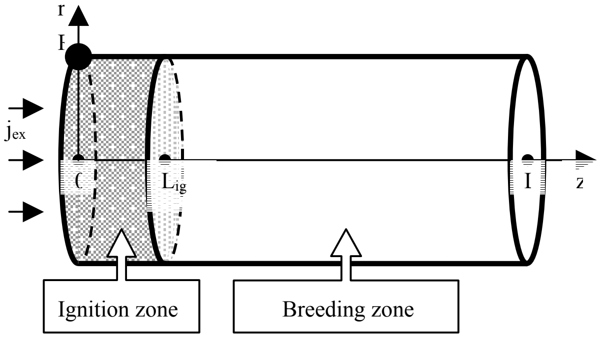

Fig. 1. *The initial critical assembly of FR*

As the neutron flux Φ_0 we choose the eigenfunction for the neutron flux in the initial critical FR normalized to a small magnitude corresponding to the average energy production density in the ignition zone equal to 10^{-8} kW cm^{-3}. The intensity of the external neutron flux that initiates the burning process has been chosen to be j_{ex}=6·10^{11} cm^{-2} s^{-1}. This external neutron flux does not irradiate the ignition zone during all the time of the reactor campaign but it was turned off at an early stage of the FR operation. The time moment t_{off} of the turning off was chosen empirically so that the neutron flux in the system had reached a level high enough for the further development of the nuclear burning process. In the calculations that are presented below the value t_{off} = 10 days has been used.

During the calculations the volume fractions of the FR components in each zone had the following values corresponding to composition of actual reactors: for the nuclear fuel F_{fu} = 44 %, the constructional material F_{Fe} = 20 % and the Pb–Bi coolant F_{cool} = 36 %.

In the calculations we take into consideration the variation of neutron flux and the effective one-group cross sections in the FR with time. Therefore, at each time layer we solve the multi-group problem for fluxes in the critical FR assembly whose composition changes according to the burn-up equations.

The group neutron fluxes $\Phi^g(z)$ found for the corresponding critical assemblies are used to obtain the effective one group cross sections. Thus, during the whole lifetime of FR the cross sections are averaged in the correspondence to the neutron spectrum alteration that occurs. In this case, the values of effective one-group cross sections at each space point are corrected at each time layer according to the fuel composition changing with time.

We have performed a set of variants of calculations of the nuclear burning evolution in reactors with different geometrical sizes. In these calculations we have studied the possibility of initiation and character of propagation of the NBW in the FR under consideration depending on the reactor radius and length. As an example, the results of calculations for the reactor length $L = 500$ cm, its radius $R = 100.5$ cm and the ignition zone width $L_{\text{ig}} = 72.12$ cm are shown in Figs. 2 and 3.

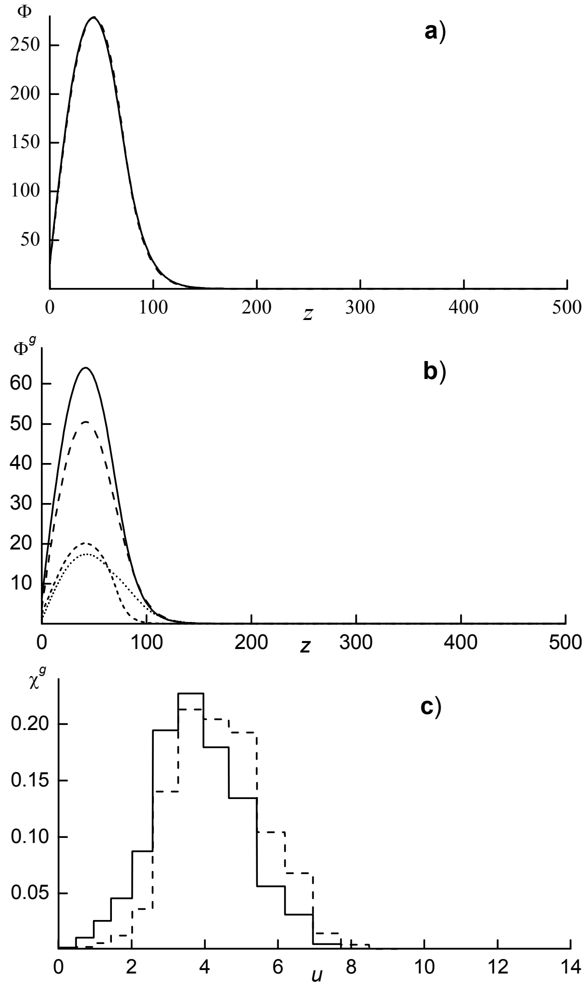

Fig. 2. *Calculations of neutron field inside initial critical FR: a) the neutron flux* Φ *(*$\times 10^{12}$ *neutron/(cm*^{2}*s)) normalized to the average energy production 0.02 kW/cm*^{3} *both summed over 26 groups (solid curve) and obtained in the one-group approach (dashed curve) vs z (in cm); b) the group neutron fluxes* Φ^g*:* $g = 5$ *(*$0.8 < E_n < 1.4$*, short dashes),* $g = 7$ *(*$0.2 < E_n < 0.4$*, solid curve),* $g = 8$ *(*$0.1 < E_n < 0.2$*, long dashes),* $g = 10$ *(*$0.0215 < E_n < 0.0465$*, dots), the bounds of energy groups (MeV) are presented in parentheses; c) the neutron zone spectrum* χ^g *(ignition zone, solid curve) and (breeding zone, dashes) vs lethargy* $u = \ln(10.5/E_n)$*,* E_n *is the neutron energy in MeV*

Fig. 2 presents the results of calculations of the scalar neutron fluxes in a initial critical assembly of FR that were carried out in the 26-group approximation. The parameter values presented above correspond to $k_{eff} = 1$ for the given variant of FR.

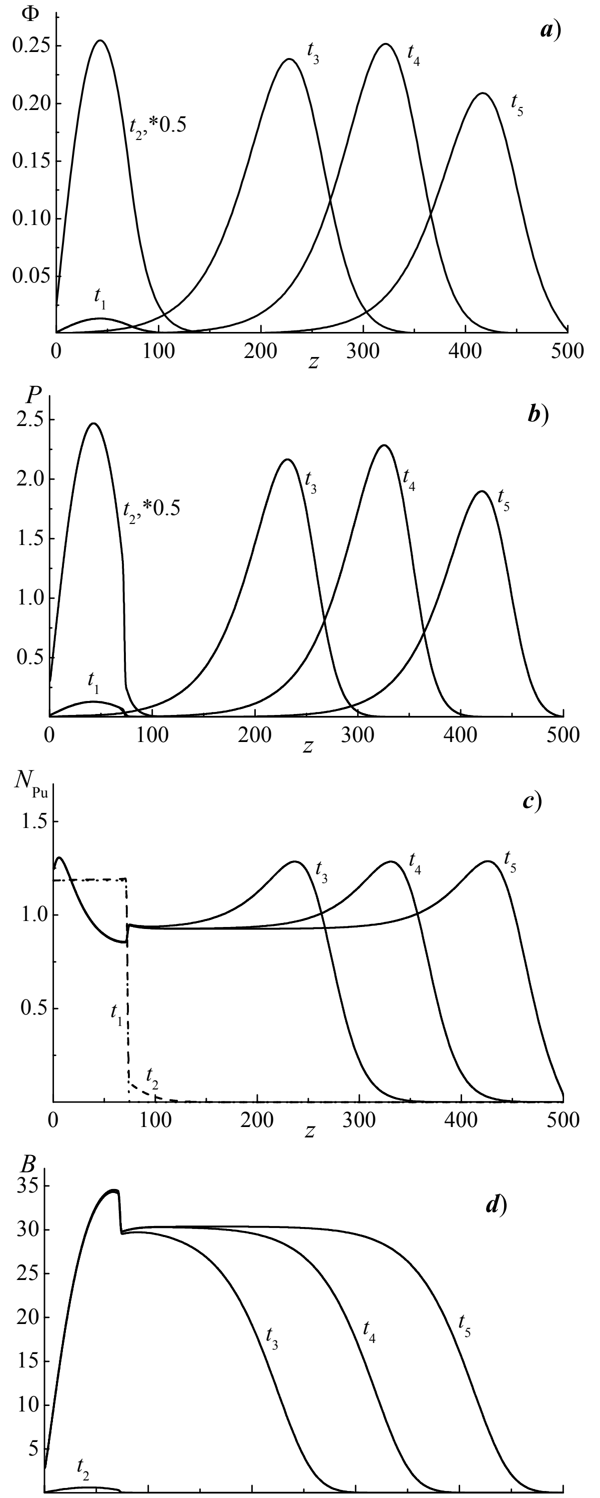

Fig. 3. *The NBW regime initiated by the external neutron flux* $j_{ex} = 6 \cdot 10^{11}$ *cm*$^{-2}$ *s*$^{-1}$ *falling onto the left boundary of FR via the reactor length z (cm): a) the scalar neutron flux* $\Phi(z)$ *(*$\times 10^{17}$ *cm*$^{-2}$*s*$^{-1}$*); b) the power density* $P(z)$ *(kW cm*$^{-3}$*); c) the* ^{239}*Pu concentration* $N_{Pu}(z)$ *(*$\times 10^{21}$ *cm*$^{-3}$*); d) the fuel burn-up depth* $B(z)$ *(%) for* $t_1=5$*,* $t_2=100$*,* $t_3=2500$*,* $t_4=4000$ *and* $t_5=5500$ *days*

In Fig. 2,a we compare the neutron flux $\Phi_S(z)$ summed over 26 groups to the corresponding flux $\Phi(z)$ calculated in the one-group approximation using the effective cross sections $\sigma_\alpha^1(z)$ averaged with 26-group fluxes $\Phi^g(z)$ (the space distributions for some of them are shown in Fig. 2,b). The results of calculations in the multigroup approach and in the considered one-group calculations are close to each other. This fact can warrant the usage of one-group calculations with taking account of the time alteration of the effective one-group cross sections (the effective multigroup approximation), as in the case of the plane one-dimensional model [11, 12], for the simulation of NBW evolution in the cylindrical FR.

The neutron energy spectra in the ignition zone and breeding one represented in Fig. 2,c, are rather hard, the thermal neutrons are practically missing there.

Fig. 3 presents the results of calculations of the main characteristics that describe the burning process in FR. It can be seen that at initial stage of the process the neutron flux Φ and the density of energy production *P* essentially increase (cf. the corresponding curves for the time moments $t_1 = 1$ day and $t_2 = 100$ days). The neutron flux has a rather sharp maximum about the 170-th day and then quickly decreases during next 250 days.

This evolution is clearly seen in Fig. 4 (see the curves for the reactor with *R*=100.5 cm) which presents the integral neutron flux Φ_I being the scalar neutron flux at the reactor axis integrated over its length.

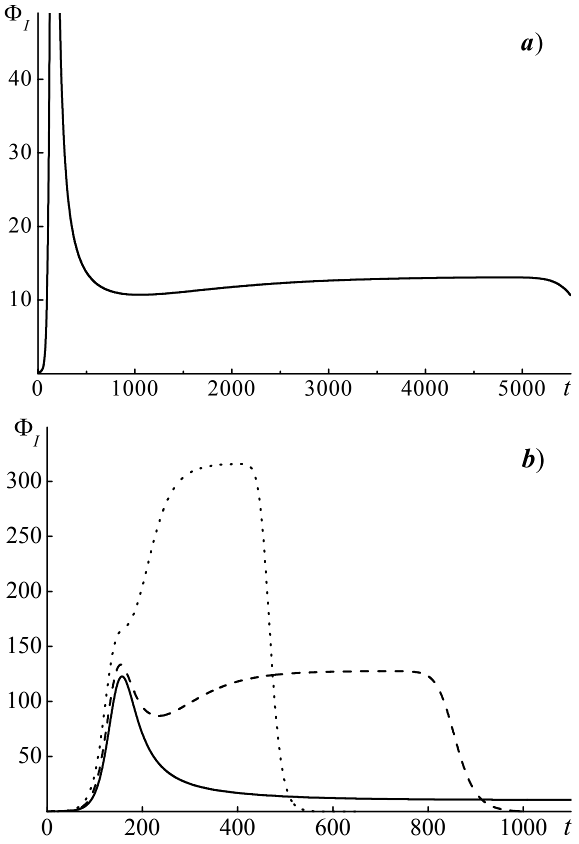

Fig. 4. *The integral (over the reactor axis) neutron flux* Φ_I *(*$\times 10^{17}$ *cm*$^{-1}$ *s*$^{-1}$*) versus time (in days): a) for the full lifetime of the reactor with radius R = 100.5 cm; b) comparison for the reactors with R=100.5 cm (solid curve);, R=110 cm (dashed curve) and R=130 cm (dotted curve)*

Further, the integral neutron flux value does not practically change during the reactor campaign that lasts about 5500 days for the considered reactor (*L*=500 cm, *R*=100.5 cm). At the same time the maxima of the spatial distributions for Φ and *P* move with a constant velocity *V* along the *z*-axis. We define the NBW velocity as the velocity of shifting the scalar neutron flux maximum whose dependence on time is shown in Fig. 5.

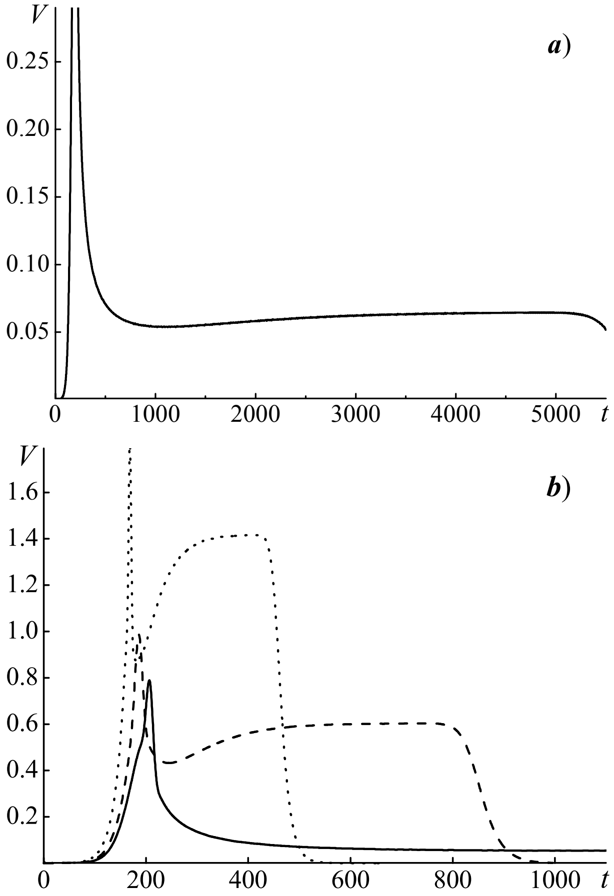

Fig. 5. *The NBW velocity V (cm d*$^{-1}$*) versus time (in days): a) for the full lifetime of the reactor with radius R = 100.5 cm; b) comparison for the reactors with R = 100.5 cm (solid curve); R = 110 cm (dashed curve) and R = 130 cm (dotted curve)*

In this figure one can observe a jump of *V* at the initial stage which corresponds to the sharp maximum of the neutron flux Φ_I and is caused by the quick formation of the NBW front. After this velocity jump a stable propagation of the NBW occurs with an almost constant velocity *V*~ 0.06 cm/d till the reactor extinction. This stage of NBW corresponds to the wave profiles for the moments $t_3 = 2500$ days and $t_4 = 4000$ days in Fig. 3. When the wave approaches the right reactor edge, the NBW velocity decreases and the stage of slow extinction of the burning process begins. The stage of the extinction lasts several hundreds days. A characteristic NBW profile for this stage is presented for the moment $t_5 = 5500$ days. This profile is featured by lower values of the neutron flux and energy production.

At the initial stage of the evolution of burning process (*t*~200 days), there are slow changes of the initial axial distribution of ^{239}Pu (see Fig. 3,c). It is gradually accumulated mainly near the right boundary of the ignition zone due to the transformation of the ^{238}U nuclei through the radiative neutron capture and two successive β-decays. This leads to the formation of the front of the future NBW.

The quickest changes that create the NBW front occur during the period between 150 and 300 days. Further, when the NBW starts to move, we observe a characteristic bump in the ^{239}Pu distribution that is situated just in the zone of the most intense burning. This bump first arises near the ignition zone boundary and then gradually shifts along the reactor axis leaving a certain residual level of plutonium concentration behind it.

When the reactor campaign is over, plutonium and the nuclear fission products are distributed with a practically uniform concentration over the whole length of the reactor, except for the regions close to its boundaries. By the extinction moment the fuel burn-up depth reaches a high level over 30 % in the whole volume of the reactor, except for the parts near the reactor boundaries (see Fig. 3,d). In the breeding zone an intense accumulation of plutonium occurs and the isotope that burns up is practically ^{238}U. This is clearly demonstrated by Fig. 6 that presents the axial distributions of the initial ($t = 0$) and final ($t = 5500$ days) fuel composition in FR.

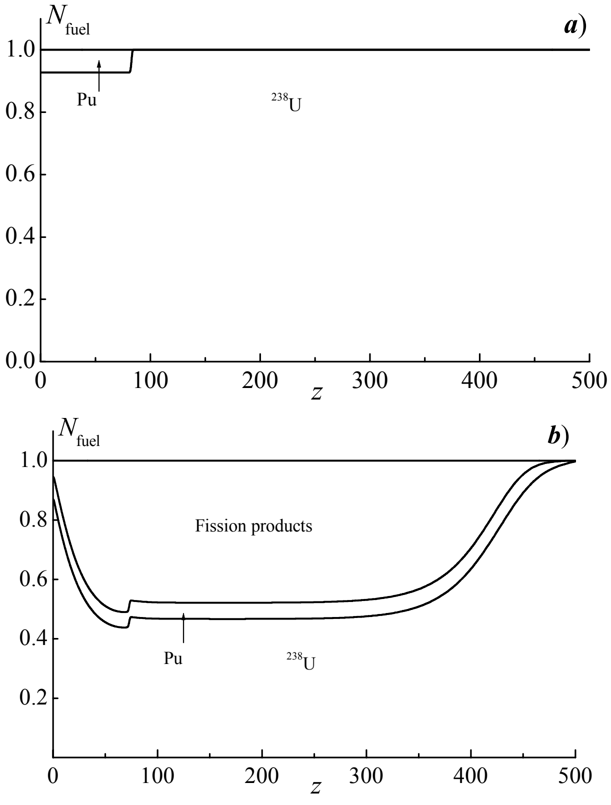

Fig. 6. *The initial (a, t = 0) and final (b, t = 5500 days) distributions of the relative fuel component concentrations in FR via the reactor length z (cm)*

The course of the evolution of nuclear burning process in the FR under consideration strongly depends on the reactor transverse size (the cylinder radius R).

Increasing R leads to the increase of the neutron flux value in the NBW and as a result to a higher NBW velocity. These effects are demonstrated in Figs. 4,b and 5,b, where are presented time dependencies of Φ_I and V for the reactors with $R = 110$ cm (ignition zone width of 70.1 cm) and $R = 130$ cm (ignition zone width of 67.2 cm) together with results for variant with $R = 100.5$ cm (the reactor length in all the cases is 500 cm).

These results show that the reactor with $R = 130$ cm is characterized by a very high neutron flux level in the NBW regime, high NBW velocity of 1.4 cm/day and a very short duration of the reactor campaign.

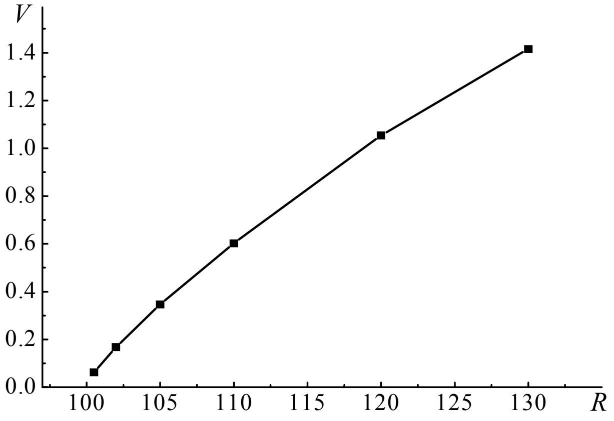

Fig. 7. *Dependence of the NBW velocity V (cm d^{-1}) on the reactor radius R (cm) at the reactor length 500 cm*

Fig. 7 presents the dependence of NBW velocity V on the reactor radius R. From this figure one can see that with R decreasing somewhat below 100.5 cm the velocity V goes to zero. This means that at such small reactor radius values the NBW cannot propagate. Our calculation for a reactor with $R = 100$ cm confirms this conclusion. In this case the neutron flux gradually decreases after reaching its maximum. At that, there is no spatial shift of the burning process along the reactor axis and NBW does not arise. Note that we have not considered the influence of the radial reflector which would lead to decrease of the transversal active zone size necessary for the NBW existence.

Of special interest is the question about the stability of the self-sustaining nuclear burning regime at its initial stage. In Fig. 8 we present the integral neutron flux Φ_I variation during the first 45 days of FR operation. The curve shown in Fig. 8 demonstrates the effect of turning the external neutron flux j_{ex} off at $t_{off} = 10$ days. It is seen that at the moment t_{off} the magnitude of the integral flux in the reactor first sharply decreases. This decrease is followed by an increase of the flux magnitude and then by the flux oscillations being gradually damped. Therefore, the arising perturbation is damped by the reactor itself during approximately 30 days. Turning the external flux off at later time moments t_{off} causes much less perturbation of the neutron flux behavior. Thus, the self-organized burning regime that arose before the moment of turning j_{ex} off is fairly stable.

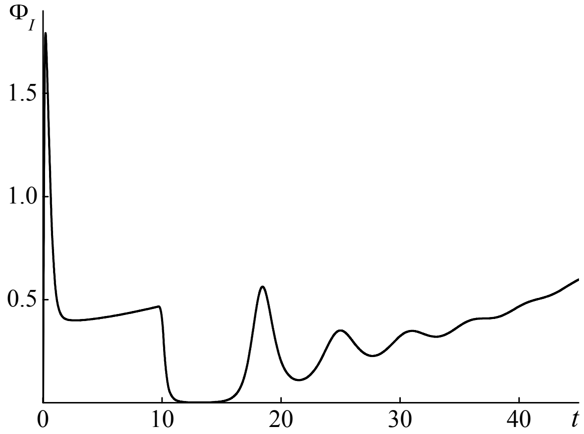

Fig. 8. *The integral (over the reactor axis) neutron flux* Φ_I *(*$\times 10^{17}$ *cm*$^{-1}$ *s*$^{-1}$*) at the initial stage of nuclear burning process versus time (days) for the reactor with radius R = 100.5 cm. The external neutron flux* j_{ex} *is turned off at the time moment* t_{off} *= 10 days. The conditions correspond to Fig. 3*

It is also interesting to consider the time evolution of such an important quantity as the equilibrium ^{239}Pu concentration N_{eq}. Fig. 9 presents the its space distributions calculated according to formulae (10) and (11) at the beginning of the nuclear burning process and at the stage of NBW propagation. It can be seen that these quantities have a considerable time and space variation. At the initial stage of the nuclear burning the values N_{eq} calculated by formulae (10) and (11) significantly differ from each other due to the essential non-stationarity of the process. When the process passes to the stationary NBW regime, the results given by formulae (10) and (11) practically coincide.

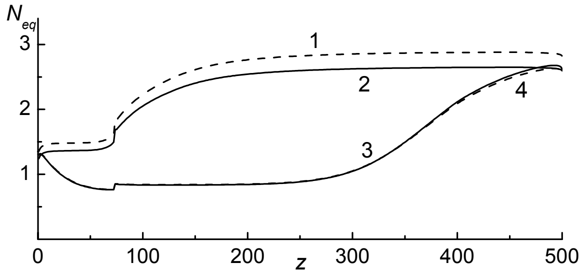

Fig. 9. *The spatial distribution of equilibrium* ^{239}*Pu concentration* N_{eq} *(*$\times 10^{21}$ *cm*$^{-3}$*), calculated by formulae (10) (dashed curves) and (11) (solid curves), for t = 5 days (1 and 2) and t = 4000 days (3 and 4)*

The results of calculations carried out in this work for the cylindrical model of FR with taking account of the transversal neutron leakage using the radial buckling concept show the essential decrease of the NBW velocity as compared to the calculation in the one-dimensional plane model [11, 12]. These results confirm the conclusion that FR in the NBW regime is automatically sustained during a long time (many years) in a state close to the critical one despite the creation of large amount of fission products.

4. CONCLUSIONS

Initiation and evolution of a self-organizing nonlinear regime in the form of a nuclear burning wave in a critical FR with metal fuel of U–Pu cycle and Pb–Bi eutectic alloy coolant has been studied on the basis of solving of the non-stationary neutron diffusion equation together with the set of fuel component burn-up equations and nuclear kinetics equations for the precursors of delayed neutrons. The calculations have been performed for a cylindrical reactor consisting of two axial homogeneous zones: the ignition zone enriched with plutonium and the breeding one. The process of nuclear burning is initiated by an external neutron flux coming into the ignition zone and then propagates along the breeding zone in the axial direction. The problem was reduced to the one-dimensional case by using the radial buckling concept that takes approximate account of the neutron leakage in transversal direction. The calculations were carried out in the framework of effective multigroup diffusion approach that allows for the variation of effective one-group cross sections according to alteration of the group neutron spectrum at each spatial point of FR with time.

The space-time evolution of the nuclear burning process has been studied for different values of radius and length of the FR under consideraion. The calculation results have demonstrated the possibility of arising of a self-organizing regime of a running NBW in the breeding zone along the reactor axis after turning the external neutron flux off at an early stage. The reactor is automatically sustained in the state close to the critical one during a long time. In a particular case of the reactor with the 5 m length and 1.05 m radius, the simulated reactor campaign lasts about 15 years. In this case at the stationary NBW stage the neutron flux reaches about $2.5\cdot 10^{16}$ n/cm^{2}s, the NBW velocity is approximately 22 cm per year, the burn-up depth takes the value more than 30 %. The increasing of the reactor radius leads to an essential increase of the neutron flux and the NBW velocity. Decreasing the reactor radius can make formation of the NBW regime impossible. For example, at the reactor radius of 1.0 m the creation of the running burning wave was not observed in our calculations.

The results obtained in this work for FR with the heavy Pb-Bi coolant are, in many respects, analogous to those obtained when using the Na coolant [13], however, the same NBW velocity values are reached at significantly lower transversal sizes of FR Besides, when Pb–Bi is used, the initial ignition zone has a smaller width than in the case of Na coolant. These facts are caused by the better neutron economy and greater conversion ratio from the fertile ^{238}U into the fissile isotope ^{239}Pu when using the heavy coolant.

The present results show that the distortion of the neutron flux caused by turning the external neutron source off at a very early stage of the process (e.g. 10 days), when the NBW is not formed yet, is damped quickly enough by the reactor itself. This notable stability of the nuclear burning regime in the FR under consideration is due to an intrinsic reactivity feedback governed by the nonlinearity of NBW regime. This feedback prevents the reactor from the runaway regime and ensures the stable evolution of the self-organizing NBW regime.

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САМОПОДДЕРЖИВАЮЩИЙСЯ РЕЖИМ ВОЛНЫ ЯДЕРНОГО ГОРЕНИЯ В U–Pu-РЕАКТОРЕ НА БЫСТРЫХ НЕЙТРОНАХ С Pb–Bi-ТЕПЛОНОСИТЕЛЕМ

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На основе решения нестационарного диффузионного уравнения переноса нейтронов совместно с уравнениями выгорания топлива и кинетики ядер – предшественников запаздывающих нейтронов описан режим волны ядерного горения в реакторе на быстрых нейтронах. Рассмотрен критический двухзонный быстрый реактор цилиндрической формы с металлическим U-Pu-топливом и Pb–Bi-теплоносителем. Для учета поперечной утечки нейтронов использовалась концепция радиального баклинга. Представлены результаты расчетов пространственно-временной эволюции нейтронного потока в этой системе, выполненные в эффективном многогрупповом приближении.

РЕЖИМ ХВИЛІ ЯДЕРНОГО ГОРІННЯ, ЩО САМОПІДТРИМУЄТЬСЯ У U–Pu-РЕАКТОРІ НА ШВИДКИХ НЕЙТРОНАХ З Pb–Bi-ТЕПЛОНОСІЄМ

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На основі рішення нестаціонарного дифузійного рівняння переносу нейтронів сумісно з рівняннями вигоряння палива та кінетики ядер – попередників запізнілих нейтронів описано режим хвилі ядерного горіння у реакторі на швидких нейтронах. Розглянуто критичний двозонний швидкий реактор циліндричної форми з металевим U-Pu-паливом та Pb–Bi-теплоносієм. Для урахування поперечної втрати нейтронів використовувалася концепція радіального баклингу. Представлено результати розрахунків просторово-часової еволюції нейтронного потоку в цій системі, що проводилися в ефективному багатогруповому наближенні.