# VERTEX CLOTHING IN QUANTUM FIELD THEORY 

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The problem of the vertex renormalization in quantum field theory is tackled via the implementation of the unitary clothing transformation method. In the model of charged spinless nucleon and scalar meson fields coupled by the Yukawa-type three-linear interaction the expression for the charge correction in the first non-vanishing (third) order in the coupling constant is derived. Being the off-energy-shell quantity, the expression can be brought to the explicitly covariant form on the energy shell, providing the momentum independence of the charge renormalization.

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## 1. INTRODUCTION

The unitary clothing transformation method proposed by Greenberg and Schweber [1] relying upon the penetrating analyses of the problems of quantum field theory performed by Van Hove [2,3] allows to overcome in a natural way some difficulties one faces in the few-body physics (see, e.g., [4]). Namely, e.g., the primary interaction vertex usually include particles which do not stay simultaneously on their mass shells, therefore the energies of intermediate states in some process can take on arbitrary values. That is why the account for the relativistic effects off the energy shell becomes of high importance while interpreting experimental data for the few-nucleon systems in wide range of energies, including bound states (see, e.g., [5]).

The clothing procedure carried out via the unitary transformation provides the transition from the representation of the initial "bare" particles and interactions towards the representation of the "clothed" particles with observable properties and physical (observed) interactions between them. As the byproducts of clothing, the mass and vertex renormalization programs are performed alongside the construction of the operators of relativistic interactions being Hermitian, energy independent and containing off-energy-shell structures in a natural way.

## 2. UNITARY CLOTHING TRANSFORMATION

The starting point of our consideration is the representation of bare particles with physical masses [6]:

$$
\begin{align*}
& H\left(\alpha_{0}\right)=H_{F}\left(\alpha_{0}\right)+H_{I}\left(\alpha_{0}\right) \\
& =H_{F}\left(\alpha_{0}\right)+V\left(\alpha_{0}\right)+M_{\text {ren }}\left(\alpha_{0}\right)+V_{\text {ren }}\left(\alpha_{0}\right), \tag{1}
\end{align*}
$$

where $H_{F}$ is the free part of Hamiltonian, $V$ is the primary interaction operator, $M_{\text {ren }}$ and $V_{\text {ren }}$ are the usual mass and vertex renormalization counterterms. Symbol $\alpha_{0}$ denotes the set of creation/destruction operators of bare particles with physical masses.

By definition, the one-bare-particle states $\left|\alpha_{0}^{\dagger} \Omega\right\rangle$ which are generated from the vacuum state $\Omega$ by bare creation operators $\alpha_{0}^{\dagger}$ are the eigenstates of the free part of Hamiltonian: However, due to the presence of inter-
action, the same one-particle states are not the eigenstates of the total Hamiltonian:

It is natural to question whether it is possible to find a new set of creation/destruction operators $\alpha_{c}$ in terms of which both free and total Hamiltonians would satisfy the requirements:

$$
\begin{align*}
& H_{F}\left(\alpha_{c}\right)\left|\alpha_{c}^{\dagger} \Omega\right\rangle=E\left|\alpha_{c}^{\dagger} \Omega\right\rangle  \tag{2}\\
& H_{c}\left(\alpha_{c}\right)\left|\alpha_{c}^{\dagger} \Omega\right\rangle=E\left|\alpha_{c}^{\dagger} \Omega\right\rangle \tag{3}
\end{align*}
$$

The set of operators $\alpha_{c}$ called clothed corresponds to particles supposed to have observable properties. Here we assume subscript " $c$ " for the Hamiltonian in terms of clothed particles to emphasize different dependence of the same Hamiltonian on particle operators:

$$
\begin{equation*}
H\left(\alpha_{0}\right)=H_{c}\left(\alpha_{c}\right) \tag{4}
\end{equation*}
$$

In order to keep observables unchanged (i.e., the Soperator intact) Greenberg and Schweber assumed the transformation which would carry out the transition towards the representation of "clothed" particles to be one of a unitary kind:

$$
\begin{align*}
& \alpha_{0}=W\left(\alpha_{c}\right) \alpha_{c} W^{\dagger}\left(\alpha_{c}\right), W W^{\dagger}=W^{\dagger} W=1, \\
& W\left(\alpha_{c}\right)=e^{R\left(\alpha_{c}\right)}, R\left(\alpha_{c}\right)=-R^{\dagger}\left(\alpha_{c}\right) \tag{5}
\end{align*}
$$

The transition between bare and clothed particle representations for an arbitrary operator $O$ having polynomial dependence on the creation/destruction operators is fulfilled in the following manner:
$O\left(\alpha_{0}\right)=W\left(\alpha_{c}\right) O\left(\alpha_{c}\right) W^{\dagger}\left(\alpha_{c}\right)=e^{R\left(\alpha_{c}\right)} O\left(\alpha_{c}\right) e^{-R\left(\alpha_{c}\right)}$
$=O\left(\alpha_{c}\right)+\sum_{k=1}^{\infty} \frac{1}{k!}\left[R\left(\alpha_{c}\right), O\left(\alpha_{c}\right)\right]^{k}$,
where we adopt the denotation for the multiple commutator:

$$
\begin{equation*}
[R, O]^{k} \equiv \underbrace{[R,[R, \ldots[R, O] \ldots]] .}_{k} \tag{7}
\end{equation*}
$$

Applying the transition recipe (6) to the total Hamiltonian operator (1), we find:

$$
\begin{align*}
& H_{c}\left(\alpha_{c}\right)=H_{F}\left(\alpha_{c}\right)+V\left(\alpha_{c}\right)+M_{r e n}\left(\alpha_{c}\right)+V_{r e n}\left(\alpha_{c}\right) \\
& +\sum_{k=1}^{\infty} \frac{1}{k!}\left[R\left(\alpha_{c}\right),\left(H_{F}\left(\alpha_{c}\right)+V\left(\alpha_{c}\right)+M_{r e n}+V_{r e n}\right)\right]^{k} \tag{8}
\end{align*}
$$

If it is supposed that total Hamiltonian (8) satisfies the requirements (2) and (3) the generator $R$ has to be chosen in such a way that the former does not contain terms, called "bad", which simultaneously do not conserve the number of particles (e.g., $M_{r e n}$ ) and prevent the one-particle states to be the eigenstates of the total Hamiltonian (e.g., $V$ ).

Extracting, collecting and removing bad terms of the increasing orders in $g$, we automatically derive the mass and charge shifts and construct the operators of relativistic interactions being Hermitian, energy independent and containing off-energy-shell structures in a natural way.

## 3. MASS AND VERTEX

## RENORMALIZATION PROGRAM

To be more specific, we are going to consider the bad terms elimination procedure in the few lowest orders in $g$. To make the following derivations more transparent, it is convenient to separate several types of operators appearing in $H_{c}\left(\alpha_{c}\right)$. We shall call "transition" the operators, denoted as $O_{t, g}^{(n)}$ and $O_{t, b}^{(n)}$ of the $g^{n}$ order, which consist of more than three creation/destruction operators of any kind. Subscripts " $g$ " and " $b$ " mark "good" operators which refer to the physical processes and "bad" operators which prevent the one-particle states to be the eigenstates of the total Hamiltonian, respectively. The notations $O_{M_{r}}^{(n)}$ and $O_{V_{r}}^{(n)}$ will be used for the "mass-" and "vertex-like" operators of the $g^{n}$ order which replicate the structures of the mass and vertex counterterms $M_{\text {ren }}$ and $V_{\text {ren }}$, respectively. Assuming the latter being expanded in orders of $g: \quad M_{\text {ren }}=\sum_{k=1}^{\infty} M_{\text {ren }}^{(2 k)}, \quad V_{\text {ren }}=\sum_{k=1}^{\infty} V_{\text {ren }}^{(2 k+1)}$, we expect the mass and charge corrections to have the same expansions.

For example, in the model of interacting nucleons and mesons, in which $b^{\dagger}(b)$ and $d^{\dagger}(d)$ are the nucleon and antinucleon creation (destruction) operators while $a^{\dagger}(a)$ state for the mesonic creation (destruction) operators respectively, the term $b^{\dagger} b a^{\dagger} a^{\dagger} a^{\dagger}$ is the bad transition operator, $b^{\dagger} d^{\dagger} b d$ is of the good transition type, $b^{\dagger} b$ and $d^{\dagger} d a^{\dagger}$ are the mass- and vertex-like operators respectively.

Taking explicitly few first terms from Hamiltonian (8), we have:

$$
\begin{aligned}
& H_{c}\left(\alpha_{c}\right)=H_{F}\left(\alpha_{c}\right)+\left[R, H_{F}\right]+V\left(\alpha_{c}\right) \\
& +M_{\text {ren }}\left(\alpha_{c}\right)+[R, V]
\end{aligned}
$$

$$
\begin{equation*}
+V_{r e n}\left(\alpha_{c}\right)+\frac{1}{2}[R, V]^{2}+\left[R, M_{r e n}\right]+\ldots \tag{9}
\end{equation*}
$$

The Hamiltonian operator (9) is expected to contain bad terms of all orders in $g$. Thus, the generator $R$ of the unitary clothing transformation, which is aimed at eliminating them, is supposed to be expanded $R=\sum_{k=1}^{\infty} R^{(k)}$ in orders of $g$ and to have the same structures as "bad" terms contained in Hamiltonian.

In the wide class of field-theoretical models the primary interaction operator $V$ consists totally of the $g^{1}$ order bad terms $H_{b}^{(1)}$. Therefore, we are going to define the generator $R^{(1)}$ in the following way:

$$
\begin{equation*}
H_{b}^{(1)}+\left[R^{(1)}, H_{F}\right]=0 \tag{10}
\end{equation*}
$$

Under this requirement, leaving terms up to the third order in $g$ and baring in mind the notation (7), we find:

$$
\begin{align*}
& H_{c}\left(\alpha_{c}\right)=H_{F}\left(\alpha_{c}\right)+\left[R^{(2)}, H_{F}\right]+\frac{1}{2}\left[R^{(1)}, V\right]+M_{\text {ren }}^{(2)} \\
& +\left[R^{(3)}, H_{F}\right]+\frac{1}{3}\left[R^{(1)}, V\right]^{2}+\left[R^{(1)}, M_{\text {ren }}^{(2)}\right]+V_{\text {ren }}^{(3)}+\ldots .(11 \tag{11}
\end{align*}
$$

To proceed in defining $R$ generator, it is necessary now to collect all of good and bad terms of the $g^{2}$ order. The $g^{2}$ order commutator $\left[R^{(1)}, V\right]$ contains the transition good part the operators of which are responsible for the physical (observable) interactions between physical particles in the second order [4] and the masslike good part the operators of which replicate structures of the good part of $M_{r e n}^{(2)}$. Besides, this commutator contains the transition bad part and the mass-like bad part which replicates structures of the bad part of $M_{\text {ren }}^{(2)}$.

Collecting good mass-like operators, we assume the following equation from which the $g^{2}$ mass corrections (contained in the $\left.M_{r e n, g}^{(2)}\right)$ can be obtained:

$$
\begin{equation*}
M_{r e n, g}^{(2)}+\frac{1}{2}\left[R^{(1)}, V\right]_{M_{r}, g}=0 \tag{12}
\end{equation*}
$$

At the same time, applying the result for the mass shifts to the bad mass-like operators (in $g^{2}$ order), we find, that in general it appears:

$$
\begin{equation*}
M_{r e n, b}^{(2)}+\frac{1}{2}\left[R^{(1)}, V\right]_{M_{r}, b} \equiv M_{r e n, b, r e s t} \neq 0 \tag{13}
\end{equation*}
$$

see Ref. [6,7] Using the outcome of the first step of mass renormalization (12) and (13), we can rewrite Hamiltonian (11) in the form containing only those bad operators of the $g^{2}$ order which are intended to be eliminated via the second clothing:

$$
H_{c}\left(\alpha_{c}\right)=H_{F}\left(\alpha_{c}\right)+\frac{1}{2}\left[R^{(1)}, V\right]_{t, g}
$$

$$
\begin{align*}
& +\left[R^{(2)}, H_{F}\right]+\frac{1}{2}\left[R^{(1)}, V\right]_{t, b}+M_{\text {ren }, b, \text { rest }}^{(2)} \\
& +\left[R^{(3)}, H_{F}\right]+\frac{1}{3}\left[R^{(1)}, V\right]^{2}+\left[R^{(1)}, M_{r e n}^{(2)}\right]+V_{\text {ren }}^{(3)}+\ldots . \tag{14}
\end{align*}
$$

The $R^{(2)}$ generator can be defined now in the similar way as $R^{(1)}$ :

$$
\begin{equation*}
H_{b}^{(2)}+\left[R^{(2)}, H_{F}\right]=0 \tag{15}
\end{equation*}
$$

where $H_{b}^{(2)}=\frac{1}{2}\left[R^{(1)}, V\right]_{t, b}+M_{\text {ren }, b, \text { rest }}^{(2)}$. Thus, after the second clothing the total Hamiltonian reaches the form which contains only good transition operators in the $g^{2}$ order:

$$
\begin{align*}
& H_{c}\left(\alpha_{c}\right)=H_{F}\left(\alpha_{c}\right)+\frac{1}{2}\left[R^{(1)}, V\right]_{t, g}+\left[R^{(3)}, H_{F}\right]+\frac{1}{3}\left[R^{(1)}, V\right]^{2} \\
& +\left[R^{(1)}, M_{r e n}^{(2)}\right]+V_{r e n}^{(3)}+\left[R^{(4)}, H_{F}\right]+M_{\text {ren }}^{(4)}+T^{(4)}+\ldots,(16 \tag{16}
\end{align*}
$$

where operators of the fourth order in $g$ are extracted:

$$
\begin{align*}
& T^{(4)}=\frac{1}{8}\left[R^{(1)}, V\right]^{3}+\frac{1}{2}\left[R^{(2)},\left[R^{(1)}, V\right]_{t, g}\right] \\
& +\frac{1}{2}\left[R^{(2)}, H_{b}^{(2)}\right]+\frac{1}{2}\left[R^{(1)}, M_{\text {ren }}^{(2)}\right]^{2}+\left[R^{(1)}, V_{\text {ren }}^{(3)}\right] . \tag{17}
\end{align*}
$$

To define the generator $R^{(3)}$ we have to collect operators of the third order in $g$. The commutator $\left[R^{(1)}, V\right]^{2}$ can be expanded as: $\left[R^{(1)}, V\right]^{2}=\left[R^{(1)}, V\right]_{t, g}^{2}$ $+\left[R^{(1)}, V\right]_{t, b}^{2}+\left[R^{(1)}, V\right]_{V_{r}}^{2}$ while $\left[R^{(1)}, M_{r e n}^{(2)}\right]$ is assumed to have only the vertex-like part. The charge shift in the $g^{3}$ order can be obtained via collecting vertex-like operators:
$\frac{1}{3}\left[R^{(1)}, V\right]_{V_{r}}^{2}+\left[R^{(1)}, M_{\text {ren }}^{(2)}\right]_{V_{r}}+V_{\text {ren }}^{(3)} \equiv V_{\text {ren,rest }}^{(3)} \neq 0$.
After renormalizing the charge, we are allowed to extract all of the bad terms in the $g^{3}$ order $H_{b}^{(3)}=\frac{1}{3}\left[R^{(1)}, V\right]_{t, b}^{2}+V_{\text {ren,rest }}^{(3)}$, and define the $R^{(3)}:$

$$
\begin{equation*}
H_{b}^{(3)}+\left[R^{(3)}, H_{F}\right]=0 \tag{19}
\end{equation*}
$$

Contrary to the Dyson-Feynman approach, the illustrated clothing procedure has a recursive character. It means that the structure of Hamiltonian in some $n$-th order in $g$ can not be specified until all the corrections of physical constants of $n$ lower orders are fixed and all the bad operators of all $n$ lower orders are removed. Thus, depending on how we determine the operators to remove (those are "bad" after Ref. [1] in our case) and choose the primary interaction, the operators corresponding to physical (observed) processes can acquire quite different forms.

## 4. FIELD THEORETICAL MODEL

Let us implement the developed technique in the simple model of quantum field theory including scalar
mesons and spinless charged nucleons. The interaction operator is chosen in the form of the Yukawa-type three-linear interaction. In this model the explicit dependencies of the operators entering the total Hamiltonian on the creation/destruction operators are as follows:

$$
\begin{align*}
& \quad H_{F}=\int d \mathbf{q} E_{\mathbf{q}} \sum_{i=-1,1} F_{\mathbf{q}}^{i \dagger} F_{\mathbf{q}}^{i}+\int d \mathbf{k} \omega_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} ;  \tag{20}\\
& V=  \tag{21}\\
& =-\frac{g}{(2 \pi)^{3 / 2}} \int \frac{d \mathbf{p} d \mathbf{q} d \mathbf{k}}{\left(8 E_{\mathbf{p}} E_{\mathbf{q}} \omega_{\mathbf{k}}\right)^{1 / 2}} \delta(\mathbf{p - q}+\mathbf{k}) \sum_{i, j} F_{\mathbf{p}}^{i \dagger} F_{\mathbf{q}}^{j} a_{\mathbf{k}}^{\dagger} \\
& +  \tag{22}\\
& \quad H_{\text {ren }}=M_{\text {ren,mes }}+M_{\text {ren,nucl }} ;  \tag{23}\\
&  \tag{24}\\
& M_{\text {ren, mes }}=\frac{\delta \mu^{2}}{4} \int \frac{d \mathbf{k}}{\omega_{\mathbf{k}}}\left(a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}+a_{\mathbf{k}}^{\dagger} a_{-\mathbf{k}}^{\dagger}\right)+H . c . ; \\
&  \tag{25}\\
& M_{\text {ren,nucl }}=\frac{\delta m^{2}}{8} \int \frac{d \mathbf{q}}{E_{\mathbf{q}}} \sum_{i, j=-1,1} F_{\mathbf{q}}^{i \dagger} F_{\mathbf{q}}^{j}+H . c . ; \\
& \\
& V_{\text {ren }}=-\frac{\delta g}{(2 \pi)^{3 / 2}} \int d \mathbf{p} d \mathbf{q} d \mathbf{k} \frac{\delta(\mathbf{p - q}+\mathbf{k})}{\left(8 E_{\mathbf{p}} E_{\mathbf{q}} \omega_{\mathbf{k}}\right)^{1 / 2}} \\
& \quad \times \sum_{i, j=-1,1} F_{\mathbf{p}}^{i \dagger} F_{\mathbf{q}}^{j} a_{\mathbf{k}}^{\dagger}+H . c .,
\end{align*}
$$

where $\delta \mu^{2}=\mu_{0}^{2}-\mu^{2}$ states for the mesonic mass shift with $\mu$ as the physical (observable) mass and $\mu_{0}$ as the bare (trial) one; $\delta m^{2}=m_{0}^{2}-m^{2}$ is the nucleonic mass shift, $m$ and $m_{0}$ are the physical (observable) and bare (trial) nucleonic masses, respectively; $\delta g=g_{0}-g$ is the charge shift depending on the physical charge $g$ and the trial one $g_{0}$. $E_{\mathbf{p}}=\sqrt{m^{2}+\mathbf{p}^{2}}$ is the energy of a nucleon with the momentum $\mathbf{p}, \omega_{\mathbf{k}}=\sqrt{\mu^{2}+\mathbf{k}^{2}}$ is the energy of a meson with the momentum $\mathbf{k}$.

In Eqs. (20)-(25) we adopt the denotations:

$$
F_{\mathbf{q}}^{i \dagger}=\left\{\begin{array}{ll}
b_{\mathbf{q}}^{\dagger} & i=1,  \tag{26}\\
d_{-\mathbf{q}} & i=-1,
\end{array} \quad F_{\mathbf{q}}^{i}=\left\{\begin{array}{cl}
b_{\mathbf{q}} & i=1, \\
d_{-\mathbf{q}}^{\dagger} & i=-1,
\end{array}\right.\right.
$$

where $b_{\mathbf{q}}^{\dagger}\left(b_{\mathbf{q}}\right)$ and $d_{\mathbf{q}}^{\dagger}\left(d_{\mathbf{q}}\right)$ are the creation (destruction) operators of nucleon and antinucleon with the momentum $\mathbf{q}$. Operators $F_{\mathbf{p}}^{i \dagger}$ and $F_{\mathbf{q}}^{i}$ satisfy the following commutation relations:

$$
\begin{equation*}
\left[F_{\mathbf{p}}^{i}, F_{\mathbf{q}}^{j \dagger}\right]=i \delta_{i j} \delta(\mathbf{p}-\mathbf{q}), i, j=1,-1, \tag{27}
\end{equation*}
$$

which follow from the usual commutation relations for the creation/destruction operators of bosons: $\left[b_{\mathbf{p}}, b_{\mathbf{q}}^{\dagger}\right]=\delta(\mathbf{p}-\mathbf{q})$ and $\left[d_{\mathbf{p}}, d_{\mathbf{q}}^{\dagger}\right]=\delta(\mathbf{p}-\mathbf{q}) \cdot a_{\mathbf{k}}^{\dagger}\left(a_{\mathbf{k}}\right)$ is the creation (destruction) operator of a meson with the momentum $\mathbf{k}$ :

$$
\begin{equation*}
\left[a_{\mathbf{k}}, a_{\mathbf{k}^{\prime}}^{\dagger}\right]=\delta\left(\mathbf{k}-\mathbf{k}^{\prime}\right) . \tag{28}
\end{equation*}
$$

## 5. CHARGE CORRECTION

With help of Eq. (10) the $R^{(1)}$ generator acquires the following form replicating the structure of $H_{b}^{(1)}$ :

$$
\begin{align*}
& R^{(1)}=\frac{g}{(2 \pi)^{3 / 2}} \int \frac{d \mathbf{p} d \mathbf{q} d \mathbf{k}}{\left(8 E_{\mathbf{p}} E_{\mathbf{q}} \omega_{\mathbf{k}}\right)^{1 / 2}} \delta(\mathbf{p - q}+\mathbf{k}) \\
& \times \sum_{i, j=1,-1} \frac{1}{i E_{\mathbf{p}}-j E_{\mathbf{q}}+\omega_{\mathbf{k}}} F_{\mathbf{p}}^{i \dagger} F_{\mathbf{q}}^{j} a_{\mathbf{k}}^{\dagger}-H . c . \tag{29}
\end{align*}
$$

$$
\begin{align*}
& \delta g^{(3)}=\frac{g^{3}}{8(2 \pi)^{9 / 2}} \int \frac{d \mathbf{k}^{\prime}}{E_{\mathbf{p}-\mathbf{k}^{\prime}} E_{\mathbf{q}-\mathbf{k}^{\prime}, \omega_{\mathbf{k}^{\prime}}}} \\
& \times\left\{\Delta\left(D_{\mathbf{q}-\mathbf{k}^{\prime}, \mathbf{q}, \mathbf{k}^{\prime}}^{1,-1} D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{q}-\mathbf{k}^{\prime}, \mathbf{k}}^{1,-D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{p}, \mathbf{k}^{\prime}}}\right)+\Delta\left(-D_{\mathbf{q}-\mathbf{k}^{\prime}, \mathbf{q}, \mathbf{k}^{\prime}}^{1,-1}, D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{p}, \mathbf{k}^{\prime}}^{-1,-1},-D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{q}-\mathbf{k}^{\prime}, \mathbf{k}}^{-1,-1}\right)\right. \\
& +\Delta\left(-D_{\mathbf{q}-\mathbf{k}^{\prime}, \mathbf{q}, \mathbf{k}^{\prime}}^{1,1}, D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{q}-\mathbf{k}^{\prime}, \mathbf{k}}^{-1,1}, D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{p}, \mathbf{k}^{\prime}}^{1,1}\right)+\Delta\left(-D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{p}, \mathbf{k}^{\prime}}^{1,}, D_{\mathbf{q}-\mathbf{k}^{\prime}, \mathbf{q}, \mathbf{k}^{\prime}}^{-1,-}, D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{q}-\mathbf{k}^{\prime}, \mathbf{k}}^{1,1}\right) \\
& \left.+\Delta\left(-D_{\mathbf{p}-\mathbf{k}^{\prime} \mathbf{p}, \mathbf{k}^{\prime}}^{1,1}, D_{\mathbf{q}-\mathbf{k}^{\prime}, \mathbf{q}, \mathbf{k}^{\prime}}^{-1,}-D_{\mathbf{p}-\mathbf{k}^{\prime} \mathbf{q}-\mathbf{k}^{\prime}, \mathbf{k}}^{-1,-1}\right)+\Delta\left(-D_{\mathbf{q}-\mathbf{k}^{\prime}, \mathbf{q}, \mathbf{k}^{\prime}}^{1,1}, D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{p}, \mathbf{k}^{\prime}}^{-1,}, D_{\mathbf{p}-\mathbf{k}^{\prime}, \mathbf{q}-\mathbf{k}^{\prime}, \mathbf{k}}^{1,1}\right)\right\}, \tag{30}
\end{align*}
$$

where we adopt the denotations: $D_{\mathbf{p}, \mathbf{q}, \mathbf{k}}^{i, j}=\frac{1}{i E_{\mathbf{p}}+j E_{\mathbf{q}}+\omega_{\mathbf{k}}}$ and $\Delta(a, b, c)=\frac{1}{3}\left(\frac{1}{a b}+\frac{1}{b c}-\frac{2}{a c}\right)$.
Each of the items in Eq. (30) corresponds to one of the six mechanisms responsible for the charge renormalization in the third order in $g$ (see Fig.).


Six mechanisms responsible for the charge renormalization in the third order in $g$

Namely, the first item refers to the diagram $a$, the second item refers to the diagram $b$, etc. The directions of arrows on these graphs differ particles from antiparticles.

Each of the non-covariant propagators $D_{\mathbf{p}, \mathbf{q}, \mathbf{k}}^{i, j}$ corresponds to the vertex on the respective diagram where the energy conservation is not assumed. Thus, the
charge shift, which is determined via the cancellation of the vertex-like operators being the off-energy-shell quantities, appears off the energy shell too.

Therefore, it is important to note that the expression for the charge shift can be presented as the following decomposition:

$$
\begin{equation*}
\delta g^{(3)}=\delta g_{\text {Feynman-like }}^{(3)}+\delta g_{\text {off }}^{(3)}(3) \text { energy-shell }, \tag{31}
\end{equation*}
$$

where the "off-energy-shell" part goes to zero on the energy shell.

The "Feynman-like" part can be brought to the explicitly covariant form on the energy shell, providing the momentum independence of the charge shift derived and giving another representation for that shift obtained within the Dyson-Feynman approach:

$$
\begin{align*}
& \delta g_{\text {Feynman-like }}^{(3)}=\frac{1}{2} \frac{g^{3}}{(2 \pi)^{3}}\left[\int \frac{d \mathbf{p}^{\prime}}{E_{\mathbf{p}^{\prime}}}\left(\frac{1}{\left(\mu^{2}-2 p^{\prime} k\right)\left(\mu^{2}-2 m^{2}-2 p^{\prime} p\right)}\right)\right. \\
& \left.-\int \frac{d^{3} \mathbf{k}^{\prime}}{\omega_{\mathbf{k}^{\prime}}}\left(\frac{1}{\left(\mu^{2}+2 k^{\prime} p\right)\left(\mu^{2}+2 k^{\prime} q\right)}\right)+\int \frac{d^{3} \mathbf{p}^{\prime}}{E_{\mathbf{p}^{\prime}}}\left(\frac{1}{\left(\mu^{2}+2 p^{\prime} k\right)\left(\mu^{2}-2 m^{2}+2 p^{\prime} q\right)}\right)\right] \tag{32}
\end{align*}
$$

where $q=\left(E_{q}, \mathbf{q}\right), p=\left(E_{p}, \mathbf{p}\right), p^{\prime}=\left(E_{p^{\prime}}, \mathbf{p}^{\prime}\right), k=\left(\omega_{k}, \mathbf{k}\right)$, $k^{\prime}=\left(\omega_{k^{\prime}}, \mathbf{k}^{\prime}\right)$. The momentum conservation for that vertex has the form $\mathbf{q}=\mathbf{p}+\mathbf{k}$.

## 6. CONCLUSION

The charge shift in the third order in the coupling constant $g$ is obtained as the byproduct of the clothing procedure by means of collecting operators off the energy shell. Six mechanisms of the charge renormalization in the third order are generated by the products of non-covariant propagators, typical of the old-fashioned perturbation theory, forming the expression for the charge correction. Each of these propagators marks the vertex on the respective diagram in which the energy conservation is not assumed.

Being an object off the energy shell, the expression for the charge shift acquires the explicitly covariant form on the energy shell, giving another representation to the respective Dyson-Feynman result and providing the momentum independence of the charge shift.

Having a recursive feature, the clothing procedure gives an expectation that the account for operators off the energy shell in the Hamiltonian could lead to new physical results in higher orders in the coupling constant, just to mention the problem of calculating the $\pi \mathrm{NN}$ form-factors in nuclear physics.

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## ОДЕВАНИЕ ВЕРШИНЫ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ

## В.Ю. Корда, И.В. Елецких

С помощью метода унитарного одевающего преобразования изучена проблема перенормировки вершины в квантовой теории поля. В модели, описывающей заряженное бесспиновое нуклонное и скалярное мезонное поля, взаимодействующие посредством трилинейной связи типа Юкавы, получено выражение для сдвига заряда в третьем порядке по константе связи. Будучи величиной вне энергетической оболочки, найденное выражение может быть представлено в явно ковариантной форме на энергетической оболочке, обеспечивая независимость перенормировки заряда от импульсов частиц.

## ОДЯГАННЯ ВЕРШИНИ В КВАНТОВІЙ ТЕОРІЇ ПОЛЯ

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За допомогою методу унітарного одягаючого перетворення досліджено проблему перенормування вершини в квантовій теорії поля. У моделі, яка описує заряджене безспінове нуклонне і скалярне мезонне поля, що взаємодіють через трилінійний зв’язок типу Юкави, знайдено вираз для зсуву заряду в третьому порядку за константою зв’язку. Розрахований вираз визначено поза енергетичною оболонкою, проте його можна подати в явно коваріантній формі на енергетичній оболонці, що забезпечує незалежність перенормування заряду від імпульсів частинок.

