Section A. QUANTUM FIELD THEORY

THE TWISTOR STRING: A NEW FRAMEWORK TO STUDY YANG-MILLS THEORIES, AND ITS SPACETIME FORMULATION

I.A. Bandos[1,2]*, J.A. de Azcárraga*[1]*, and C. Miquel-España*[1]

[1]*Dept.of Theoretical Physics, Valencia University, and IFIC (CSIC-UVEG) 46100-Burjassot (Valencia), Spain;*
[2]*on leave from ITP, NSC KIPT, Kharkov, Ukraine;*
e-mails: bandos@ific.uv.es; j.a.de.azcarraga@ific.uv.es; cesar.miquel@ific.uv.es

We give a brief review of the twistor string approach to supersymmetric Yang-Mills theories with an emphasis on the different formulations of (super)string models in supertwistor space and their superspace form. We discuss the classical equivalence among the Siegel closed twistor string action and the Lorentz harmonics formulation of the (N=4) tensionless superstring, and notice the possible relation of the twistor string to the (D=10) Green-Schwarz superstring action, as well as to models in the enlarged, tensorial superspaces that are relevant in higher spin theories.

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1. INTRODUCTION

At the end of 2003, E. Witten looked again [1] at the connection between N=4 supersymmetric Yang-Mills (SYM) theory and string theory using the twistor approach [2]. In contrast to the conventional AdS/CFT correspondence (between string theory in five-dimensional anti-de Sitter space [AdS] and conformal field theories [CFT] in four-dimensional Minkowski space, the conformal boundary of AdS_5) [4], the relation above establishes a link between the weak coupling limits on both sides and, hence, it can be checked perturbatively. This led to the development of a new technique to compute gauge theory amplitudes [3] (the Cachazo-Svrček-Witten [CSW] or MHV diagram rules, see below) and renewed the interest in the Penrose twistor program [2] of replacing spacetime by twistors.

2. MHV YANG-MILLS AMPLITUDES, TWISTORS AND SUPERTWISTORS

2.1. MHV AMPLITUDES THROUGH BOSONIC SPINORS

The amplitude for the scattering of *n* gauge bosons with 2 positive and (*n* − 2) negative helicities has the +form (see [5, 6] and refs. therein)

$$A(1,2,\ldots,n) = 2^{\frac{n}{2}} i g^{n-2} Tr(t^{a_1}\ldots t^{a_n}) \frac{<IJ>^4}{<12><23>\ldots<n1>}, \quad (1)$$

where $2^{\frac{n}{2}} i$ is a numerical factor, *g* is the YM coupling constant, $Tr(t^{a_1}\ldots t^{a_n})$ is the trace of the product of *n* gauge group generators, and the 'one-particle matrix elements' <12 >, ... , < *n*1 > are expressed through the contraction of two bosonic spinors, one for each gluon state, *e.g.*,

$$<12>= \lambda^{\alpha 1}\lambda^2_\alpha = \varepsilon^{\alpha\beta}\lambda^1_\beta\lambda^2_\alpha \equiv -\lambda^{\alpha 2}\lambda^1_\alpha = -<21>. \quad (2)$$

Specifically, *I* and *J* in (1) ($A(1,2,\ldots,n) \propto <IJ>^4$) refer to two positive helicity gluons, while the gluons with $n \neq I, J$ are assumed to have negative helicity.

Notice that the maximally helicity violating or MHV amplitude (1) is holomorphic: it depends on the λ's through their contractions <12>, etc. In contrast, the complex conjugate spinors $\bar\lambda^1_{\dot\alpha} := (\lambda^1_\alpha)^*, \ldots, \bar\lambda^n_{\dot\alpha} := (\lambda^n_\alpha)^*$ etc., the contractions of which are denoted by $[1,2] := \bar\lambda^{\dot\alpha 1}\bar\lambda^2_{\dot\alpha} \equiv -[2,1]$, are not present in (2) (they are involved in the conjugate, 'mostly plus', MHV amplitudes where all the *n* gluons but two have positive helicity).

One may ask, why the amplitudes can be expressed in terms of just *n* bosonic spinors λ^i_α ? The answer is that a bosonic spinor can be used to describe the *on-shell* momentum and the helicity of a massless particle. The momentum of such particle is light-like, $p^2 := p_\mu p^\mu = 0$, a condition that is solved in spinor space by the Penrose expression which gives p_μ in terms of a single bosonic spinor [2],

$$p_{\alpha\dot\alpha} := p_\mu \sigma^\mu_{\alpha\dot\alpha} = \lambda_\alpha \bar\lambda_{\dot\alpha} \Leftrightarrow p_\mu = \frac{1}{2}\lambda\sigma_\mu\bar\lambda := \frac{1}{2}\sigma_{\mu\alpha\dot\alpha}\lambda^\alpha\bar\lambda^{\dot\alpha}. \quad (3)$$

The light-likeness of the vector p_μ in (3) follows from $\sigma^\mu\tilde\sigma^\nu + \sigma^\nu\tilde\sigma^\mu = 2g^{\mu\nu}$ for the relativistic Pauli matrices $\sigma^\mu := \sigma^\mu_{\alpha\dot\alpha}, \tilde\sigma^\mu := \tilde\sigma^{\mu\dot\alpha\alpha}$ which gives $p_{\alpha\dot\alpha}\tilde p^{\dot\alpha\beta} = p^2\delta^\beta_\alpha = 0$ since $p_{\alpha\dot\alpha}\tilde p^{\dot\alpha\beta} = \lambda_\alpha\bar\lambda_{\dot\alpha}\bar\lambda^{\dot\alpha}\lambda^\beta$ vanishes identically,

$$[\bar\lambda, \bar\lambda] = \bar\lambda^{\dot\alpha}\bar\lambda_{\dot\alpha} \equiv \varepsilon^{\dot\alpha\dot\beta}\bar\lambda_{\dot\alpha}\bar\lambda_{\dot\beta} = 0 \; (\varepsilon^{\dot\alpha\dot\beta} = -\varepsilon^{\dot\beta\dot\alpha}).$$

The gluon polarization vector ε^μ is defined to be lightlike and orthogonal to the lightlike momentum, $\varepsilon^2 = 0, \varepsilon^\mu p_\mu = 0$. This can be again solved in terms of the same bosonic spinor λ or its complex conjugate $\bar\lambda$. The two basic solutions corresponding to negative and positive helicity particles,

$$\varepsilon^{-i}_{\alpha\dot\alpha} = \frac{\lambda^i_\alpha \bar u_{\dot\alpha}}{\left[\bar\lambda^{(i)},\bar u\right]}\ , \qquad \varepsilon^{+i}_{\alpha\dot\alpha} = \frac{u_\alpha \bar\lambda^{(i)}_{\dot\alpha}}{< u, \lambda^{(i)} >}\ , \tag{4}$$

are constructed in terms of λ and $\bar\lambda$ and contain a constant reference spinor $\bar u_{\dot\alpha}$ ($u_\alpha = (\bar u_{\dot\alpha})^*$). The relativistic invariance condition corresponds to the requirement that the observable quantities are independent of the choice of the reference spinor u.

Thus, all the kinematical information on the *on-shell* state of a gauge boson, its helicity and its lightlike momentum, are encoded in a single bosonic spinor. This is the reason why the scattering vertex amplitude for n gauge bosons can be condensed in an expression written in terms of bosonic spinors, as (1) above.

2.2. THE CACHAZO-SVRČEK-WITTEN OR MHV DIAGRAM TECHNIQUE INSPIRED BY THE TWISTOR STRING

The diagram technique proposed by Cachazo, Svrček and Witten [3] consists in cutting the Feynman diagram in MHV pieces (which is always possible [3]) and then treating them as vertices connected by scalar propagators. Clearly, there is an immediate problem: by cutting a Feynman diagram into MHV pieces, one gets generally subdiagrams in which one or more legs correspond to virtual particles, *i.e.* particles that are off-shell and for which the basic Penrose representation (3) does not hold. The prescription proposed in [3], and checked to be true inside and beyond [6] the domain of the original N=4 supersymmetric Yang-Mills (SYM) context, is to associate to a virtual particle the bosonic spinor defined by $\lambda_\alpha(p) = p_{\alpha\dot\alpha}\varpi^{\dot\alpha}$, where $\varpi^{\dot\alpha}$ is an arbitrary reference spinor. Relativistic invariance then requires that the amplitude A is independent of the choice of the reference spino, $\partial A/\partial\varpi = 0$. This condition has been shown to hold for tree and one-loop diagrams in N=4 SYM theory, and checked for some two-loop and some non- and less supersymmetric theories [6].

The equivalence of the MHV and the Feynman diagram calculus was originally proved for the tree diagrams of N=4 SYM theory [3]. It was then extended to one-loop diagrams [3] and also checked for some higher-loop ones as well as for less supersymmetric (N=2, N=1 and non-supersymmetric N=0) YM theories (see [6] for a recent review). However, the original version was developed for N=4 SYM theories and was inspired by the twistor string model [1], which possesses N=4 supersymmetry and is a string model formulated in the space of N=4 supertwistors. The action principles which lie beyond the twistor string [1,7,8] and its spacetime (superspace) formulation will be the main subject here.

To begin our discussion, we should briefly address two questions: 1) what is a twistor? and 2) what is a supertwistor?

2.3. PENROSE TWISTORS AND FERBER SUPERTWISTORS

A twistor [2] can be understood as a Dirac spinor; it has four complex components in two Weyl spinors, $\mathrm{Y}^{\hat\alpha} = (\lambda_\alpha, \mu^{\dot\alpha}) \in \mathrm{C}^4$, and provides the spinorial representation for the conformal group $SO(2, 4)$ as well as the fundamental representation for the (locally isomorphic) SU(2, 2) ≈ Spin(2, 4) group.

Twistor space is considered to be a projective space because twistors that differ in a complex scale parameter z are identified,

$$\mathrm{Y}^{\hat\alpha} \approx z\mathrm{Y}^{\hat\alpha} \Rightarrow \mathrm{Y}^{\hat\alpha} = (\lambda_\alpha, \mu^{\dot\alpha}) \in \mathrm{CP}^3\,. \tag{5}$$

The reason for this identification is the obvious complex scale invariance of the Penrose incidence relation

$$\mu^{\dot\alpha} = x^{\dot\alpha\alpha}\lambda_\alpha\ , \quad x^{\dot\alpha\alpha} := x^\mu \tilde\sigma^{\dot\alpha\alpha}_\mu\ , \tag{6}$$

which defines a spacetime point x^μ or, more precisely, a lightlike line in Minkowski space, $\hat x^{\dot\alpha\alpha}(\tau, x) = x^{\dot\alpha\alpha} + \tau\lambda^\alpha\bar\lambda^{\dot\alpha}$. The incidence relation (6) provides the general solution for the constraint $\bar{\mathrm{Y}}_{\hat\alpha}\mathrm{Y}^{\hat\alpha} := \bar\lambda_{\dot\alpha}\mu^{\dot\alpha} - \bar\mu^\alpha\lambda_\alpha = 0$. [In a twistor formulation of the massless superparticle, the quantum counterpart of this constraint fixes the helicity s, which appears as a constant, $2s$, in its *r.h.s.*].

Supertwistors, the supersymmetric generalization of the Penrose twistors, were introduced by Ferber [9]. The N-extended supertwistor,

$\mathrm{Y}^\Sigma := (\mathrm{Y}^{\hat\alpha}, \eta_i) = (\lambda_\alpha, \mu^{\dot\alpha}, \eta_i) \in \mathrm{C}^{(4|N)}$, ($i$=1,…,N), includes, in addition to the two Weyl spinors in the twistor $\mathrm{Y}^{\hat\alpha}$, N fermionic (Grassmann odd) variables $\eta_i, i = 1,\ldots,N$, and defines the fundamental representation space of SU(2,2|N).

Using the complex scaling $\mathrm{Y}^\Sigma \approx z\mathrm{Y}^\Sigma$ as an equivalence relation, the supertwistors become homogeneous coordinates of the projective superspace $\mathrm{CP}^{(3|N)}$,

$$\begin{aligned} &\mathrm{Y}^\Sigma := (\mathrm{Y}^\alpha, \eta_i) = (\lambda_\alpha, \mu^{\dot\alpha}, \eta_i) \in \mathrm{CP}^{(3|N)};\\ &\eta_i\eta_j = -\eta_j\eta_i;\\ &i = 1,\ldots,N. \end{aligned} \tag{7}$$

The above scaling $\mathrm{Y}^\Sigma \approx z\mathrm{Y}^\Sigma$ appears as a symmetry of the Penrose-Ferber incidence relations,

$$\begin{aligned} &\mu^{\dot\alpha} = x_L^{\dot\alpha\alpha}\lambda_\alpha, \quad \eta_i = \theta^\alpha_i\lambda_\alpha\\ &x_L^{\dot\alpha\alpha} := x_L^\mu\tilde\sigma^{\dot\alpha\alpha}_\mu := x^{\dot\alpha\alpha} + 2i\theta^\alpha_i\bar\theta^{\dot\alpha i}, \end{aligned} \tag{8}$$

which involve the coordinates $Z^M := (x^\mu, \theta^\alpha_i, \bar\theta^{\dot\alpha i})$ of N-extended D=4 superspace and define a (1|N)-dimensional subsuperspace $R^{(1|N)}$ in this N-extended superspace

$$\begin{aligned} &\hat x^{\dot\alpha\alpha} = x^{\dot\alpha\alpha} + \tau\lambda^\alpha\bar\lambda^{\dot\alpha}, \quad \hat\theta^\alpha_i = \theta^\alpha_i + \kappa_i\lambda^\alpha,\\ &\left\{(\tau, \kappa^i)\right\} = R^{(1|N)}. \end{aligned} \tag{9}$$

where the κ_i are N fermionic parameters. This $R^{(1|N)}$ is the Sorokin-Tkach-Volkov-Zheltukhin worldline superspace [10], the simplest example of the superworldvolume of the superembedding approach to superbranes [11, 12]; superworldlines of this type were first introduced in the context of the spinning superparticle [13].

3. TWISTOR STRING ACTION(S)

The basic worldsheet fields of the twistor string models are the supertwistors (7). At present there are three main versions of the twistor string action: (i) the constrained $CP^{(3|4)}$ sigma model by Witten [1]; (ii) the open string model by Berkovits [7] involving *two* supertwistors; and (iii) the simplest one, proposed by Siegel in [8] (see [14] for further discussion and references).

The action for the Siegel closed string model is given by [8]

$$S = \int_{W^2} e^{++} \wedge \bar{Y}_\Sigma \nabla Y^\Sigma + d^2\xi L_G = \int d^2\xi \left[\sqrt{|\gamma(\xi)|}\bar{Y}_\Sigma(\xi)\nabla_{--}Y^\Sigma(\xi) + L_G\right], \quad (10)$$

where $\bar{Y}_\Sigma := (Y^\Pi)^* \Omega_{\Pi\Sigma} = (\lambda_\alpha, -\mu^\alpha; 2i\bar{\eta}^i)$ is the SU(2,2|N)-adjoint of Y^Σ (eq. (7)), $e^{\pm\pm} = d\xi^m e_m^{\pm\pm}(\xi)$ are the worldsheet zweibein one-forms and $e^{++} \wedge e^{--} = d^2\xi\sqrt{|\gamma|}$ is the invariant surface element of the worldsheet W^2. The covariant derivative $\nabla = e^{++}\nabla_{++} + e^{--}\nabla_{--} = d - iB$ involves the U(1)-connection *B*, which serves as a Lagrange multiplier for the constraint

$$\bar{Y}_\Sigma Y^\Sigma = \bar{\lambda}_{\dot\alpha}\mu^{\dot\alpha} - \bar{\mu}^\alpha \lambda_\alpha + 2i\bar{\eta}^i\eta_i = 0 . \quad (11)$$

Finally, in (10), L_G is the Lagrangian for the worldsheet fields that are used to construct the Yang-Mills symmetry current. As noted in [7], one can use *e.g.*, the worldsheet fermionic fields ψ^I in the fundamental representation of the gauge group. Then,

$$d^2\xi L_G = \frac{1}{2} e^{++} \wedge (\bar{\psi}_I d\psi^I - d\bar{\psi}_I \psi^I)$$

in the notation of [14].

The Lagrangian of the open string model (ii) by Berkovits [7] is given by

$$S = \int_{W^2} e^{++} \wedge \bar{Y}^-_\Sigma \nabla(Y^{-\Sigma}) - e^{--} \wedge \bar{Y}^+_\Sigma \nabla(Y^{+\Sigma}) + \int_{W^2} d^2\xi (L^L_G + L^R_G) .$$

It contains two supertwistors, one left-moving $Y^{-\Sigma}$ and one right-moving $Y^{+\Sigma}$, and also two copies of the 'YM current' degrees of freedom, which are 'glued' by boundary conditions on ∂W^2. The Lagrangian form integrated over the open worldsheet W^2 is actually the sum of Siegel's Lagrangian in (10) and its right-moving counterpart. Finally, the original action for the $CP^{(3|4)}$ twistor string model (i) by Witten [1], expressed in the present notation, can be found in [14].

4. THE TWISTOR STRING AS A TENSIONLESS SUPERSTRING

As it was shown in [14], an equivalent form of the action (10) is given by the tensionless superstring action from [15,16] (called twistor-like Lorentz-harmonics formulation of the null superstring for reasons explained in [14]). This means that, ignoring its YM part, the action (10) can be written in D=4, N=4 superspace as

$$S = \int_{W^2} e^{++} \wedge \hat{\Pi}^{\alpha\dot\alpha} \bar{\lambda}_{\dot\alpha}\lambda_\alpha = \int_{W^2} e^{++} \wedge (dx^{\alpha\dot\alpha} - id\theta^\alpha_i \bar{\theta}^{\dot\alpha i} + i\theta^\alpha_i d\bar{\theta}^{\dot\alpha i})\bar{\lambda}_{\dot\alpha}\lambda_\alpha , \quad (12)$$

where $\hat{\Pi}^{\alpha\dot\alpha} \equiv d\xi^m \Pi_m^{\alpha\dot\alpha} \equiv d\tau \Pi_\tau^{\alpha\dot\alpha} + d\sigma \Pi_\sigma^{\alpha\dot\alpha}$ is the pull-back to the worldsheet W^2 of the flat supervielbein on D=4, N=4 superspace,
$\Pi^{\alpha\dot\alpha} := dx^{\alpha\dot\alpha} - id\theta^\alpha_i \bar{\theta}^{\dot\alpha i} + i\theta^\alpha_i d\bar{\theta}^{\dot\alpha i}$. This action possesses an irreducible κ-symmetry

$$\delta_\kappa x^{\alpha\dot\alpha} = i\delta_\kappa\theta^\alpha_i \bar{\theta}^{\dot\alpha i} - i\theta^\alpha_i \delta_\kappa \bar{\theta}^{\dot\alpha i} , \quad \delta_\kappa \theta^\alpha_i = \kappa_i \lambda^\alpha , \quad \delta_\kappa \bar{\theta}^{\dot\alpha i} = \bar{\kappa}^i \bar{\lambda}^{\dot\alpha} , \quad (13)$$

$$\delta_\kappa \lambda^\alpha = \delta_\kappa \bar{\lambda}^{\dot\alpha} = \delta_\kappa e^{++} = 0 .$$

This is obtained from the infinitely reducible κ-symmetry [17], with $\delta_\kappa \theta^\alpha_i = \kappa_{\dot\alpha i}\Pi^{\alpha\dot\alpha}_{--}$, $\delta_\kappa \bar{\theta}^{\dot\alpha i} = \Pi^{\alpha\dot\alpha}_{--}\bar{\kappa}^i_\alpha$, by using the relation $\Pi^{\alpha\dot\alpha}_{--} \approx \bar{\lambda}^{\dot\alpha}\lambda^\alpha$ which provides the general solution of the equations of motion for the bosonic spinor field λ (which is an auxiliary field in the action (12)). The κ-symmetry reduces the number of degrees of freedom to the same 8+(16/2) of the twistor string (10).

The simplest way to check the equivalence [14] of (12) to the first, supertwistor part of the Siegel twistor closed action (10), is to use Leibniz's rule to move the derivative to act on the bosonic spinors λ and to take into account that the Penrose-Ferber incidence relations (8) provide the general solution of the constraint (11).

The fact that the twistor string is tensionless [8] can be understood by observing the conformal invariance of the action (10) or (12), which implies the absence of any dimensionful parameters in it. [If such parameters were introduced by changing the dimensions of the basic variables, they could equally be removed by a suitable redefinition of the fields].

The Berkovits open twistor string model (ii) is equivalent to the tensionless superstring moving in the direct product of two copies of the D = 4, N = 4 superspace [14].

5. ON THE TENSIONFUL PARENT OF THE TWISTOR STRING

The tensionless nature of the twistor string was first noticed by Siegel [8], who also posed the question of the existence of a possible tensionful parent. This can also be understood [14] as a consequence of the results in [15,16] according to which the mass spectrum of the intrinsically tensionless or null string is continuous, while the also tensionless quantum twistor string [1,7,8] is assumed to describe the Yang-Mills theory amplitudes and, hence, must have massless fields in the quantum state spectrum.

In fact, since in the conformal algebra the dilatation operator does not commute with the square of the momentum operator, a continuous mass spectrum or a zero-masses one are the only alternatives for a conformally invariant theory. The quantization of the tensionless superstring, which leads to massless fields in the spectrum [18], is formulated in terms of stringy oscillators, which are the suitable variables for the tensionful string. In contrast, those of the null-string are rather the spacertime coordinates and momenta. As a result, the quantization of the twistor string should correspond to the quantization of the tensionless limit of a tensionful superstring, rather than that of the intrinsically tensionless, or null, superstring.

In [8], Siegel discussed the possible tensionful parents of the twistor superstring in a purely bosonic context, proposing a tensionful QCD string [19] as its bosonic part. The inclusion of fermions brings in new questions. In particular, as far as we assume that the tensionless limit has to be a smooth one, the number of degrees of freedom should not change in this limit and, thus the number of gauge symmetries should be the same, including the number of fermionic κ-symmetries already mentioned.

A detailed discussion of these questions can be found in [14]. In short, if we were interested just in the N=1, 2 counterparts of the tensionless twistor string, their tensionful counterparts would be the D=4, N=1,2 Green-Schwarz superstrings, as can be seen in the framework of the spinor moving frame or Lorentz harmonics formulation [20]. In the more interesting N=4 case, the tensionful parent action requires an extension of the bosonic sector of superspace so that, to obtain the twistor string, the tensionless limit has to be accompanied by dimensional reduction. One could conjecture that the tensionful parent of the twistor string is given by the D=10 Green-Schwarz superstring; to describe such a relation in a simple way one would have to use the spinor moving frame or the Lorentz harmonics formulation of the D=10 Green-Schwarz superstring [20,21].

6. CONCLUDING REMARKS

We would like to mention that, at the present level of our understanding, the Green-Schwarz superstring does not appear as the only possible candidate for a tensionful parent of the twistor string. As discussed in [14], one can also consider supersymmetric string models in enlarged tensorial superspaces [22], which have found applications in higher spin theories [23].

According to [18], the quantization of a tensionless limit of the superstring should result in a higher spin theory. On the other hand, in the light of our identification of the twistor string with the zero-tension superstring, it turns out that the twistor string description of the Yang-Mills amplitudes [1,7] should be related to the quantization of such a tensionless superstring. Thus, it seems that there are three possible ways of quantizing the zero-tension superstring: (a) that in [15,16] of the intrinsically tensionless or null superstring (which gives an unphysical continuous mass spectrum); (b) the quantization in [18], which leads to higher spin theory and, finally (c) the ones associated to the twistor string path integral [1,7,8]. In view of the previous discussion, the quantizations (b) and (c), which should lead to massless fields, have to be associated with the tensionless limit of a tensionful superstring. These two quantizations are, nevertheless, seemingly inequivalent. It would be interesting to understand the interrelations and differences between these quantizations.

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ТВИСТОРНАЯ СТРУНА: НОВЫЙ ФОРМАЛИЗМ ДЛЯ ИЗУЧЕНИЯ ТЕОРИЙ ЯНГА-МИЛЛСА И ЕЁ ПРОСТРАНСТВЕННО-ВРЕМЕННЫЕ ФОРМУЛИРОВКИ

И.А. Бандос, Х.А. де Аскаррага, Ц. Микель-Испания

Представлен краткий обзор так называемого подхода твисторной струны к исследованию суперсимметричных теорий Янга-Миллса. Особое внимание уделяется различным формулировкам моделей (супер)струн в пространстве супертвисторов и их суперпространственной форме. Мы обсуждаем классическую эквивалентность действия твисторной струны в форме Зигеля и Лоренц-гармонической формулировки (N=4) суперструны без натяжения (нуль суперструны), а также отмечаем возможную связь твисторной струны с десятимерной суперструной Грина-Шварца и с моделями в расширенном суперпространстве, которое использовалось ранее для описания высших спинов.

ТВІСТОРНА СТРУНА: НОВИЙ ФОРМАЛІЗМ ДЛЯ ВИВЧЕННЯ ТЕОРІЙ ЯНГА-МІЛЛСА І ЇЇ ПРОСТОРОВО-ТИМЧАСОВІ ФОРМУЛЮВАННЯ

І.А. Бандос, Х.А. де Аскаррага, Ц. Микель-Іспанія

Представлено короткий огляд так званого підходу твісторної струни до дослідження суперсиметричних теорій Янга-Міллса. Особлива увага приділяється різним формулюванням моделей (супер)струн у просторі супертвісторів й їхній суперпросторовій формі. Ми обговорюємо класичну еквівалентність дії твісторної струни у формі Зігеля й Лоренц-гармонійного формулювання (N=4) суперструни без натягу (нуль суперструни), а також відзначаємо можливий зв'язок твісторної струни з десятивимірною суперструною Гріна-Шварца й з моделями в розширеному суперпросторі, що використовувалося раніше для опису вищих спінів.