GENERATION OF THE MAGNETIC FIELD IN THE COMPRESSIBLE PLASMA STREAMS

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Results of the numerical simulations of the compressible plasma flows in the modified quasi-steady plasma accelerator with longitudinal magnetic field are presented. The investigations are carried out within the framework of the one-component two-dimensional MHD-model of the axial-symmetrical plasma flows taking into account of the finite conductivity of the medium. Features of the compressible streams at an outlet from accelerator are revealed in the presence of a longitudinal field and rotation of plasma. Generation of a magnetic field on a conic shock wave is observed.

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1. INTRODUCTION

The quasi-steady plasma accelerators (QSPA) with an azimuthal magnetic field [1,2] are the multipurpose systems owing to an opportunity of their work both in the accelerating and in the compressible modes. The possible use of the accelerators as the electrojet engines assumes the optimum organization of the accelerating modes in the channel. Plasma is accelerated along an axis of system due to the Ampere force $\frac{1}{c}[\mathbf{j}, \mathbf{H}]$ where a plasma current $\mathbf{j}$ between two coaxial electrodes has mainly radial direction. The accelerating modes answer also to problems of generation of the high-energy plasma streams for the various applications including thermonuclear ones. The compressible flows at an outlet from accelerator with the truncated central electrode of the special form allow to receive the high values of the density and temperature in a vicinity of a system axis due to the plasma compression. The compressible flow modes are of interest for the various plasma technologies.

The powerful plasma streams with a high level of stability and azimuthal symmetry in the moving plasma have been received in experiments on the QSPA [3-6]. The theoretical and numerical researches of processes in the accelerators for the relatively dense plasma were executed within the framework of the MHD-models (see, for example, [1,2,7,8]). The history of researches of the compressible flows in the plasma accelerators with an azimuthal magnetic field begins with work [9].

The new direction of researches in the QSPA is connected with an introduction in the system of an additional longitudinal magnetic field. It leads to the rotation of plasma. The theoretical investigations [10] have revealed that owing to a longitudinal field it is possible to reduce essentially the influence of the Hall effect in the accelerator channel. The theoretical analysis of processes in the channel of the QSPA with a longitudinal field was added by a series of the numerical researches [11]. The main peculiarities of the plasma flows and the estimations of the efficiency of the acceleration process in the channel of the QSPA with a longitudinal field are presented in [12].

The experimental research of a compression zone [6] specifies what even the weak longitudinal magnetic field influences essentially on a compressible plasma streams.

2. MHD-EQUATIONS IN TERMS OF VECTOR POTENTIAL OF MAGNETIC FIELD

We consider a plasma flow in the channel between two coaxial profiled electrodes (see Figure) and at an outlet from the accelerator in case of when the central electrode is shorter external one. Whole three components of the magnetic field $\mathbf{H} = (H_z, H_r, H_\varphi)$ and velocity $\mathbf{V} = (V_z, V_r, V_\varphi)$ participate in model in the presence of a longitudinal field and the arising rotation. To be specific, we investigate a plasma formed from atomic hydrogen when the inertia of electrons ($m_e << m_i = m$) is neglected. The medium is assumed to be quasi-neutral $n_i = n_e = n$. The construction of the model within framework of the one-component approximation ($\mathbf{V}_e = \mathbf{V}_i = \mathbf{V}$) is based on the traditional MHD-equations taking into account of the conductivity. We have the following equations in the dimensionless form

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{V}) = 0 \quad ; \qquad \rho \frac{d \mathbf{V}}{d t} + \nabla P = \mathbf{j} \times \mathbf{H}$$

(1)

$$\rho \frac{d\varepsilon}{d t} + P \nabla \bullet \mathbf{V} = \nu \mathbf{j}^2 ; \qquad \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{H}) - \nabla \times (\nu \mathbf{j})$$

Here, $P = P_i + P_e = \beta \rho T$ is the total pressure; $\varepsilon = \beta T / (\gamma - 1)$ is the intrinsic energy per unit mass; $\mathbf{j} = \nabla \times \mathbf{H}$ is the electric current. We will restrict our attention to the case of a single-temperature mixture $T_i \approx T_e = T$. In accordance with Ohm's law, the electrical field is given by the relation

$$\mathbf{E} = \nu \mathbf{j} - \mathbf{V} \times \mathbf{H} \qquad (2)$$

The initial quantities are related to the dimensionless parameters of the problem as follows: $\beta = 8\pi P_o / H_o^2$ is the ratio between the gas-kinetic and magnetic pressure at inlet, $P_o = 2 k n_o T_o$; $\nu = 1/\mathrm{Re}_m = c^2 / 4\pi L V_o \sigma$ is the magnetic viscosity which is inversely proportional to the Reynolds number with the Spitzer conductivity $\mathrm{Re}_m = \sigma_o T^{3/2}$. $H_o = 2 J_p / c R_o$, R_o - the radius of the external electrode, J_p - the discharge current, $V_o = H_o / \sqrt{4\pi \rho_o}$ - the characteristic velocity.

It is necessary to satisfy a condition $\nabla \bullet \mathbf{H} = 0$ for a magnetic field at the numerical integration of a multivariate MHD problem. In the case of the axial flow symmetry ($\partial / \partial \varphi = 0$) it is possible to introduce the vector potential $\mathbf{A}$ ($\mathbf{H} = \nabla \times \mathbf{A}$) so that $\nabla \bullet \mathbf{H} = 0$ (see, e.g., [10-12]). The azimuthal component A_φ of a vector **A** defines the components of a magnetic field

$$H_r = -\frac{\partial A_\varphi}{\partial z}, \qquad H_z = \frac{1}{r}\frac{\partial (r A_\varphi)}{\partial r} \tag{3}$$

With allowance for above remarks, the equations (1) can be written in the explicit form in terms of azimuthal components A_φ and H_φ. In accordance with (3), we obtain the transformed equations in which the right parts contain the azimuthal component of the plasma current $j_\varphi = \partial H_r / \partial z - \partial H_z / \partial r$. The equivalent equation for entropy $S = \ln\left(T / \rho^{\gamma - 1}\right)$ is used instead of the equation for the internal energy (1). As a result, we have the system of seven equations for seven variables ρ, S, V_z, V_r, V_φ, H_φ and A_φ.

3. BOUNDARY CONDITIONS

The conditions at the channel inlet $(z = 0)$ correspond to subsonic plasma inflow with the known distributions of the density $\rho(r) = f_1(r)$ and the temperature $T(r) = f_2(r)$. We assume that the total electric current flowing in the system is maintained constant. This assumption brings about the boundary condition at the inlet for the azimuthal magnetic field $j_z = 0$ or $r H_\varphi = r_o = const$, where $r_o = R_o / L$. The inflow is carried out along the coordinate lines, for example. We suppose also that the plasma at the inlet cross section $z = 0$ is non-rotating and $V_\varphi = 0$. We specify longitudinal field at entrance $H_z \neq 0$. In accordance with [10], for a cold plasma $\beta << 1$ in case of the radial-equilibrium case we have $H_z = H_z^o = const$ at $z = 0$. Integrating (3), we obtain $\psi = r A_\varphi = H_z^o r^2 / 2$.

At the outlet $z = 1$ the boundary conditions correspond to a free supersonic plasma flow.

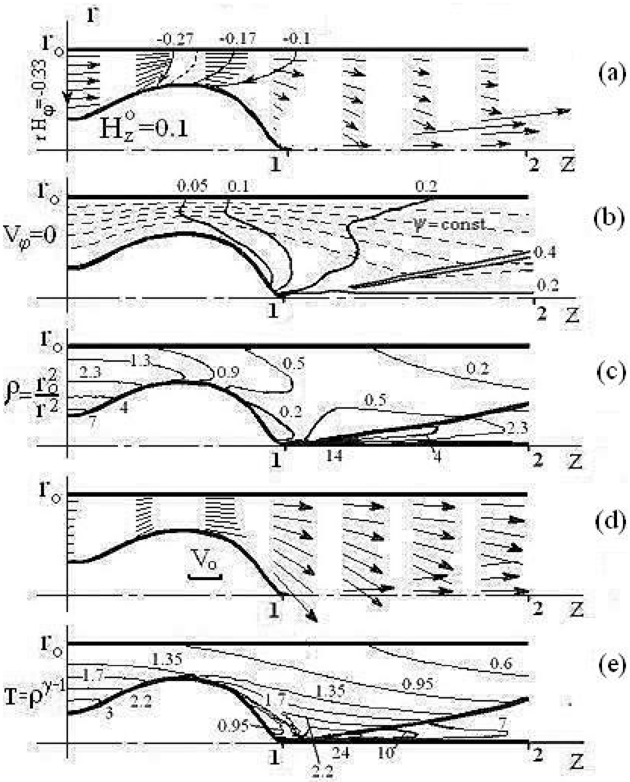
We suppose that the electrodes with the given profiles $r = r_k(z)$ and $r = r_a(z)$ are equipotential $E_\tau = 0$ and non-penetrable $V_n = 0$ surfaces. The Hall effect does not take into account in the one-fluid model. Therefore, this model is suitable for the qualitative description of the both regimes of the ion and electron current transport.

It is necessary an additional relation in the presence of longitudinal magnetic field. The equality $H_n = 0$ is the typical and natural condition in plasmadynamics. This condition leads to the conservation law of the magnetic flux along the channel.

On an axis of system ($r = 0$) we have the obvious conditions: $V_r = 0$; $V_\varphi = 0$; $H_\varphi = 0$; $H_r = 0$. Methods of the numerical solution were described in detail in [11].

4. RESULTS OF CALCULATIONS

A series of the numerical experiments is carried out on the basis of the presented model. A choice of values of the concentration n_o, temperature T_o, discharge current J_p and lengths of channel L correspond to the experiments under the QSPA program [3-6]. If $n_o = 8.1 \cdot 10^{14} cm^{-3}$, $T_o = 2\,eV$, $J_p = 300\,kA$, $L = 60\ cm$, $R_o = 20\,cm$, the dimensionless parameters of a problem are equal $\beta = 0.007$ and $\sigma_o = 678$. Besides it is necessary to set a value of a longitudinal magnetic field at an inlet in the channel, for example, $H_z^o = 0.1$. Such value of H_z^0 is low enough. It gives the possibility to produce the transonic flow in the channel in accordance with the analytical model [10,12]. The channel geometry (see Figure) also corresponds to the analytical research of the plasma flows. In this case the density at the inlet varies according to the law $\rho(z = 0, r) = r_o^2 / r^2$. Assuming also that the plasma injected into the channel is isentropic ($s = c_v \ln P / \rho^\gamma = const$) we have $T = \rho^{\gamma - 1}$ at $z = 0$. It is possible to consider the quasi steady-state solutions obtained by means of the relaxation method.



The compressible plasma flow in the presence of the longitudinal magnetic field

The compressible plasma flow in the presence of a longitudinal field $H_z^o = 0.1$ is presented in Figure under the condition of the non-uniform inflow of plasma at the inlet according to the analytical model. Here we have plotted: (a) the distribution of the magnetic field **H** in the (r, z) plane and the plasma current (j_r, j_z) (solid contour lines of function $r H_\varphi$); (b) the level lines of function $\psi = r A_\varphi$ or magnetic flux (dashed curves) and the contour lines of the azimuthal velocity $V_\varphi(r, z)$ (solid curves); (c) the distribution of the density $\rho(r, z)$; (d) the projections of the velocity vector **V** onto the (r, z) plane; and (e) the contour lines of temperature $T(z, r)$. The dashed curve in Figure (a) shows that in the central part of the accelerator channel the flow velocity increases from below to above the local speed of a fast magnetosonic wave. The level lines of function $r\,H_\varphi = const$ in Figure (a) define a direction of an electric current depending on a polarity of the electrodes. To be specific, we consider that an external electrode is the anode. The length of vectors in Figure d is equal to the dimensionless value of velocity at the given point. The scale of vectors is defined by the characteristic velocity V_0 indicated in the Figure.

Also as well as in the absence of a longitudinal field, the conic shock wave is distinctly observed. There are a break of lines of a plasma stream (Figure (d)) and of a magnetic flux (Figure (b)) and the corresponding spasmodic change of a tangential component of a longitudinal magnetic field (Figure (a)). To similarly previous case on a conic shock wave there is a spasmodic change of the density, temperatures and pressure. Accordingly, the temperature, density and pressure increase on a shock wave. As a whole the compression zone represents an region of the compressed and heat plasmas.

However, under the influence of a longitudinal magnetic field the density and temperature noticeably decrease on an axis of the system. According to Figure (b) the rotation of plasma in a vicinity of a conic shock wave occurs to rather high speed. Besides under the influence even of a weak longitudinal field we observe the increase in a corner between the forming of the conic surface of a shock wave and an axis of system. This corner increases with the increase in a longitudinal field. That circumstance is essential also that behind a shock wave the region with rather high values of a longitudinal field is formed. There is a spasmodic change of the tangential component of a magnetic field on a conic shock wave.

The presented numerical model takes into account the finite conductivity of plasma. In spite of the fact that the known relations on the MHD-breaks including the shock waves answer the ideally conducting plasma, these relations can be used for an investigated shock wave. The analysis of processes on the shock transition has shown in particular that according to the elementary theoretical constructions on a shock wave we have $\mathbf{H}_\tau^R \parallel \mathbf{H}_\tau^L \parallel \{\mathbf{V}_\tau\}$ where the jump is $\{\mathbf{V}_\tau\} = \mathbf{V}_\tau^L - \mathbf{V}_\tau^R$. In this case we assume that the condition to the right of break (R) belongs to the flow before a shock wave.

5. CONCLUSIONS

The peculiarities of the compressible plasma streams at an outlet from the channel of the quasi-steady plasma accelerators (QSPA) with a longitudinal magnetic field are revealed. As a result of the numerical experiments, the essential influence of a weak longitudinal field on the compressible plasma streams is discovered. The zone of a compression contains a conic shock wave on which there is a sharp change of the density and temperature. The characteristic break of lines of a plasma stream and lines of a magnetic flux is observed on a shock wave. Under the influence of a weak longitudinal field the plasma starts to rotate with the rather high speed in a vicinity of the conic shock wave. We observe a generation of a longitudinal magnetic field with rather high values behind a shock wave. Such parameters of plasma as the density and temperature decrease noticeably on an axis of system under the influence of a longitudinal field.

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ГЕНЕРАЦИЯ МАГНИТНОГО ПОЛЯ В КОМПРЕССИОННЫХ ПОТОКАХ ПЛАЗМЫ

А.Н. Козлов

Представлены результаты численного исследования компрессионных потоков плазмы в модифицированном квазистационарном плазменном ускорителе с продольным магнитным полем. Исследования проведены в рамках одножидкостной двумерной МГД-модели осесимметричных течений плазмы с учетом конечной проводимости среды. Выявлены особенности компрессионных потоков на выходе из ускорителя при наличии продольного поля и вращения плазмы. Наблюдается генерация магнитного поля на конической ударной волне.

ГЕНЕРАЦІЯ МАГНІТНОГО ПОЛЯ У КОМПРЕСІЙНИХ ПОТОКАХ ПЛАЗМИ

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Представлено результати чисельного дослідження компресійних потоків плазми у модифікованому квазістаціонарному плазмовому прискорювачі з подовжнім магнітним полем. Дослідження проведено у рамках однорідинної двомірної МГД-моделі вісесиметричних течій плазми з урахуванням кінцевої провідності сердовища. Виявлено особливості компресіонних потоків на виході з прискорювача при наявності подовжнього поля й обертання плазми. Спостережується генерація магнітного поля на канонічній ударній хвилі