# HIGH ENERGETIC ELECTRON DISTRIBUTION TAILS FOR NONLINEAR COLLISIONAL KINETIC EQUATION IN THE PRESENCE OF HEATING

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The time-depended solutions of collisional electron kinetic equation with the heating term allowing the solutions in self-similar variables are considered. A broader class of the heating terms resulting in enhancement of the tail of the distribution function in comparison with Maxwellian is analyzed both analytically and numerically. The results obtained can be used for the assessment of the impact of superthermal electrons on heat transport for divertor plasma as well as for the benchmarking of sophisticated kinetic codes.

PACS:52.25.Dg 52.55.Fa 52.65.-y

In many important cases the applicability of a fluid description of plasma may be questionable and one should treat plasma transport kinetically. Examples are: the electron heat transport in inertial confinement fusion (e.g. see Refs. [1-6] and the references therein); propagation of the heat bursts, caused by edge localized mode (ELM), into scrape-off layer (SOL) of tokamak e.g. see Refs. [7-10] and the references therein). In more general sense the solutions of the Fokker-Planck equation, which one of the key ingredient of plasma kinetic equation, have much broader interest ranging from plasma physics to stellar dynamics (e.g. see Refs. [11, 12] and the references therein)

In practice, kinetic solution of plasma transport problems (e.g. 1D2V problem of ELM burst propagation along the magnetic field lines) can be done only numerically, which is very difficult and time consuming. In addition, complex nonlinear kinetic codes require careful benchmarking, which is not a trivial problem on it's own.

However, in many cases, besides the considering the complex multi-dimensional kinetic problem, it worthwhile to analyze simpler models, solution of which, nevertheless, exhibit some important features of the problem of interest (we notice that such models also help to benchmark complex kinetic codes). For example, the transport of electrons along the magnetic field lines from hot upstream region of the SOL to cold divertor region during ELM burst (which is a 1D2V problem), resulting in the enhancement of the tail and anisotropization of electron distribution function in divertor (e.g. see Refs. [7-10]), can be mimicked by a proper heating term in a much simpler time-dependent model:

$$\frac{\partial f}{\partial t} = \hat{C}(f,f) + \hat{H}(f), \qquad (1)$$

where $f(v,t) \equiv f(\varepsilon,t)$ is the electron distribution function, $v$ is the velocity, $\varepsilon$ is the variable "energy", $\hat{C}(f,f)$ is the electron Coulomb collision term, $H(f)$ is the heating term

$$\hat{H}(f) = \nabla_{\vec{v}} \cdot \left(\hat{D}(v,t)\nabla_{\vec{v}} f\right), \qquad (2)$$

The distribution function is normalized to unity, i.e., density (number of particles) is

$$n = \int_0^\infty dv v^2 f(v,t) = 1.$$

The distribution function tends to zero $f \to 0$ as $v \to \infty$. The energy, the temperature, and the thermal velocity are expressed through the distribution function in the following way:

$$e(t) = \int_0^\infty dv v^4 f(v,t),\ T(t) = 1/3e,\ v_{th} = \sqrt{2e/3}\,.$$

During the whole process under consideration the electron density $n$ remains constant and the energy $e$ increases in consequence of heating operator action. The local Maxwell distribution $f_{Maxw}(v,t)$ corresponding to the time-dependent thermal velocity is

$$f_{Maxw}(v,t) = n\frac{4}{\pi^{1/2}} v_{th}^{-3} \exp\left[-\frac{v^2}{v_{th}^2}\right].$$

The equation (1) was considered in [13] with taking into account the heating operator with the diffusion coefficient $D(v,t)$ localized in the cold region $v << 1$. In this case the self-similar solutions have strongly depleted tails as $v \to \infty$ in comparison to Maxwellian ones. However, the situation changes drastically when $\hat{D}(v,t) \propto v^{2p-3}(T(t))^{1-p}$ increases with increasing velocit, where the normalization constant $5/2 > p > 0$ is an adjustable parameter.

At the instant $t = 0$, the initial function $f_0(v,t) = f(v,0)$ is located in the thermal velocity region. In order to verify our analytic results we solve

Eq.(1) numerically for the diffusion coefficient in the heating term, which correspond to the self-similar solutions:

$$F(\xi \to \infty) \propto \exp\left\{-\frac{\Gamma(p+1)}{\Gamma(5/2)}\frac{\xi^{5/2-p}}{(5/2-p)}\right\},$$

$3/2 \le p < 5/2$; and $F(\xi \to \infty) \propto \xi^{-5/2}$, $p = 5/2$ ( $\xi = \varepsilon / T(t)$, $\Gamma(x)$ is the Gamma function).

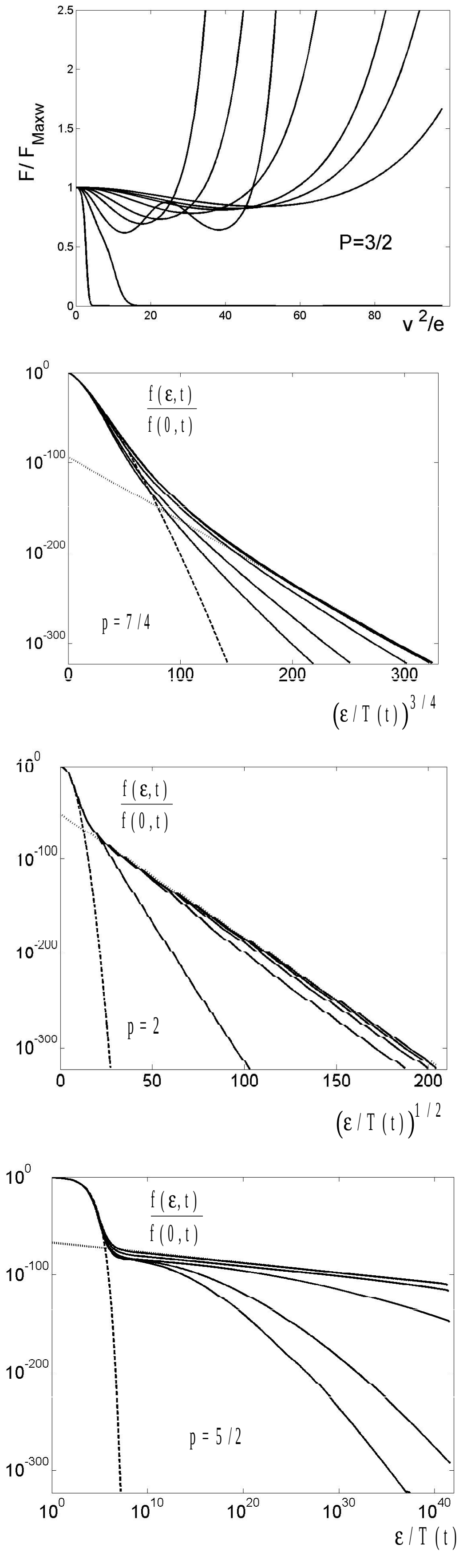
In numerical modeling we utilize finite difference schemes developed in [14], which conserve density and energy of the system and allow large time steps and velocity intervals without error accumulation. The results of the solutions are shown in Figs. 1-4 for the diffusion coefficient corresponding p=3/2, 7/4, 2, and 5/2.The initial distribution is approximated on the mesh in the usual way, that is,

$$f(v_i,0) = \begin{cases} 1/h, & if \quad v_i = 1, \\ 0, & otherwise. \end{cases}$$

Our investigation is concentrated on the evolution and formation of the distribution function tails for long times $t >> 1$, as $v \to \infty$ and establishment of the asymptotic self-similar solutions. For this period the relaxation in the thermal velocity region is practically finished. Very rapidly, the solution acquires a quasi equilibrium form in the thermal velocity region $(0 \le v < v_{th})$ at the instant $t_e \sim 1$ that corresponds to the so-called collision time. In this region the distribution functions are close to each other throughout the entire relaxation process for different functions $D(v,t)$. The main difference is observed in the region of the distribution tails for $v >> v_{th}$.

For the case p=3/2 the solution of (1)-(2) is Maxwellian. At the beginning the tail has Coulombian character and then spreads into superthermal region following the diffusion law. During the process the local Maxwell distributions approach the Maxwellian solution $f_{Maxw}(v)$ in the thermal region. In the high energetic region (it can be approximately estimated from the plots as $v > 20v_{th}$) the electron distribution manifests the self-similar solution and coincide with the analytic solution. Figure shows the distribution function tail formation for different time moments.

We consider time-depended solutions of collisional electron kinetic equation with the heating term allowing the solutions in self-similar variables. Such feature is typical for divertor plasma during ELM burst. Therefore, the results we obtained can be used for the assessment of the impact of ELMs on heat transport and sheath parameters. In addition, our analytic and, confirming them, numerical results can be used for the benchmarking of more sophisticated kinetic codes.




*The distribution function is normalized on the Maxwellian distribution for p=3/2. For p=7/4, p=2, and p=5/2, the distribution function is normalized on its value at zero velocity*

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*Article received 16.10.08.*

### ВЫСОКОЭНЕРГЕТИЧНЫЕ ХВОСТЫ ЭЛЕКТРОННОГО РАСПРЕДЕЛЕНИЯ ДЛЯ НЕЛИНЕЙНОГО СТОЛКНОВИТЕЛЬНОГО КИНЕТИЧЕСКОГО УРАВНЕНИЯ ПРИ НАЛИЧИИ НАГРЕВА

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Рассматриваются зависящие от времени решения столкновительного электронного кинетического уравнения с нагревом, которые допускают представление в автомодельных переменных. Анализируется аналитически и численно широкий класс операторов нагрева, который приводит к росту хвоста функции распределения по сравнению с максвелловским. Полученные результаты могут быть использованы для оценки влияния сверхтепловых электронов на перенос тепла для диверторной плазмы, а также в качестве теста для сложных кинетических кодов.

### ВЫСОКОЕНЕРГЕТИЧНІ ХВОСТИ ЕЛЕКТРОННОГО РОЗПОДІЛУ ДЛЯ НЕЛІНІЙНОГО ЗІШТОВХУВАЛЬНОГО КІНЕТИЧНОГО РІВНЯННЯ ПРИ НАЯВНОСТІ НАГРІВАННЯ

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Розглядаються залежні від часу рішення зіштовхувального електронного кінетичного рівняння з нагріванням, що допускають представлення в автомодельних змінних. Аналізується аналітично і чисельно широкий клас операторів нагрівання, що приводить до росту хвоста функції розподілу в порівнянні з максвеллівським. Отримані результати можуть бути використані для оцінки впливу надтеплових електронів на перенос тепла для диверторної плазми, а також як тест для складних кінетичних кодів.