NONRELATIVISTIC DYNAMICS OF THE CHARGED PARTICLES AT CYCLOTRON RESONANCES

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It is shown that transition to the chaotic dynamics of the charged particles at cyclotron resonances can take place on an unusual scenario - not through overlapping of nonlinear resonances, but through qualitative periodic tuning of topology of phase space. It is shown that such scenario can be realized at excitation of low-frequency oscillations by dense flows of the charged particles in the strong magnetic field.
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1. INTRODUCTION

Now conditions of transition from regular dynamics to chaotic are received for all known resonances of interaction a wave-particle [1-3]. Thus a source of chaotic dynamics is crossing homoclinic or heteroclinic trajectories. In physical language this fact is more convenient for expressing as overlapping of nonlinear resonances. The received results are correct at interaction of a particle with a separate electromagnetic wave with constant amplitude. In real physical problems of waves amplitude vary, except for that waves can be several. Generally such complication of physical system does not lead to any qualitative changes of processes chaotic. Simply there are additional resonances appear or width of existing resonances varies. Therefore the studied scenario of transition to chaos is rather universal. Besides its universality is caused by that fact, that interaction of the charged particles with electromagnetic waves can be considered as perturbation in Hamilton formalism. Thus, as is known, the truncated equations in a vicinity of resonances are described by the equation of a mathematical pendulum. Therefore it seemed natural what exactly the scenario of overlapping of nonlinear resonances should describe practically all variety of conditions of interaction wave-particle. However, as we shall see below, at cyclotron resonances, for the description of dynamics nonrelativistic particles, the truncated equations are described not by the equation of a mathematical pendulum, and system of the equations topological similar to Duffing oscillator. Duffing oscillator unlike the mathematical pendulum has two free parameters. This fact leads to that at change of amplitude of the wave in which the particle moves, its phase portrait (the phase portrait Duffing oscillator) can qualitatively vary. Presence of such qualitative change of the phase portrait, as we shall see below, is the reason of occurrence of chaotic dynamics. Change of amplitude of the wave in which particles move, can be caused by external factors. Besides as is known, at beams instabilities in a nonlinear mode of amplitude of excited waves also periodically vary. Depth of amplitude modulation essentially depends on density of the electron beam. Below it is shown, that at enough big density of the beam this modulation appears sufficient for realization of the chaotic regime in the isolated cyclotron resonance. The mechanism responsible for a randomness in this case is qualitative change of topology of phase space.

2. STATEMENT OF THE PROBLEM. THE BASIC EQUATIONS

We shall consider a problem about excitation of an electromagnetic field by a monoenergetic beam of oscillators with function of distribution:

$$f_0 = \frac{N_b}{2\pi p_\perp}\delta(p_\perp - p_{\perp 0})\delta(p_\parallel) \qquad (1)$$

where $p_\perp$, $p_\parallel$ - perpendicular and parallel axes z components of pulse , N_b - equilibrium beam density.

The beam moves in the constant magnetic field directed along z axis. We shall consider excitation of wave which propagates perpendicularly to the magnetic field. Considered statement of the problem does not differ from that which has been formulated in works [1-3]. Moreover, we shall take advantage of many results received in these works. The self-consistent system of the equations which describes dynamics of excited fields and dynamics of the charged particles consists of Maxwell equations and the equations of motion of separate particles.

The full system of the equations is written out in [2,3]. Below we shall write out the truncated system of the equations describing dynamics of particles and fields in the isolated cyclotron resonance with number s :

$$\frac{dp_\perp}{d\tau} = iJ'_s(\mu)e^{i\theta_s}\varepsilon ,$$
$$\frac{d\theta_s}{d\tau} = \frac{s\omega_H}{\gamma} - 1 + \frac{1}{\omega_H}\left(1 - \frac{s^2}{\mu^2}\right)J_s(\mu)e^{i\theta_s}\varepsilon , \qquad (2)$$
$$\frac{d\varepsilon}{d\tau} = i\frac{\omega_b^2}{2\pi}\int_0^{2\pi} d\theta_{s0}\frac{p_\perp}{\gamma}J'_s(\mu)e^{-i\theta_s}$$

where - $p_\perp = p_\perp / mc$, $\mu = p_\perp / \omega_H$, $\gamma = \sqrt{1+\mu^2\omega_H{}^2}$, $\omega_H = eH_o / mc\omega$, $\omega_b^2 = 4\pi e^2 n_b / m_e$ $\varepsilon = eE / mc\omega$.

The system of the equations (2) differs from what have been analyzed in works [2,3], presence of last equation term in the right part for phase. For relativistic particles this term can be neglected. In this case first two equations of system (2) at constant intensity of the wave's field ($\varepsilon = const$) represent the equation of a mathematical pendulum. These equations have been used in works [2,3] for the finding of conditions of local instability occurrence. For not relativistic particles this term can be essential and as we shall see below, it leads to the scenario of stochasticity occurrence distinct from the scenario of overlapping of nonlinear resonances.

3. DYNAMICS OF NONRELATIVISTIC PARTICLE MOTION IN THE FIELD WITH CONSTANT AMPLITUDE

If the amplitude of the field does not vary, the third equation in system (2) can be not considered. Dynamics of particles is described by first two equations. Such system has Hamiltonian

$$H(\theta_s, I) = \frac{s}{\omega_H}\gamma - I + \frac{\varepsilon}{\omega_H} 2I \frac{d}{dI}\left(J_s(\sqrt{2I})\right)\cos(\theta_s), \quad (3)$$

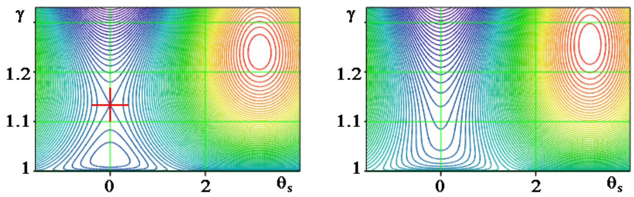
where $I = \mu^2 / 2$.

The phase portrait of system with Hamiltonian (3) topologically is similar to the phase portrait Duffing oscillator. Really, on a phase plane ($p_\perp, \theta_s$), in general case, there are three critical points: ($\theta_s = 0$, $p_{\perp 1} = \Pi$), ($\theta_s = 0$, $p_{\perp 2} = 1 - \Pi / 2$), ($\theta_s = \pi$, $p_{\perp 3} = 1 + \Pi / 2$). Here $\Pi = \varepsilon / p_{\perp,0}^3$, $p_{\perp,0}$ - initial impulse of particle. Where, two of these critical points (the second and the third) represent points of type "center", and one (first) – saddle point. Such kind of phase space is realized at small amplitude of an external wave ($\Pi << 1$). If the amplitude is large enough ($\Pi >> 1$), that two critical points, namely saddle point and the point of type "center" (the first and second critical points) merge and disappear. There is only one special point - a point of type "center". All these features of phase space are similar to features of phase space Duffing oscillator.

However it is necessary to pay attention to that fact, that oscillations of Duffing oscillator are potential, and for the equations considered by us it is not possible to find potential. The important feature of phase space topology of considered system is that fact, that the closed trajectories in a vicinity of a critical point of type "center" can identify the trapped particles. Unclosed trajectories which surround the closed trajectories, it is possible to identify passing particles. We have got used, that the trapped and passing particles are divided by separatrix, i.e. homoclinic or heteroclinic trajectories. In this case such trajectories are absent. It is necessary to tell, that absence of such trajectories leads to different dynamics of particles which pass through the region dividing region of trapped and passing particles.

4. NUMERICAL ANALYSIS OF DYNAMICS OF PARTICLES AND FIELDS

By numerical methods, first of all, had been investigated dynamics of particles in the field of external electromagnetic wave with constant and with periodically varying amplitude. Besides self-consistent dynamics of particles and fields excited by these particles has been investigated. As a whole the received results will well be agreed with available representations about studied processes. So in Fig. 1 the phase portraits are presented for case $\Pi << 1$ (Fig.1, a) and for $\Pi > 1$ (Fig.2, b). Straight lines correspond to initial coordinates of testing particles. Apparently from Fig.1(a) on the phase plane, in full conformity with the results obtained above, is available three critical points: two type of the center and saddle point.



a *b*

Fig.1. Phase trajectories at a) ε =0.08; b) ε =0.12

At increasing of the wave amplitude two points ("saddle" and "center" at $\theta_s = 0$) come together and disappear (Fig.1,b). If the amplitude of the external field periodically varies in the vicinity of the merge point the dynamics of particles turns out chaotic: the spectra of particles motion is wide, correlation functions quickly enough fall down (Fig. 2).

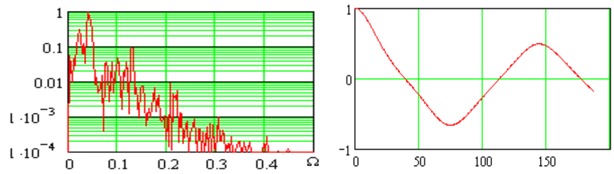


Fig.2. Spectrum of the impulse near saddle point (left); correlation function near saddle point (right)

If the amplitude of field varies in region where there is no qualitative change of the phase portrait the dynamics of particles undergoes small changes (see Fig. 3).

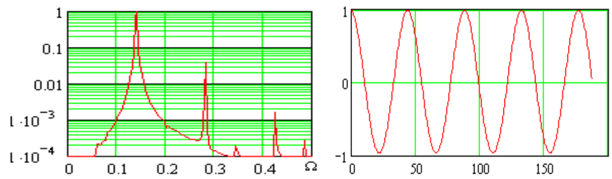


Fig.3. Spectrum of the impulse far from saddle point (left); correlation function far from saddle point (right)

The received results confirm the above formulated assumption about chaotization mechanism which is caused by periodic qualitative change of the phase portrait.

By numerical methods had been investigated also self-coordinated dynamics of particles and fields, which are excited by these particles (system of the equations (2)). Dynamics of studied processes thus completely corresponds to the qualitative picture described above. Really, if density of the beam particles is not too large, so the amplitude of excited field satisfies to an inequality $\Pi << 1$, the dynamics of particles and fields is similar to dynamics of particles and fields at beam-plasma interaction (Fig.4, a).

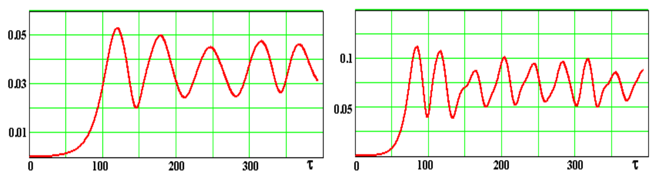


Fig.4. Amplitude of field versus time at: a) small beam density: ω_b^2 =0.02; b) at large beam density: ω_b^2 =0.1

If the density of particles becomes such that the amplitude of the wave excited by beam and depth of its modulation become such, that occur qualitative change of the phase portrait described above then irregular dynamics as fields, and particles is appeared much earlier, than for the case of relatively small amplitudes (Fig.4, b).

5. CONCLUSIONS

Thus, transition from regular dynamics to chaotic one at interactions of the wave-particle type can be realized not only as result of overlapping nonlinear resonances, but also as the result of phase portrait topology changing. Above we have considered such mechanism of the local instability occurrence for cases of nonrelativistic charged particles interaction with electromagnetic wave in conditions of the isolated cyclotron resonance. Clearly, that similar mechanisms can be realized in the large number of other physical systems. It will be those systems which motions can be described by motion of Duffing oscillator. These are numerous systems. Really, as is known, to the analysis of dynamics of the mathematical pendulum it is possible to reduce studying of the Hamiltonian systems under periodic disturbance. Hamiltonian of the mathematical pendulum in this connection name the first fundamental Hamiltonian. To represent the fact, that to the analysis of the Duffing oscillator dynamics is reduced the investigation of many physical systems, Hamiltonian of the Duffing oscillator name the second universal Hamiltonian.

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НЕРЕЛЯТИВИСТСКАЯ ДИНАМИКА ЗАРЯЖЕННЫХ ЧАСТИЦ ПРИ ЦИКЛОТРОННЫХ РЕЗОНАНСАХ

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Показано, что переход к хаотической динамике заряженных частиц при циклотронных резонансах может происходить по необычному сценарию - не через перекрытие нелинейных резонансов, а через качественную периодическую перестройку топологии фазового пространства. Показано, что такой сценарий может реализоваться при возбуждении низкочастотных колебаний плотными потоками заряженных частиц в сильном магнитном поле.

НЕРЕЛЯТИВІСТСЬКА ДИНАМІКА ЗАРЯДЖЕНИХ ЧАСТИНОК ПРИ ЦИКЛОТРОННИХ РЕЗОНАНСАХ

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Показано, що перехід до хаотичної динаміки заряджених частинок при циклотронних резонансах може відбуватися за незвичайним сценарієм - не через перекриття нелінійних резонансів, а через якісну періодичну перебудову топології фазового простору. Показано, що такий сценарій може реалізуватися при збудженні низькочастотних коливань щільними потоками заряджених частинок в сильному магнітному полі.