METHOD FOR PARTICLE ENSEMBLES CONTROL UNDER UNCERTAINTY

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The paper is devoted to the issues related to nonlinear dynamics of controlled intensive charged particle beams. Study the beams possessing exceptionally large inherent fields and extreme high impulse intensity and energy calls for new cybernetic approaches aimed to developing controlling algorithm of optimal behavior. We describe the beam distribution in particle accelerations (a Vlasov-Maxwell scheme) in the framework of differential games and control problem. This paper proposes an optimal approach for the well-known partial differential equation of Vlasov involving semi-group contraction and dissipative operators in Hilbert space.

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THE FUNDAMENTAL OF SCIENCE PROBLEM

The paper is devoted to the issues related to nonlinear dynamics of controlled intensive charged particle beams. Study the beams possessing exceptionally large inherent fields and extreme high impulse intensity and energy calls for new cybernetic approaches aimed to developing controlling algorithms of optimal behavior.

The inverse problem of electrodynamics is discussed in [1,3]. Its gist consists in the following. The particle velocity fields are given in the phase space the former chosen on the basis of particular goal of study. For example, the problem of acceleration and focusing of a charged-particle beam is treated. In this case the velocity field as well as the particle trajectory fields specifies the electromagnetic field. This field keeps the motion of particles along prescribed trajectories under the action of Lorentz force.

V.I. Zubov [2] was first who suggested such problem statement for the electromagnetic field synthesis. He deduced the differential equation for potential of the vector field and substantiated this electromagnetic fields actual causing the motion of the charged particles in accordance with the given field of velocities. However, only the case of noninteractive particles was studied that is way his results could not be directly applied to investigate the intensive particle ensembles.

It is well known that the evolution of a single particles distribution density function $f(t,r,v)$ in impulse-configuration space is described by the self-consistency Vlasov-Maxwell system [4]:

$$\frac{\partial f}{\partial t} + div_r f + div_v \langle \dot{v} \rangle f = 0, \quad (1)$$

$$\langle \dot{v} \rangle = \frac{\int \dot{v} f(r,v,\dot{v},t) d\dot{v}}{f(r,v,t)} = \frac{e}{m}\{E + \frac{1}{c}(\bar{v} \circ H)\},$$

$$divE = 4\pi e \int_{\infty} f dv, rotH - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{4\pi e}{c}\int_{\infty} v f dv, \quad (2)$$

$$(\text{divH}=0, \quad \text{rotE}+\frac{1}{c}\frac{\partial H}{\partial t} = 0). \quad (3)$$

The notation has become a standard in this subject. Where E and H are the electric and magnetic fields, v is the nonrelativistic velocity, and c is the speed of light. The charged density g and current j are given by

$$g = 4\pi e \int f dv + 4\pi\rho, \quad j = e\int v f dv.$$

Here e and ρ are charge density of interior and exterior respectively.

Thus we have Vlasov's equation (1) which is the equation of the motion of the beam and Maxwell's equations (2-3) which are defined the electromagnetic fields.

In studies of sophisticated dynamics of intensive beams moving in combined exterior and interior electromagnetic fields, problems of controllabillity of an equalibrium configurations and their stability arise, first of all. To examine these issues, a new approach is suggested, based on rigorous Vlasov-Maxwell mathematical model, for which the synthesis is performed using principles of the positional differential games. We describe the one in the frame work of the optimal control problem using the position differential games.

Thus, using Zubov's approach (see above) to the equation (1) we obtain the following its controlled model:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial r} v + \frac{\partial f}{\partial v}(U_1 + U_2) = 0, \quad (4)$$

where U_1 and U_2 are interior and exterior of the Lorentz's force respectively.

It should be observed that the free beam (U_2=0) decays. This reasoning yields the following conclude.

For the beam focusing we play under Nature (U_1) by means of the exterior force $U_2 \neq 0$.

The problem can be formulated in the terms of the position differential games [3].

STATEMENT OF THE PROBLEM

We shall seek a solution to (4) in the form $f(t,r,v) = \exp(-\lambda t)\varsigma(r,v)$. It may be noted that the function $f[t]$ in solutions of the following equation

$$\frac{dx}{dt} = X(x), \quad (5)$$

$x = (r,v)$, $\quad X = (v, eE + \frac{1}{c}[v \times H])$ as well known, is to be constant or $\exp(-\lambda t)\varsigma(x) = const(n)$, where n is number of trajectory. Thus eq.(4) can be simplified by this substitution in it. We obtain the following equation

$$\frac{\partial\varsigma}{\partial r}v+\frac{\partial\varsigma}{\partial v}(U_1+U)_2=\lambda\varsigma\,. \qquad (6)$$

PROBLEM

To find a strategy (a distribution exterior charged) $\rho(t,r)$, providing meeting

$\{\tau,r,v)\in Mo\subset\Omega,\ \ t\le\tau$, and $\{t,r,v\}\notin Mc.$

Where Mc is focus size is given beforehand and Ω is a phase volume.

The problem consists in the following. The field, generated by the beam charged $f(t,r,v)$ strikes to scatter the particles while the exterior charged should work to intercepting that is to assemble them in pre-assigned target set – the focus. In essence the problem can be carry out in the same way. Let us we have be the given set Mo as an inner part of the given closed set No, $Mo\subset No$. We will suppose in the following that into the interval $t\le\vartheta$ of the time there exists some function $\varsigma(t,r,v)$ such that

$$\varsigma(x)>\alpha \ \text{ if } \ x\notin M_\alpha. \qquad (7)$$

Further suppose that the function ς satisfy the conditions:

1. The one have partial derivatives

$$\frac{\partial\varsigma}{\partial t},\ \frac{\partial\varsigma}{\partial r},\ \frac{\partial\varsigma}{\partial v} \qquad (8)$$

in domain $t<\alpha$, (8) $\varepsilon<\varsigma(t,r,v)<\varepsilon+\beta$, $(\beta-const)$.

2. In the domain (7) holds true an inequality:

$$\min_{u\in U_2}\max_{u\in U_1}([\frac{\partial\varsigma}{\partial x}]^* f(t,x,u,\rho)+\frac{\partial f}{\partial t})\le 0\,. \qquad (9)$$

It can be proved [2]

Lemma. Under conditions (8-10) the set $W=\{t,x\}$, $(t<\tau)$ satisfies the condition

$$\varsigma(r,v)\le\alpha$$

and forms $u-$stabile bridge which extends from M_0 to M_α.

The lemma and conditions (8-9) are more appropriate for solvability questionthe the another approaches, but this situation requireds of knowledge which to find the function $\varsigma(r,v)$ on the set W. Contemporary is main problem to the position differential games.

In this point we give some results which was used above. We consider the problem via dissipative operator of compact dynamical system (1) (in short Σ) and semi-group contraction in Hilbert space L^2 [5-6].

For this aim we introduce some definitions and notation. Observe that the Σ introduce into consideration in $L(\Omega)^2$ a differential operator

$$L_0 g = X\partial g/\partial x$$

real continuous, and continuously differentiable with respect to the components of the vector x as well known such functions. This implies that the domain $D(L_0)$ of the operator L_0 is dense in the real space. Although this operator is linear operator it differs from a linear finite operator (matrix) very much. But the operator L_0 as in case of a linear finite operator may be of the dissipative operator.

Definition. A linear operator K, reflecting domain $D\,(\mathrm{K})\subseteq L^2\to L^2$ is called the dissipative operator if $\mathrm{Re}(Kg,g)\le 0$ for all $g\in D(K)$.

A. M. Lyapunov has proved that the steady state x=0 of the linear system $x\in R^n$ is asymptotically stable if and only if there exist such a metric by matrix C in R^n that yields the operator K for it metric will be the dissipative operator, i.e.

$$\sigma(CK+K^*C)<0\,,$$

where the matrix C is such (Cx,x)>0 if $\|x\|\ne 0$ |.

As well known that the matrix C is the matrix C of solution the equation $CK+K^*C=B$ where a matrix B is some given negative-definable matrix, (Bx,x)<0 if $\|x\|\ne 0,\ \ B=B^*$.Thus there exists metric that the operator P becomes of the dissipative operator and asymptotically stable is then transformed into contraction. We will extensions of the notion of the dissipative operator P for the operator L_0, and study the determination of the region of attraction of the asymptotically stable steady state x=0 under this approach. As well know, the dissipative operator yields a theory of semi-group of compression.

Thus, we introduced into consideration the Problem a new class of operators – the dissipative operators. We find it useful to define a concept of the Problem in terms of semi group of compression. Introducing a mathematical concept of compression into a physically problem we introduce the one in the power Theory of Hille-Yosida. One of the fundamental problems in kinetic theory is to find out wether or not we con control of transport into given conditions for the full three-dimensional Vlasov-Maxwell system. From sated above followes: in mathematical sens the focusing yields to Lyapunv problem [7].

The fundamental Lyapunov method assert that in region of attraction of the asymptotically stable seady-state there exists some function (function of Lyapunov). It is proved [5] that for this case there exists an optimal operator of Hilbert-Schmidt [6]. Then using the obtained optimal operator, we will the constructive approach resolve the Problem [5].

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МЕТОД ДЛЯ УПРАВЛЕНИЯ АНСАМБЛЯМИ ЧАСТИЦ

В. Задорожный

Статья посвящена нелинейной динамике управления интенсивными пучками заряженных частиц. Мы описываем распределение пучка при ускорении частиц (схема Власова-Максвелла) в системе дифференциальных игр и проблемы управления. Эта работа предлагает оптимальный подход для хорошо известного дифференциального уравнения Власова в частных производных, использующий полугрупповую контракцию и диссипативные операторы в гильбертовом пространстве.

МЕТОД ДЛЯ КЕРУВАННЯ АНСАМБЛЯМИ ЧАСТИНОК

В. Задорожний

Стаття присвячена нелінійній динаміці керування інтенсивними пучками заряджених частинок. Ми описуємо розподіл пучка при прискоренні частинок (схема Власова-Максвелла) у системі диференційних ігор та проблеми керування. Ця робота пропонує оптимальний підхід для добре відомого диференційного рівняння Власова у частинних похідних, що застосовує напівгрупову контракцію та дисипативні оператори в гільбертовому просторі.