COHERENT STACKING OF PICOSECOND LASER PULSES IN A HIGH-Q OPTICAL CAVITY FOR ACCELERATOR APPLICATIONS

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We have performed the harmonic analysis of the steady-state coherent pulse-stacking process in a high-Q Fabry-Perot cavity. The expression for the stacked pulse shape is obtained as a function of both the laser cavity and pulse-stacking cavity parameters. We have also estimated the pulse power gains attainable in the laser-optical system of NESTOR storage ring, which is under development at Kharkov Institute of Physics and Technology. It is shown that high power gains ($\sim 10^4$) can be, in principle, achieved in a cavity, formed with low-absorption, high reflectivity ($R \sim 0.9999$) mirrors, if the laser cavity length will differ exactly by half wavelength from the pulse-stacking cavity length. It implies development of the sophisticated frequency stabilization loop for maintaining the cavity length constant within a sub-nanometer range. At the same time, power gains of $\sim 10^3$ can be obtained with medium reflectivity mirrors ($R \sim 0.999$) at considerably lower cost.

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1. INTRODUCTION

Over the last decade a number of projects has appeared where Compton scattering (CS) has been considered as a tool for production of the intensive X-ray beams in low-energy (E_0=10 − 200 MeV) compact electron storage rings [1, 2]. There are also the most challenging schemes of using CS in linear super-colliders [3, 4].

The common feature of all these facilities is the problem of obtaining the intensive laser pulses with pulse width of ∼10 ps and pulse energy of ∼10 mJ (for $\gamma\gamma$-colliders these values are ∼1 ps and ∼1 J, respectively) that follow at a repetition rate of 100 – 500 MHz. Henceforth, we will refer to them as quasi-CW (quasi-continuous wave) laser beams. The feasible solution is coherent stacking of laser pulses with required time characteristics in a high-Q optical cavity system. The simplest scheme is an open cavity (Fabri-Perot cavity) formed with two high-reflectivity mirrors [5, 6].

Today the stacking of (CW) laser beams in a Fabry-Perot cavity is a well-established technique, while its implementation for quasi-CW laser beams is encountered with difficulties. The essential requirement to the resonance optical system, intended for stacking of short laser pulses, is coherence of pulse-summing process that ensures accumulation of energy in the laser pulse without deterioration of its temporal characteristics. To meet this requirement, the axial-mode spectrum of the pulse-stacking cavity (PSC) has to match the harmonic spectrum of quasi-CW laser beam, the latter being coincident with the mode spectrum of the laser cavity with active element.

Feasibility of coherent pulse stacking in a high-Q open cavity was demonstrated by R.J. Loewen at SLAC [5]. Iteratively adjusting the cavity length eventually enabled the laser to lock in 2 − 3 ms intervals to the peak axial mode in a 6.7 kHz bandwidth cavity (mirrors reflectivity $R = 0.9998$). The accumulation factor was estimated to be ≈ 4500. It should be noted that the natural, manufacturer-specified, laser pulse width of 7 ps was stretched up to 25−30 ps in order to eliminate the effect of dispersion mismatch between the laser cavity and PSC.

In this paper we present the results of sequential harmonic analysis of the steady-state pulse-stacking process in a high-Q Fabry-Perot cavity together with some estimations for NESTOR storage ring, which is under development at KIPT [2]. Absorption in cavity mirrors is not taken into account, because it does not affect the time characteristics of the stored pulse while essentially complicates the derived expressions. Detailed description is given elsewhere [7].

2. GENERAL DESCRIPTION

A match between the laser and the PSC means that not only cavity lengths have to be matched, but also their frequency spectra (combs) have to be identical. The main causes that hamper the matching are dispersion in optical elements and the essential difference in reflector parameters that form these cavities. The laser cavity reflectors have a large curvature radius in order to form a wide beam thus ensuring effec-

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tive interaction with the lasing medium. Reflectivity of the mirror, through which the laser beam is extracted, has to be low ($R \sim 0.9$) in order to attain a reasonable efficiency of the laser system and to provide the required output power.

The mirrors of the PSC have to meet practically contrary requirements. They should be high-reflectivity mirrors ($R = 0.999 - 0.9999$), and the radius of curvature ρ_c should be rather small ($\rho_c \approx L_c/2$, where L_c is cavity length) in order to focus the laser beam to the required spot size in the interaction point.

The axial mode spectrum f_q of the symmetric cavity, formed with two mirrors with complex reflectivity $\dot{r}_c = r_c \cdot \exp(-i\varphi_c^{\dot{r}})$, is given by [8]:

$$f_q = f_{FSR}\left[q + 1 + \frac{2}{\pi}\arctan\left(\frac{\rho_c}{L_c/2} - 1\right)^{-\frac{1}{2}} - \frac{\varphi_c^{\dot{r}}}{\pi}\right], \quad (1)$$

where $q = 2\ L_c/\lambda$ is the longitudinal index of the axial cavity mode TEM_{00q}; λ is the laser wavelength; $f_{FSR} = c/(2L_c)$ is the free spectral range and c is the speed of light.

For the real parameters of the laser and pulse-stacking cavities their frequency combs are shifted one against the other.

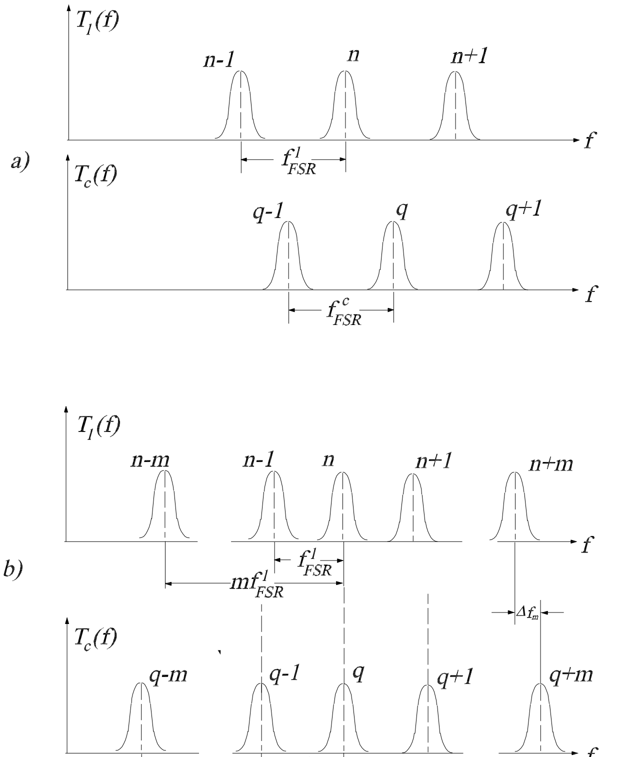

Fig. 1. *Schematic spectra (frequency combs) of the laser cavity modes and PSC modes: a) $L_l = L_c$, no mode locking; b) after adjusting PSC length ($L_l \neq L_c$) for mode locking ($f_n^l = f_q^c$)*

This effect is illustrated in Fig.1,a where the spectra of two cavities, namely, laser cavity (L_l, ρ_l, $\varphi_{\dot{r}}^l$, $n = 2L_l/\lambda$) and PSC (L_c, ρ_c, $\varphi_{\dot{r}}^c$, q) are sketched. Here the laser cavity with active element is presented with some equivalent two-mirror cavity. The cavity lengths are matched, i.e. $L_l = L_c$. To drive the PSC at the laser carrier frequency $\omega_0 = 2\pi f_0 = 2\pi c/\lambda$, which corresponds to the cavity mode with longitudinal index n, one has to lock this frequency to the frequency of the PSC mode with longitudinal index q or to the frequency of any of PSC modes that lay in close proximity to this one. It can be done either by changing the carrier frequency via a proper adjustment of the laser cavity length or by changing the PSC length. In both these cases the free spectral ranges for two cavities become unequal ($f_{FSR}^l \neq f_{FSR}^c$), so the mode frequencies in two cavities get different shifts against the matched mode frequency $f_n^l = f_q^c$. Fig.1,b illustrates this effect. So, the quasi-CW laser beam sidebands will drive the PSC at frequencies that correspond to the wings of its resonance curves instead of their peaks. It can lead to pulse lengthening that finally results in lower accumulation factors achieved.

3. HARMONIC ANALYSIS OF THE PULSE-STACKING CAVITY

The infinite periodical sequence of electromagnetic pulses of arbitrary shape propagating in z-direction with group velocity v and repetition rate f_{rep} can be presented in the time domain with the infinite sum of pulses:

$$S(t,z) = \sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} s(\tau)\delta\left[\tau - \left(t + \frac{n}{f_{rep}} - \frac{z - z_0}{v}\right)\right] d\tau. \quad (2)$$

Each of them presented with the function $s(\tau) = f(\tau)cos(\omega_0\tau + \delta_0)$, where ω_0 and δ_0 are carrier frequency and carrier initial phase, respectively. The pulse structure is displayed in Fig. 2.

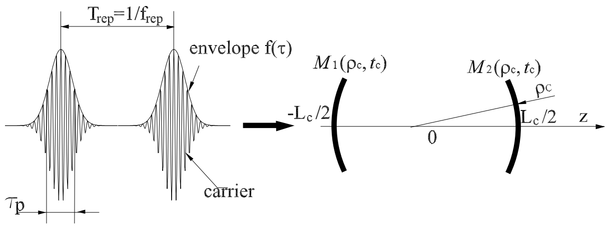

Fig. 2. *The laser pulse structure and the PSC parameters*

Let then assume that this sequence of pulses is incident on the symmetrical two-mirror PSC with complex mirror reflectivity $\dot{r}$ and complex transmission factor $\dot{t}$. Note that $R = \dot{r}^2$, $T = \dot{t}^2$ and $R + T = 1$.

The incident laser beam has to be presented in the frequency domain at some fixed point along z-axis. In our case it is the point $z = -L_c/2$ where the left mirror of the PSC is located:

$$s\,(t,-L_c/2)=\frac{1}{2}\left\{\sum_{m=-\infty}^{\infty}\dot{C}_m \exp\left\langle i\left[(\omega_0+m\omega_{rep})t+\delta_0\right]\right\rangle+\sum_{m=-\infty}^{\infty}\dot{C}_m \exp\left\langle -i\left[(\omega_0-m\omega_{rep})t+\delta_0\right]\right\rangle\right\}\ , \quad (3)$$

where $\omega_{rep} = 2\ \pi/T_{rep}$, and Fourier coefficients $\dot{C}_m$ for Gaussian pulses are given by:

$$\dot{C}_m=\sqrt{\frac{\pi}{2}}\cdot\frac{\tau_p}{T_{rep}}\exp\left[-\left(m\pi\frac{\tau_p}{T_{rep}}\right)^2/2\right]d\tau\ , \quad (4)$$

where τ_p is the pulse duration. The beam stored in the PSC can be presented at M_1 location with the following infinite Fourier series [8] :

$$B\,(t,-L_c/2)=\sum_{m=-\infty}^{\infty}\dot{C}_m\dot{T}_m^{+}\cos\left[(\omega_0+m\omega_{rep})\,t+\right.$$
$$\left.+\delta_0+\Psi_m^{+}\right]\ , \quad (5)$$

where T_m^+ and Ψ_m^+ are, respectively, the modulus and the phase of the transfer function $\dot{T}_m^+ = T_m^+ \exp(i\Psi_m^+)$. They can be obtained from the following relations:

$$T_m^{+}=\frac{k_{ph}^{0}}{\sqrt{1+\left[\frac{2r_c\sin\left(m\pi\Delta L/L_l\right)}{1-r_c^2}\right]^2}}\ , \quad (6)$$

$$\Psi_m^{+}=\arctan\frac{r_c^2\sin\left(2m\pi\Delta L/L_l\right)}{1-r_c^2\cos\left(2m\pi\Delta L/L_l\right)}-\varphi_c^{\dot t}\ , \quad (7)$$

where:$\varphi_c^{\dot t}$ is the phase of the transmission factor $\dot{t}$, $k_{ph}^0 = t_c/\left(1-r_c^2\right)$ is the amplitude gain in the PSC for the first harmonic of the laser beam and $\Delta L/L_l = (L_l - L_c)/L_l$ can be obtained with:

$$\Delta L/L_l=1-\frac{q}{n}+\frac{1}{\pi n}\left[2\arctan\sqrt{\frac{\rho_l}{L_l/2}-1}-\right.$$
$$\left.-\arctan\sqrt{\frac{\rho_c}{L_c/2}-1}-\left(\varphi_l^{\dot r}-\varphi_c^{\dot r}\right)\right]\ , \quad (8)$$

where indices c and l refer to the pulse-stacking and laser cavities, respectively. By substituting Eqs. (6), (7), and (8) in Eq. (5) one can obtain the final solution for the stored pulse at $z = -L_c/2$.

4. RESULTS AND DISCUSSION

At the first stage we have performed calculations by using the derived formulae for parameters which are relevant to NESTOR facility [2] ($\lambda = 1064\ \mu m$, $E_{laser} = 10\,W$, $\tau_p = 7$ ps) The influence of the following parameters was studied:
– PSC length,
– radius of curvature of laser mirrors,
– reflectivity of the PSC mirrors.

Two possible PSC configurations were considered: 0.42 m (short) cavity and 2.52 m (long) cavity. The first one corresponds to facility operation with 18 electron bunches and a high laser pulse repetition rate $f_{rep} = 350$ MHz. The radius of curvature of PSC mirrors $\rho_c = 21.4$ cm was chosen so as to obtain the transverse beam-waist size $2w_0 = 200\ \mu$m. The same parameter for laser mirrors was varied so as to obtain the beam-waist size in the laser cavity 1 mm and 2 mm. The reflectivity of the laser mirrors was taken to be 0.95, while PSC mirrors reflectivity ranged from 0.999 to 0.9999. The incident laser pulse width 7 ps (FWHM) corresponds to the specified value for the commercially available mode-locked lasers with SESAM mirrors [9].

Some calculation results are presented in Figs.3,4. Fig.3 illustrates how a time profile of the stored laser pulse changes when the PSC cavity length changes so as to lock the laser mode with longitudinal index n to corresponding cavity modes. The significant pulse distortion in the PSC with high-reflectivity mirrors can be seen for non-optimal matching.

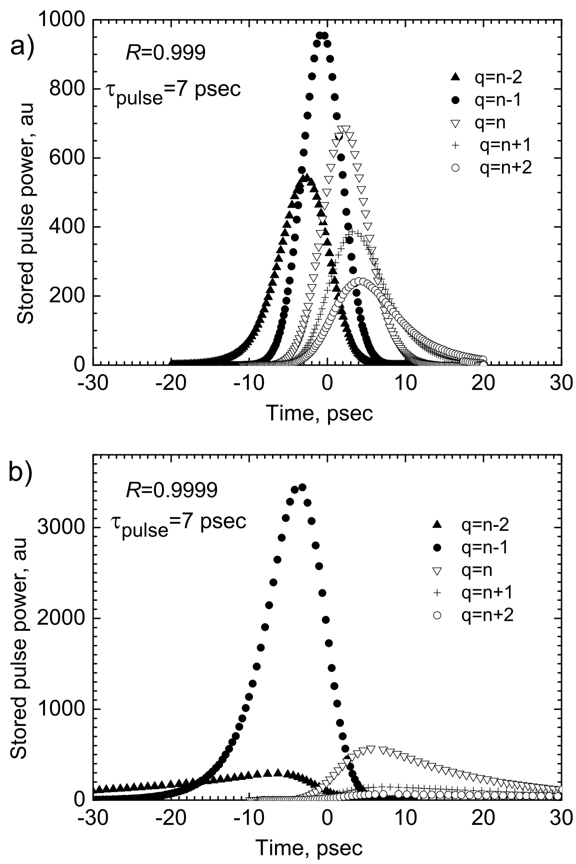

Fig. 3. *Time profiles of the 7 ps laser pulse stored in 0.42 m PSC with different mirror reflectivity: a)* $R = 0.999$*; b)* $R = 0.9999$

Calculations have shown:

– maximal gain is obtained for the case, when the PSC axial mode TM_{00q} is locked to the laser cavity mode TM_{00n-1} (optimal matching);

– for medium-reflectivity mirrors we come very close to ideal matching, i.e. $(k^0_{ph})^2 \sim 10^3$, while for the PSC with high-reflectivity mirrors we are far from the goal value of 10^4;

– in the PSC with high-reflectivity mirrors only one cavity mode can provide the reasonable (for given mirrors) pulse amplitude gain; a cavity with medium-reflectivity mirrors permits one to use several modes, thus simplifying mode matching and widening the operation range;

– the long-cavity version yields to the short-cavity one, this disadvantage becomes more pronounced with increasing of mirror reflectivity;

– pulse-shape distortion and pulse widening is conspicuous only for the non-optimal matching, this effect is more noticeable for the cavity with high-reflectivity mirrors.

The last effect is seen clearly in Fig.4 where the shape of the stored pulse is given for different reflectivity of the PSC mirrors and two different radii of curvature of the laser mirrors. Only the optimal inter-cavity mode shift $q = n - 1$ is presented. Solid symbols correspond to large curvature radius of the laser mirror $\rho_l = 41.7$ m providing 2 mm beam waist in the laser crystal, while the open symbols correspond to $\rho_l = 3.2$ m that provides 1 mm beam waist. In the last case the essential decreasing of the stored pulse peak power is seen for high-reflectivity mirrors. The noticeable widening of the stored pulse for the PSC with high-reflectivity mirrors is also seen: by factor 1.5 and 1.8 for $\rho_l = 41.7$ m and $\rho_l = 3.2$ m, respectively.

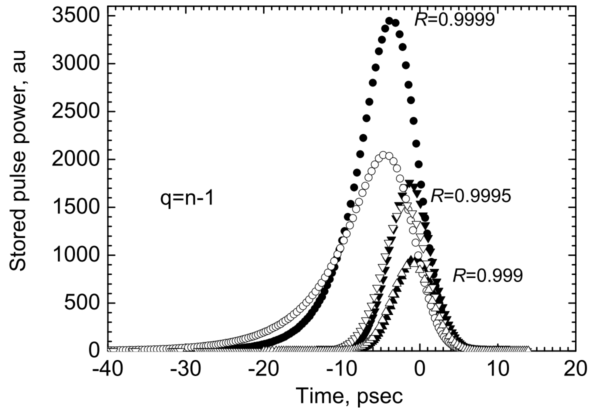

Fig. 4. *Time profiles of the 7 ps laser pulse stored in 0.42 m PSC. Solid symbols correspond to ρ_l=41.7 m, open symbols refer to ρ_l=3.2 m*

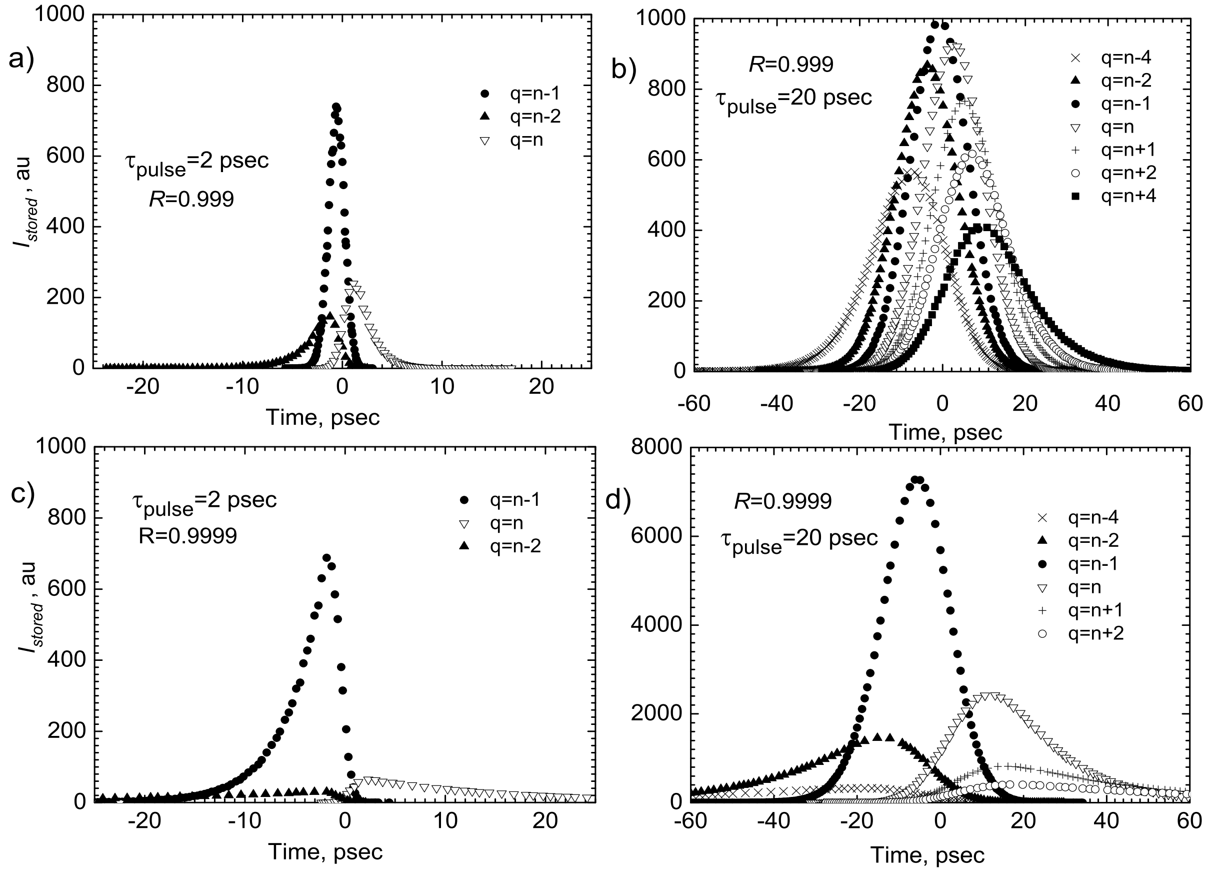

Fig. 5. *Time profiles of the 2 ps (a, c) and 20 ps (b, d) laser pulse stored in the PSC with medium-reflectivity (a, b) and high-reflectivity (c, d) mirrors. The family of curves in each picture corresponds to different shifts between longitudinal indices of the laser mode and locked PSC mode*

At the second stage we performed calculations for the 20 ps and 2 ps incident laser pulses in order to understand how much the considered effects will be alleviated for longer pulses (or aggravated for shorter ones).

The results for 20 ps pulses show:

– a number of PSC axial modes which can be used for pulse stacking in the medium-reflectivity version increases while in the high-reflectivity version only $q = n - 1$ case is admissible;

– the pulse amplitude gain in the high-reflectivity version essentially increases against the case of 7 ps pulses, and for optimal matching $q = n - 1$ it approaches the ideal matching value.

It can be seen in Fig.5 where the time profiles of the stored pulse is presented both for the short 2 ps pulse (a, c) and long 20 ps pulse (b, d) and for medium-reflectivity (a, b) and high-reflectivity (c, d) mirrors. In vertical scale the instant power in stored laser pulse I_{stored} is plotted in arbitrary units. The incident laser pulse amplitude is unity, so from a comparison of the peak power value from the figure with the square of k^0_{ph} we can deduce how close to the ideal matching we approach. Note, that $(k^0_{ph})^2$ is equal to 10^3 and 10^4 for (a, b) and (c, d), respectively.

Calculations show that for 2 ps pulse width and high-reflectivity mirrors (see Fig.5c) the stored pulse is greatly widened against the incident one. The amplitude of the stored laser pulse decreases drastically and even for optimal matching attains only 0.07 of the ideal matching value. The same value of I_{stored} can be obtained in the PSC formed with mirrors with reflectivity $R = 0.999$ (see Fig.5,a).

The pulse amplitude gain doesn't take into account pulse widening of the stored pulse. When this widening is within controllable limits it is useful to define the power gain factor k_{ph} as a figure of merit that describes a pulse-stacking efficiency:

$$k_{ph} = \int_{-T_{rep}/2}^{T_{rep}/2} I_{stored}(t)dt / \int_{-T_{rep}/2}^{T_{rep}/2} I_{incident}(t)dt \ , \quad (9)$$

where $I_{stored}(t)$ and $I_{incident}(t)$ are power intensities of the stored and incident laser pulse, respectively. This parameter is displayed in Fig. 6 versus cavity mode shift $q - n$ for two values of PSC mirrors reflectivity and three pulse durations. One can see that the long-pulse results show a systematical excess over the short-pulse data except the case of optimal matching for $R = 0.999$ and $\tau_p = 7\,ps$ and $20\,ps$, where both points coincide giving maximal value of 10^3 (no dispersion effects).

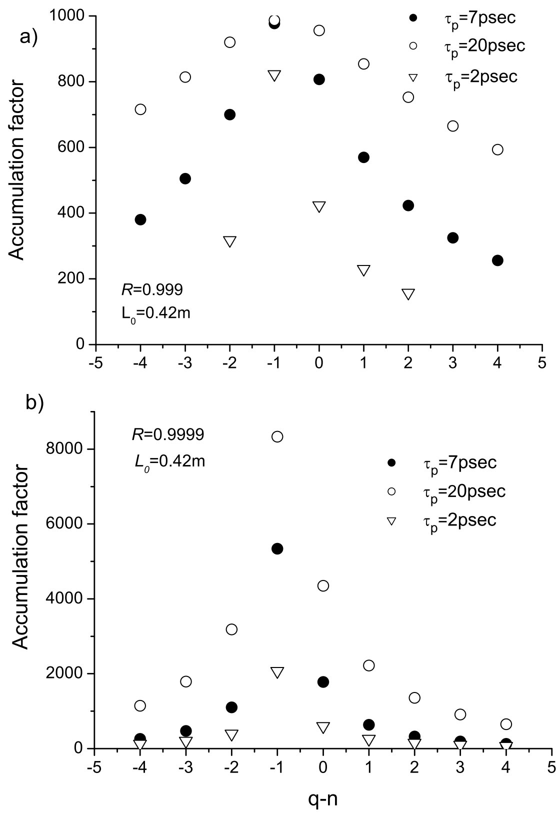

Fig. 6. *Power gain factor as a function of the cavity mode shift q-n for medium-reflectivity mirrors (a) and high-reflectivity mirrors (b)*

5. CONCLUSION

We have analyzed the effects of matching between axial mode spectra of the laser cavity and pulse-stacking cavity upon the stored pulse width and stored power. It is shown that for high reflectivity mirrors ($R \sim 0.9999$) high power gains can be obtained only if the laser carrier frequency coincides with the definite axial mode frequency of the PSC, namely, with that second to the peak mode. It means that both cavity lengths have to differ exactly by one half of the laser wavelength, and these conditions have to be maintained during all period of generation. This task is the present state-of-the-art of laser optics technology, and it requires much efforts, both financial and scientific, to develop a laser pulse stacking system with power gains up to 10^4.

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КОГЕРЕНТНОЕ НАКОПЛЕНИЕ ПИКОСЕКУНДНЫХ ЛАЗЕРНЫХ ИМПУЛЬСОВ В ВЫСОКОДОБРОТНОМ ОПТИЧЕСКОМ РЕЗОНАТОРЕ ДЛЯ ПРИМЕНЕНИЯ В УСКОРИТЕЛЯХ

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Мы выполнили гармонический анализ стационарного процесса когерентного накопления лазерных импульсов в высокодобротном резонаторе Фабри-Перо. Получены выражения, описывающие форму накопленного импульса в зависимости от параметров лазерного и накапливающего резонаторов. Мы также оценили коэффициент накопления мощности в импульсе, который может быть получен в лазерно-оптической системе накопителя НЕСТОР, разрабатываемого в ННЦ ХФТИ. Показано, что коэффициент накопления ($\sim 10^4$) может быть, в принципе, получен в резонаторе, образованном зеркалами с малым поглощением и высоким коэффициентом отражения ($R \sim 0.9999$), если длина лазерного резонатора будет отличаться от длины накапливающего резонатора ровно на половину длины волны лазерного излучения. Это подразумевает необходимость создания очень сложной и дорогой системы стабилизации частоты, которая способна непрерывно контролировать длину резонатора с суб-нанометровой точностью. В то же время, коэффициент накопления $\sim 10^3$ может быть получен с зеркалами, имеющими коэффициент отражения $R \sim 0.999$, с существенно меньшими затратами.

КОГЕРЕНТНЕ НАКОПИЧЕННЯ ПІКОСЕКУНДНИХ ЛАЗЕРНИХ ІМПУЛЬСІВ У ВИСОКОДОБРОТНОМУ ОПТИЧНОМУ РЕЗОНАТОРІ ДЛЯ ВИКОРИСТАННЯ У ПРИСКОРЮВАЧАХ

В.П. Андросов, І.М. Карнаухов, Ю.М. Телегін

Ми виконали гармонічний аналіз стаціонарного процесу когерентного накопичення лазерних імпульсів у високодобротному резонаторі Фабрі-Перо. Одержані співвідношення, що описують форму накопиченого імпульсу в залежності від параметрів лазерного резонатору та резонатору, що накопичує. Ми також оцінили коефіцієнт накопичення енергії в імпульсі, що може бути одержаний у лазерно-оптичній системі накопичувача НЕСТОР, що розробляється в ННЦ ХФТІ. Показано, що коефіцієнт накопичення ($\sim 10^4$) може бути, в дійсності, одержаний у резонаторі, що створений дзеркалами с малим поглинанням і високим коефіцієнтом віддзеркалення $R \sim 0.9999$, якщо довжина лазерного резонатора буде відрізнятись від довжини резонатора, що накопичує, рівно на половину довжини хвилі лазерного випромінювання. Це розуміє необхідність створення дуже складної та дорогої системи стабілізації частоти, що має безперервно контролювати довжину резонатора з суб-нанометровою точністю. У той же час, коефіцієнт накопичення $\sim 10^3$ може бути одержаний з дзеркалами, що мають коефіцієнт віддзеркалення $R \sim 0.999$, зі значно меншими витратами.