THE QUASI-COOLING EFFECT OF RELATIVISTIC CHARGED PARTICLES BEAMS

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(Received March 20, 2007)

A possibility of realization of a new, of principle, physical phenomenon is shown. This is the phenomenon of energy equalization of charged particle beams in the time their acceleration in the crossed undulative electric and magnetic fields (EH-fields). The phenomenon predicted is referred to as the quasi-cooling effect. Main properties and features of the effect are studied. In particular, it is shown that it would be experimentally observed for the existing today level of acceleration technologies.

PACS: 52.75.-d

1. INTRODUCTION

It is well known that the problem of forming of essentially "cold" charged particle beams (for instance, the cooling effect in the storage rings) are very topical for many branches of modern applied electrodynamics [1, 2]. They are accelerators of special class, various technologies for generation and amplification of especially "cold" electromagnetic signals, etc. However, really satisfactory physical mechanisms for realization of practical systems of such kind are not found till today. In this connecting we chose the main purpose of this article the looking for such mechanisms. As a result of the work accomplished the mechanism of *energy equalization of charged particles* in beams during their acceleration in the crossed undulative electric and magnetic fields (EH-fields) has been found. The phenomenon predicted is called conditionally the *quasi-cooling effect*. It should be noted that, strongly saying, the quasi-cooling of charged particle beams, which move in the accelerating EH-fields, looks as an unusual phenomenon from many points of view. Including, it is known that accord-ing to the second law of thermodynamics any heat cannot be transferred from any colder object to other hotter one. Therefore, at the first sight, it looks that existing the quasi-cooling effect in Nature is impossible, in principle. However, really this is not quite correct, because this law is applicable strictly for thermodynamically closed systems only. It is well known that any charged particle beam moving in external electromagnetic fields is an essentially open system. Therefore, really the second thermodynamic law does not prohibit realization of the discussed type of phenomena. The second "doubtful" position concerns the Liouville theorem. Let us remind that, in accordance with this theorem, the *phase volume* of any dynamical system conserves its magnitude always. In our case, the phase volume of the accelerated electron bunch is determined in the six-dimensional space of three space coordinates x, y, z and three components of canonical momentum P_x, P_y , P_z . According to opinion of potential our critics, the momentum components of a charged particle bunch cannot decrease (i.e., the beam can not be cooled), in principle, because of the Liouville theorem. Unfortunately, it is a widespread mistake. The *Liouville theorem really forbids the changing phase volume only.* Or, in other words, this prohibition is not correct if it is respected to part of coordinates of the phase volume only (in our case they are momentum coordinate components). The point is that the momentum coordinates may change really if the spatial coordinates change simultaneously, too. It is obvious that the phase volume, as a whole, should be conserved in such situation. This could be treated as the *phase volume could turn in six-dimension space*, as a whole, conserving its total magnitude. We can accomplish such turn, for instance, in such a manner that the momentum sizes of the phase volume (i.e., momentum spread on coordinates P_x, P_y , P_z) *decreases* (the process the bunch energy equalization occurs). And, simultaneously, the spatial size (coordinate spread width of on the spatial coordinates) *increases* synchronously. Therein, the magnitude of the phase volume does not change during this process. Hence, the Liouville theorem holds really in such case. It should be mentioned that just this situation is realized in the case of considered quasi-cooling effect.

2. ESSENCE OF THE QUASI-COOLING EFFECT

The physical meaning of the quasi-cooling effect

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is illustrated in Fig. 1. There the charged particle (electrons) moves in the EH-accelerator [1] in the EH-field. One readily can be made certain that the electron trajectories has a sinus-like shape and the electrons moving in such field are accelerated [1]. And the last, we can find that the amplitude of electron undulations depends on its energy. This means that the electron with less energy (or that is the same, with less relativistic mass) moves along the trajectories with greater oscillation amplitude (see item 2 in Fig .1). Correspondingly, the trajectory of the electron with larger energy is characterized by less amplitude (see item 5 at the same place). Then, let us turn the reader attention that the acceleration of electrons occurs within the space (gap) between two neighboring magnetic poles (see item 1 in Fig.1) under action of the transversal electric field $\vec{E}$ (see item 4 in the same place). As a result, the electrons with lager amplitudes 2 get the lager acceleration and vice versa. However, as it noted above, the electrons with lager amplitude possess the less energy. Therefore the electrons with less energy are accelerated more than the electrons with lager energy. Therein, particle 2 in Fig.1 as well electron 5 (in the same place) equalize their energy with the energy of benchmark electron 3. This means that the discussed model has a peculiar *phenomenon of equalization of the electron energies.* Just this phenomenon, as it is mentioned above, has been referred by authors to as the quasi-cooling effect.

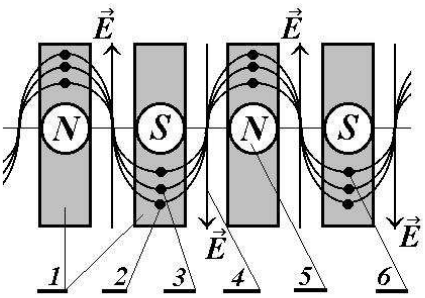

Fig.1. *Trajectories of three typical electrons, differing by energies, moving in the crossed undulative magnetic and electric fields (EH-fields). Here: 1 are the magnetic poles, 2 is the electron which has the smallest energy ε_1, 3 is the electron which we regard as a benchmark particle (whose energy $\varepsilon_2 > \varepsilon_1$), 4 are the vectors of intensity of transverse electric field $\vec{E}$, 5 are the directions of the vectors of undulative electric field $\vec{B}$, 6 is the electron which has largest energy*

Saying about possible practical realizations of the model shown in Fig.1, we can turn the reader attention on the following. Firstly, the transverse electric field $\vec{E}$ can be generated by two different methods. The first is the inductional one. In this case the magnetic field $\vec{B}$ (see item 5 in Fig.1) is a slowly varying on time function. The undulative transverse electric field $\vec{E}$ (see item 4 in the same figure) in this case is generated owing to the well-known effect of electromagnetic induction. In such situation we have the so-called *non-stationary system.* In the case when the electric field $\vec{E}$ is generated by some other independent (with respect to the undulative magnetic field 5) method we have the *stationary system.* Secondly, independently with the method of generation of the electric field, we, in turn, have (with respect to the amplitudes of undulative fields) the *homogeneous* and *inhomogeneous systems*, respectively. Then, let us discussed shortly characteristic features of the system of these types.

3. THE HOMOGENEOUS NON-STATIONARY SYSTEMS

In what follows, let us continue our analysis of physical essence of the quasi-cooling effect. The weak-relativistic homogeneous circularly polarized EH-models is chosen as a convenient object for this purposes. The dependencies of relativistic factors for some ten electrons distinguishing by their initial energies $mc^2\gamma(j)_0$ are shown in Fig.2 (here: $j = 1, 2, \ldots, 10$ are electron numbers; $(j)_0$ are their initial relativistic factors).

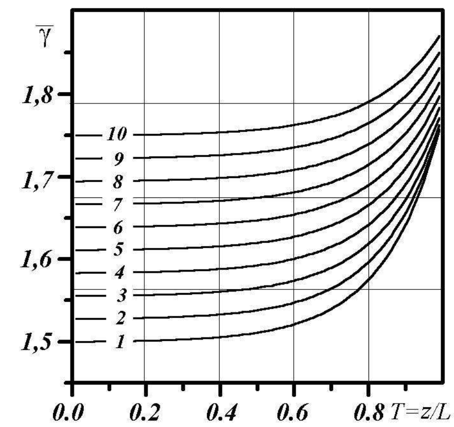

Fig.2. *Dependencies of the averaged relativistic factors $\bar{\gamma}$ for ten electrons, which have different initial energies $mc^2\gamma(j)_0$. Here: $j = 1, 2, \ldots, 10$ are the electron numbers; the magnetic field amplitude $B = 3\,kGs$; the electric field intensity is $E = 100\,kV/m$; the undulation period $\Lambda = 5\,cm$; the system length $L = 1\,m$*

The result of numerical calculations confirms that the quasi-cooling effect indeed could be realized in the EH-accelerators. The initial energy spread $\Delta\varepsilon_0\Delta\gamma_0 mc^2 \sim 122\,keV$ at the system input (see in Fig.2) transforms into the output spread $\Delta\bar{\varepsilon}_{output} \sim 35\,keV$, i.e. and the decreasing of initial energy would be obtained in $\sim$ 3.5 times. However, it is shown also that the real dynamics of electrons in the EH-fields is more complicated than it seems at first sight. First, the quasi-cooling effect manifests its "quasi-cooling properties" with respect to some separated group of electrons only (just for the electrons with numbers from 1 till 5, see Fig.2). At the same time, it ap-pears rather feeble for the rest electrons, i.e., for the electrons with numbers from 6 till 10.

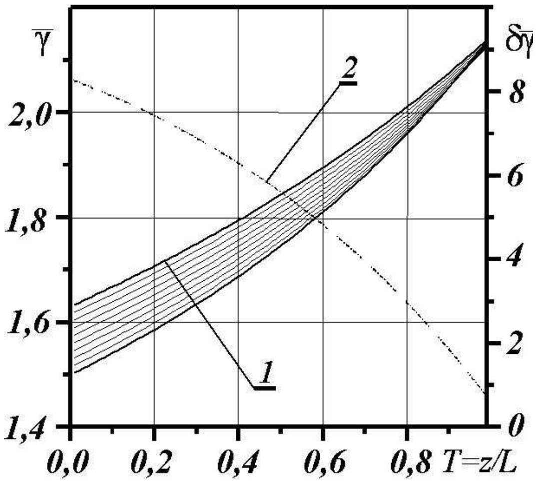

Fig.3. *Dependencies of the averaged relativistic factors for the ten electrons (curve 1), which possess different initial energies (initial spread takes place with respect to all three components of electron motion). Curve 2 shows the dependency of relative decreasing of energy spread* $\delta\overline{\gamma} = \triangle\gamma/\langle\gamma\rangle \cdot 100\%$ *on the non-dimension coordinate* $T = z/L$. *Here: the induction of magnetic component of the EH-undulated field* $B = 3\,kGs$, *the intensity of vortex electric field* $E = 150\,kV/m$, *the period of undulations* $\Lambda = 5\,cm$, *the system length* $L = 1\,m$

As analysis shows, the effectiveness of such *selection mechanism* essentially depends on system parameters, including, the field amplitudes, the initial energies of electrons, etc. Second, from the practical point of view, the scale of quasi-cooling in the considered example does not make an impression. Therefore we should look for ways of in-creasing the practical attractiveness of the considered phenomenon. As a first step in this direction, let us to make clear the main physical causes of the shortcoming mentioned. In particular, we will study the effectiveness of quasi-cooling mechanism depending of the width of electron spread for initial energies. Previously let us introduce a few new specific designations. The relativistic factors of electrons 1 and 10 (see Fig. 2) can be written as

$$\begin{aligned} \gamma_{(1)} &= \left[1 - \left(v_x^2 + v_y^2 + v_z^2\right)/c^2\right]^{-1/2} \qquad (1)\\ \gamma_{(10)} &= \Big\{1 - \Big[(v_x + \Delta v_x)^2 + (v_y + \Delta v_y)^2 + \\ &\quad + (v_z + \Delta v_z)^2\Big]/c^2\Big\}^{-1/2}, \end{aligned}$$

where Δv_x, Δv_y, Δv_z are initial electron spreads with respect to their velocities. Taking in view ordinary common sense, let as accept the following condition for the "relative coldness" of cooled beam:

$$|\Delta v_{x,y,z}| << v_{x,y,z}. \qquad (2)$$

Relevant numerical estimations show that condition 2 is satisfied for the considered example for electrons with numbers from 1 till 5 (see Fig. 2). With the condition 2 after corresponding expansion in a power series we write for the width of spread with respect to electron relativistic factors:

$$\begin{aligned} \Delta\gamma &= \gamma_{(10)} - \gamma_{(1)} \approx \qquad (3)\\ &\approx \gamma_{(1)}^3\left(v_x\Delta v_x + v_y\Delta v_y + v_z\Delta v_z\right)/c^2 + \\ &+ \gamma_{(1)}^3\left[(\Delta v_x)^2 + (\Delta v_y)^2 + (\Delta v_z)^2\right]/c^2 + ... \end{aligned}$$

Besides that, the definitions are introduced for the partial relativistic factors $\gamma_{(j)x,y,z}$:

$$\gamma_{(j)x,y,z} = \left(1 - v^2{}_{x,y,z}/c^2\right)^{-1/2}. \qquad (4)$$

Therein, to avoid misunderstandings, we should point out especially that $\gamma_{(j)} \neq \gamma_{(j)x} + \gamma_{(j)y} + \gamma_{(j)z}$, i.e., *partial relativistic factors* $\gamma_{(j)x,y,z}$ *have no direct energetic sense.* They are only the normalized (in specific way) squares of electron velocities on different coordinates. Then, expression 3 after some transformation can be written as

$$\begin{aligned} \Delta\gamma &\approx \frac{\gamma_{(1)}^3}{\gamma_{(1)x}^3}\Delta\gamma_x + \frac{\gamma_{(1)}^3}{\gamma_{(1)y}^3}\Delta\gamma_y + \frac{\gamma_{(1)}^3}{\gamma_{(1)z}^3}\Delta\gamma_z + \qquad (5)\\ &+ O\left[(\Delta\gamma_{x,y,z})^2\right], \end{aligned}$$

where only the linear terms are taken into account to calculated with respect to the differences of partial relativistic factors

$$\begin{aligned} \Delta\gamma_{x,y,z} &= \gamma_{(10)x,y,z} - \gamma_{(1)x,y,z} \approx \qquad (6)\\ &\approx \gamma_{(1)x,y,z}^3\left(v_{x,y,z}\Delta v_{x,y,z}/c^2\right). \end{aligned}$$

As it will be demonstrated below, expressions 3,5, explicitly explains all mentioned specific features of the quasi-cooling effect. As it is accepted above, the electrons have initially only longitudinal non-zero velocity spread Δv_{z0}. In spite of this supposition, the corresponding dynamic transverse spread $|\Delta\gamma_{x,y}| > 0$ appears during the acceleration process because of nonlinear relation of transverse and longitudinal electron motions. But, let us discuss this peculiar physical mechanism in more details. The mentioned transverse spread appears just due to the dependency of electron energy on all velocity components at the same time. Electrons, which have larger initial longitudinal relativistic mass, get, as it mentioned already, the less energy of transverse oscillations (see item 2 in Fig. 1 and corresponding explanations). Inasmuch as it has been assumed earlier that electrons in the input are differed by longitudinal relativistic factors $\gamma_{(j)0z}$ only (i.e., initially $\Delta\gamma_z > 0$, $\Delta\gamma_{x,y}(z = 0) = 0$, see Fig. 2) that the corresponding addends to the transverse current energy are found to be formally *negative*: $\Delta\gamma_{x,y}(z > 0) < 0$. In view of 5, it, in turn, means that the appearance of transverse electron spread is accompanied by decreasing the total energy spread $mc^2\Delta\gamma$. Or, in other words, there is a paradox that in *spite of the appearance of additional transverse spread in the considered model, the decreasing total energy spread occurs.* It is interesting to note that in the case if the initial transverse spread is given nonzero that it should also decrease during the acceleration process. Explanation of this paradox is rather simple. The point is that the initial energy spread (including the transverse one) is always positively determined value, whereas the above-discussed transverse additions, as it mentioned already, are characterized by negative sign: $\Delta\gamma_{x,y}(z > 0) < 0$. Further, come to discussion about behavior of the electron group represented by particles 6 - 10 in Fig. 2. It should be mentioned that formula 5 is not applicable in this case. This could be explained by that electrons of this group are characterized by relatively large initial energy spread. As a result, at list the condition for the longitudinal motion component like 2 is not satisfied in this case. More precise (than 5) expression for the total electron spread can be obtained taking into account the next (quadratic) terms $\sim (\Delta v_{x,y,z})^2$ of corresponding expansions. However, the addends of this type to the right side of 5 turn out to be always positive. There-fore, the work of the above-discussed compensation mechanism (based on the negative signs of linear addends $\Delta\gamma_{(j)x,y}$ in 5) changes for the worse. Therein, this occurs more essential then wider initial energy is spread $\Delta\varepsilon = \Delta\gamma_0 mc^2$, i.e. the stronger is the influence of quadratic addends $\sim (\Delta v_{x,y,z})^2$. The dynamics of electrons $6 - 10$ in Fig.2 illustrates just this physical situation. Then let us discuss more general homogeneous model whose quasi-cooling dynamics is illustrated in Fig.2. We consider that initial spread in this model takes place with respect to all three components of electron motion, and the model is more relativistic. On the contrary to Fig.2, it has been assumed that the magnitude of initial energy spread is not too large (i.e. we will study dynamics of the electrons analogous to that taking numbers from 1 until 5 in Fig.2). The dynamics of the total averaged relativistic factors $\bar{\gamma}_{(j)}$ of ten different electrons (i.e. $j = 1, 2, \ldots, 10$) and the total averaged spread width $\Delta\bar{\gamma}$ are shown in Fig.3. It is seen that the averaged spread width $\Delta\bar{\gamma}$ eventually decreases more than 6 times, i.e. the gain of quasi-cooling (in comparison with the previous case (see Fig.2)) is approximately 1,7 times. The explanation of this fact is simple because this result could be got due to the above-accepted assumption that the initial energy spread $mc^2\Delta\gamma_0$ is chosen as moderate one.

4. THE HOMOGENEOUS STATIONARY EH-COOLER

Example of the design-schemes of stationary linearly-polarized EH-coolers is shown in Fig. 4 (one period only, for simplicity). In distinct from the non-stationary EH-cooler, the vortex electric field (see item 7 in Fig. 4) in the work bulk of the stationary EH-cooler is generated by special external inductors. The latter are placed in the space between magnetic poles 4.

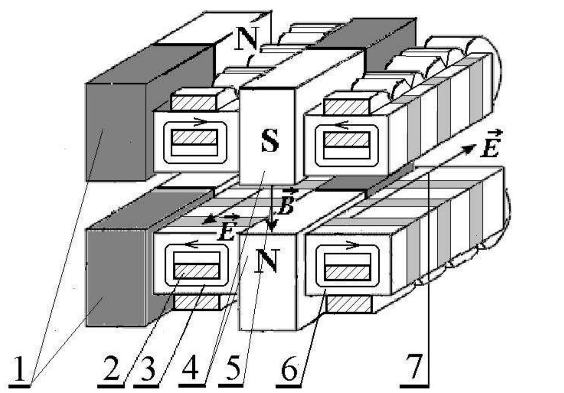

Fig.4. *Design scheme of the linearly polarized stationary EH-cooler (one period only). Here: 1 are the ceramic inserts, 2 are the inductor windings, 3 are magnetic fluxes within cores of the ferrite inductors, 4 are the permanent magnet poles, 5 is the vector of undulative magnetic field, 6 are the ferrite cores of inductors, 7 is the vector of intensity of the vortex electric field*

Each of these inductors consists of a ferrite core 6 and windings 2 (one or a few coils). The specific feature of the discussed design is utilization of a special magnetic screen. Due to this we can avoid some practical problems, connected with influence of the boundary magnetic field. In a practice, the magnetic poles 4 are made of some magnetic materials in the region only, where the turning of the accelerated (cooled) bunch occurs. The rest part of the poles 1 could be made of some nonmagnetic (ceramic) dielectric material.

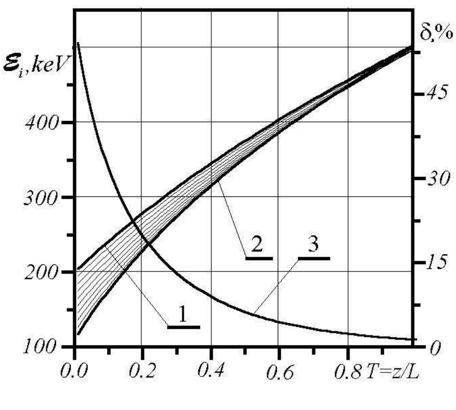

Fig.5. *Dependencies of the averaged kinetic particle energy $\bar{\varepsilon}_1$ and the relative energy spread $\delta = (\varepsilon_{max} - \varepsilon_{min})/\langle\varepsilon\rangle$ for ten large particles ($i = 1, 2, ..., 10$) on normalized longitudinal coordinate $T = z/L$. All particles different by the initial kinetic energy, therein curve 1 and 2 correspond to particles with maximal and minimal initial energy, respectively. Curve 3 describes the dependency of the relative energy spread $\delta = \delta(T)$. Here: the intensity of electric field is $1.32\,MV/m$, the induction of magnetic field is $180\,Gs$, the initial energy of electrons is $160\,keV$*

Then, let us analyze dynamics of the quasi-cooling process in the considered model. The methods of large particles and hierarchic version of the Bogolyubov method 1,3 are used for quantitative analysis. It is assumed that number of the large particles is ten. After tabulation of corresponding analytical expressions result of the analysis are represented in a graphical form (see Fig.5). First of all, it should be noted that, in contrast to the non-stationary EH-systems, the capture effect 1 does not realize in the stationary model. This allows eliminate a number of limitations on the system parameters, including, to increase the work length of the system. In turn, it opens more promising prospects for obtaining higher levels for the beam quasi-cooling. The materials of Fig. 5 evidently demonstrate these hopes. This is made on the example of system with the averaged input electron energy $\sim 160\,keV$ for the initial energy spread $\sim 52\%$. Correspondingly, the output energy spread is $\sim 1\%$ for the averaged energy $0.5\,MeV$. Thus, there is the electron bunch quasi-cooling at ~ 52 times in this particular case. Besides that, in distinct from the case of non-stationary EH-cooler (see the previous paragraph), in the discussed case we get a stable electron bunch, i.e., the bunch whose electron velocities do not depend on their output time.

5. THE INHOMOGENEOUS STATIONARY EH-COOLER

Some results of quantitative analysis of the EH-coolers with longitudinal and transversal inhomogeneities of amplitudes of the electric and magnetic fields are illustrated in Fig.6 and Fig.7.

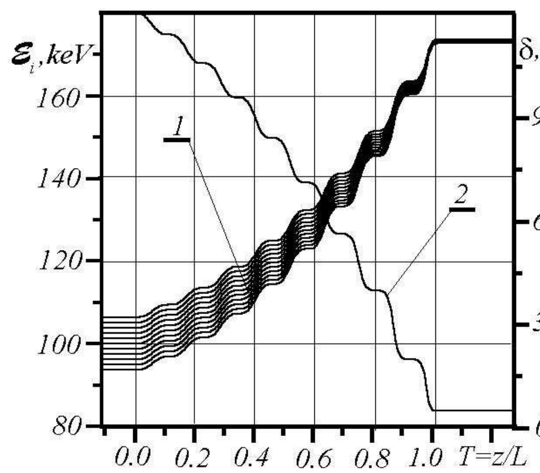

Fig.6. *Dependencies of the electron kinetic energy ε_i for ten electrons ($i = 1,...,10$) differing their initial energy (curves 1), and the relative electron spread δ (curve 2) on the normalized longitudinal $T = z/L$. Here: the electric field intensity $E = 190\,kVm$, the undulation period is $20\,cm$, and the system length $L = 1\,m$. The longitudinally inhomogeneous system*

It is readily seen that the above-formulated conclusion about the practical promising of the EH-coolers can be confirmed in the case of inhomogeneous models also. Namely, unique systems for forming especially high-qualitative electron beams can be designed on the basis of standard low-qualitative electron injectors. In particular, for the initial energy spread 12% eventually the electron beam is obtained with the energy spread 0.3%, i.e., the energy spread can be reduced in 40 time and more. Therein, the well-known beam transportation problem is solved here effectively by means of focusing property of the inhomogeneous real EH-model (see corresponding results in Fig.7).

We calculate the trajectories of all large particles for study of behavior of the electron beam, as whole. As a result we obtain a possibility to control the evolution its width during the quasi-cooling process. The beam width $\langle\Delta x\rangle$ could be determined as a result of statistical averaging on transverse coordinates of all particles in each beam cross-section. Correspond-ing results of such calculations are represented in Fig.7. It is readily seen that owing to introducing the transverse inhomogeneity the above mentioned transportation problem could be successfully solved. It is important to note that, as analysis shows, the introducing the transverse inhomogeneity allows additionally amplifying the affectivity of the quasi-cooling process in some special system arrangements.

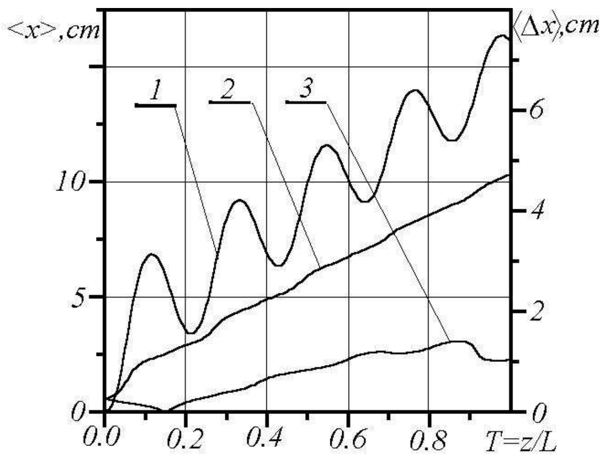

Fig.7. *Evolution of the averaged beam width $\langle\Delta x\rangle$ and trajectory of the benchmark particle $\langle x\rangle$ during the quasi-cooling process in the transversely inhomogeneous system. Here: curve 1 describes the non-averaged (on the undulations) trajectory of the benchmark particle, curve 2 illustrates the evolution of the averaged beam width $\langle\Delta x\rangle$ in the case of an equivalent homogeneous model, and curve 3 shows the same dependency in the case of considered inhomogeneous model; the induction of magnetic field is $180\,Gs$, the intensity of electric field is $920\,kV/m$, the input electron energy is $100\,keV$, the inhomogeneity coefficient $\chi' = 0,04\,cm^1$*

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ЭФФЕКТ КВАЗИ-ОХЛАЖДЕНИЯ ПУЧКОВ РЕЛЯТИВИСТСКИХ ЗАРЯЖЕННЫХ ЧАСТИЦ

В.В. Кулиш, И.В.Губанов, Нгуен Ай Вьет, А.Ю.Брусник

Описан новый физический эффект - эффект выравнивания энергий заряженных частиц пучка при их ускорении в скрещенных электрическом и магнитном ондуляторных полях (ЕН-полях). Данный эффект трактуется, как эффект эффективного охлаждения. Изучены основные особенности и характеристики данного эффекта. В том числе показано, что данный эффект может быть экспериментально реализован на существующем уровне ускорительных технологий.

ЕФЕКТ КВАЗІ-ОХОЛОДЖЕННЯ ПУЧКІВ РЕЛЯТИВІСТСЬКИХ ЗАРЯДЖЕНИХ ЧАСТИНОК

В.В. Куліш, І.В. Губанов, Нгуєн Ай В'єт, А.Ю. Бруснік

Описано новий фізичний ефект - ефект вирівнювання енергій заряджених частинок пучка під час їх прискорення в схрещених електричному та магнітному ондуляторних полях (ЕН-полях). Даний ефект трактується як ефект ефективного охолодження. Вивчено загальні властивості та основні характеристики даного ефекту. Зокрема показано, що дане явище може бути експериментально реалізовано на існуючому рівні прискорювальних технологій.