TO THE CHARGING OF SPHERE IN A EHD GAS FLOW

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The interaction between conducting sphere and ions has been investigated in a presence of external electric field and electrohydrodynamic (EHD) gas flow. Gas is considered as weakly ionized under atmospheric pressure. Diffusion is assumed to be weak and the trajectory assumption is taken into consideration. Ion currents are obtained analytically and investigated numerically for the collinear electric field and gas flow vectors. It is shown, that sphere charging regimes depend on the key parameter $\xi_\pm$ - the relation of ion drift velocity far away from sphere to gas velocity. So, the cases $|\xi_\pm| < 1$ and $|\xi_\pm| > 1$ yield to different charging regimes. For the potential flow, the ion current has been found analytically in continuous $\xi_\pm$-parameter space.
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TASK FORMULATION

A conducting sphere moves through the weakly ionized gas with a fixed terminal velocity U_p. External electric field is collinear over sphere motion vector. All ions which attach sphere surface recombinate on it. The selectivity properties of the sphere surface are discarded. The equation for ion trajectories in a spherical coordinates may be written as

$$r\frac{V_{r\pm}}{V_{\theta\pm}} = \frac{dr}{d\theta} = -2\frac{\Psi_\pm \cos\theta + \eta_\pm}{(\Psi_\pm)'_r \sin\theta}, \qquad (1)$$

where $\Psi_\pm = \Psi_{1\pm} cos\alpha + \Psi_0$ assuming $\alpha = 0$ for direct flow or $\alpha = \pi$ for contrary one,

$$\Psi_{1\pm} = \xi_\pm(r^2 + \frac{2p_\varepsilon}{r}),\ \Psi_0 = r^2 - \frac{1}{r} \qquad (2)$$

and

$$\eta_\pm = \frac{qB_{i\pm}}{R^2 U_\infty},\ \xi_\pm = \frac{B_{i\pm}E_\infty}{U_\infty},\ p_\varepsilon = \frac{\varepsilon - 1}{\varepsilon + 2}, \qquad (3)$$

where r is the radial spherical coordinate normalized on particle radius R, θ is the azimuth, $B_{i\pm} = \pm|B_{i\pm}|$ are ions mobilities, E_∞ is the magnitude of the external uniform electric field and is assumed positive. U_∞ is the magnitude of the neutral gas velocity far away from the sphere. The value p_ε reflects particle polarization, for high static relative permittivities, ε \ 1, $p_\varepsilon \to 1$, as for ideal conducting sphere, for "transparent" particle we have p_ε Z 1.

ION TRAJECTORIES AND SPECIAL POINTS

Steady states for ions are defined from the equations: $V_{r\pm} = V_{\theta\pm} = 0$. For the neutral particle, only ions which are being resisted by the external electric field have stationary points with $V = 0$. In direct flow, $\alpha = 0$, that is negative ions. The limiting trajectories form a separatrix envelope - sphere with a radius R_S for $|\xi_-| < 1$, which is determined from the equation $V_{r-} = 0$:

$$R_s = \left(\frac{1 - 2\xi_- p_\varepsilon}{1 + \xi_-}\right)^{\frac{1}{3}} \qquad (4)$$

The same envelope forms and for positive ions in contrary flow, $\alpha = \pi$. We need to substitute $-\xi_+$ except ξ_- in (4) to determine R_S. Both saddle points, back ($r = R_{S2}$) and front ($r = R_{S1}$) lie on the sphere $r = R_S$, but have different azimuths: $\theta = \pi$ is for front saddle point, $\theta = 0$ is for back one. When sphere acquires the charge then the separating sphere $r = R_S$ disintegrates on two independent separatrix surfaces, each is connected with a corresponded saddle point.

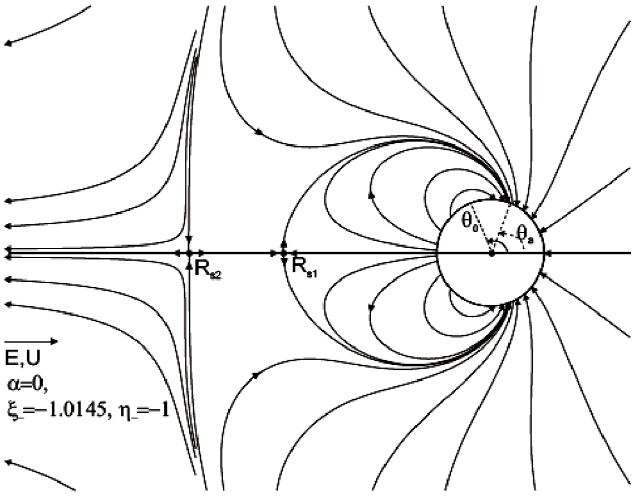

Fig. 1. Negative ions tracks (potential flow), $\xi_-=-1.0145$, $\eta_- = -1$

One important exclusion concerns the case of potential flow with $|\xi_\pm| \to -1$. Let us consider negative ions first. Then $R_{S2}{\cdot}R_{S1} \to \infty$ and $R_{S2} > R_S$, $R_{S1} < R_S$, where R_S is the spherical separatrix radius with the same ξ_- but with $\eta_-= 0$. And in the range $|\xi_\pm| > 1$, no any stationary points exist for the Stokes flow. But for the potential flow the saddle point R_{S1} moves to the limit value $R_{S1} = -3/\eta_-$ when $\xi_- \to -1$ while R_{S2} tends to infinity. When ξ_- turns through the point $\xi_- = -1$, the saddle point R_{S2} returns from infinity but from the opposite direction, $\theta = \pi$, see Fig.1.

This state with two saddles with the same azimuth $\theta_m = \pi$ contracts to one saddle ring under $\eta = \eta_m$, $\xi = \xi_m$ for positive particle charge ($\eta_m < 0$):

$$\eta_m = \frac{3}{2^{2/3}}(1 + \xi_m)^{1/3}(2\xi_m - 1)^{2/3}. \qquad (5)$$

At this special point, $r = R_m = 3(2\xi_m - 1)/2\eta_m$ we have $V_{r\pm} = \partial V_{r\pm}/\partial r = 0$, where "+" is for $\alpha = 0$ and "-" is for $\alpha = \pi$. The bifurcation in this point, (ξ_m, η_m), gives rise the saddle ring, see Fig. 2, and in the bifurcation point we have: $R_{S1} = R_{S2} = R_{S\pm}$.

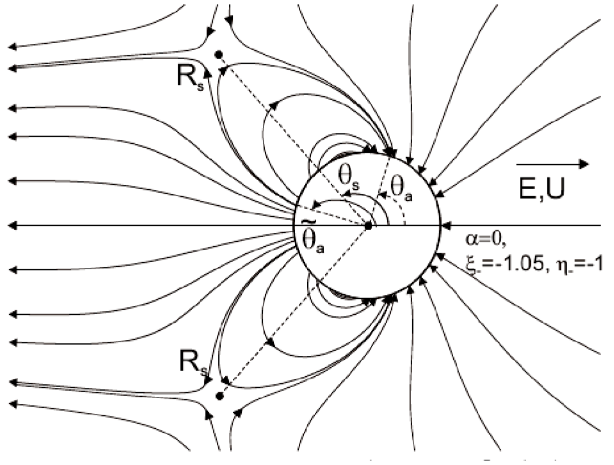

Fig. 2. Negative ions tracks (potential flow), $\xi_- = -1.05$, $\eta_- = -1$

Coordinates of the saddle rings ($R_{S\pm}$, $\theta_{S\pm}$), for the negative ions and direct flow, $\alpha = 0$, and for the positive ions in opposite flow, $\alpha = \pi$, are defined from the system $V_{r\pm} = V_{\theta\pm} = 0$:

$$R_{s\pm}^3 = \frac{2\xi_\pm \pm 1}{2(\xi_\pm \mp 1)}, \ \cos\theta_{s\pm} = R_{s\pm}\frac{2\eta_\pm}{3(2\xi_\pm \pm 1)}. \quad (6)$$

When electric field became large, $|\xi_\pm| \gg 1$ and the sphere charge is fixed then the saddle ring (R_{S-}, θ_{S-}) turns to sphere equator (1, $\pi/2$) asymptotically. The transition of the point (R_{S-}, θ_{s-}) through equator occurs when particle charge has become positive (for negative ions). Positive ions have the same tacks structure in the contrary flow, $\alpha = \pi$ and the bifurcation point (ξ_m, η_m) is determined by expression (5) with substituting $-\xi_m$ against ξ_m. As it will be showed below, the dislocation of saddle ring has an influence on charging ion flux.

ION CURRENT, $\xi_\pm < 1$

The solution of the equation (1), with boundary conditions at infinity, $r \sin\theta|_{\theta\to\pi,\, r\to\infty} \to h$ be as following:

$$\Psi_\pm = \frac{2\eta_\pm(\cos\theta + 1) + h^2(1+\xi_\pm)}{\sin^2\theta}. \quad (7)$$

It is supposed that $|\xi_\pm| < 1$, otherwise the limit $\theta \to \pi$ will be valid only for positive ions, for negative one it might be $\theta \to 0$. To find the cross-section, we have to choose those ions from the flow, which are collected by the sphere: $\Psi_\pm(1) = (2p_\varepsilon + 1)\xi_\pm$. With this, we obtain an expression connecting the impact angle $\theta_{i\pm}$ with the shoot parameter $h_\pm$:

$$\cos\theta_{i\pm} = -\tilde{q} \mp \sqrt{(1-\tilde{q})^2 - h_\pm^2 \frac{1+\xi_\pm}{(2p_\varepsilon+1)\xi_\pm}}, \quad (8)$$

where sign "+" is for the positive ions flow while "−" for the negative ions, $\tilde{q} = q/q_U$, where q_U is the maximum charge acquired by the isolated ideal conducting sphere in the unipolar ions environment in the electric field, $q_U = (2p_\varepsilon + 1)E_\infty R^2$, see [1]. Let us consider, first, positive ions. With varying h_+ from 0 to h_{t+} the impact angle θ_{i+} varies from π to $\theta_{a+} = \arccos(-\tilde{q})$.

So, the threshold shoot distance h_{t+} is obtained from the (8) with $\cos\theta_{i+} = -\tilde{q}$:

$$h_{t+}^2 = (2p_\varepsilon + 1)\xi_+ \frac{(1-\tilde{q})^2}{1+\xi_+}. \quad (9)$$

If $\tilde{q} > 0$ then $B_{i+}E_r(1, \theta) > 0$ for any θ and, as consequence, all positive ions will be repelled from the aerosol surface and $\tilde{J}_+ = 0$, see (10). Oppositely, if $\tilde{q} < -1$ then $B_{i+}E_r(1, \theta) < 0$ for any azimuths θ and all ions in the vicinity of aerosol particle will be attracted by it and $\tilde{J}_+ = -4\tilde{q}$. The corresponding collision cross-section is: $\pi h_{t+}^2 = -4\pi\eta_+/(1 + \xi_+)$. Negative ions are resisted by the external electric field for $\alpha = 0$, and it gives income to the current if only $q > 0$. Then negative ions bend around the particle and impact the surface at the range $\theta_{i-} \in [0, \theta_{a-}]$, where $\cos(\theta_{a-}) = 1 - 2\tilde{q}$, as it follows from the (8). It defines the ring at the particle surface which is connected by the separatrix paths with the front saddle point (r_S, π). Ions from the maximum distance h_{t-} move to the back saddle point and attach the sphere surface with $\theta_{i-} = 0$. As it follows from the equation (8) the cross-section be as:

$$\pi h_{t-}^2 = -4\pi\eta_-/(1 + \xi_-).$$

So, the expressions for $h_{t\pm}$ determine positive and negative charging current. The dimensional current, $J_\pm$, may be expressed through the collision cross-section πh_{t+}^2:

$$J_\pm = j_\pm \pi h_{t\pm}^2 R^2, \ j_\pm = U_\infty(1+\xi_\pm)n_{0\pm}e, \quad (10)$$

or

$$J_\pm = \tilde{J}_\pm J_{0\pm}, \ J_{0\pm} = q_U \nu_{i\pm} \quad (11)$$

where $J_{0\pm}$ is the maximum charging current. Summarizing the results, the expressions for the currents be as:

$$\tilde{J}_+ = \begin{cases} -4\tilde{q}, & \tilde{q} < -1; \\ (1-\tilde{q})^2, & |\tilde{q}| < 1; \\ 0, & \tilde{q} > 1; \end{cases} \quad \tilde{J}_- = \begin{cases} 0, & \tilde{q} < 0; \\ 4\tilde{q}, & \tilde{q} > 0. \end{cases} \quad (12)$$

After the similar arguments as in the case $\alpha = 0$ charging currents for $\alpha = \pi$ may be expressed as:

$$\tilde{J}_+ = \begin{cases} -4\tilde{q}, & \tilde{q} < 0; \\ 0, & \tilde{q} > 0; \end{cases} \quad \tilde{J}_- = \begin{cases} 0, & \tilde{q} < -1; \\ (1+\tilde{q})^2, & |\tilde{q}| < 1; \\ 4\tilde{q}, & \tilde{q} > 1. \end{cases} \quad (13)$$

The formulae (12), (13) are the same as obtained in [2] by another way. Actually, the method of integration over the sphere surface (to sum all ions tracks and deduce the current) was used in [3] for fast ions $|\xi_\pm| > 1$. Following this assumption and integrating the value $3\xi \cos\theta + \eta$ over the sphere surface from $\theta = 0$ till $\theta = \theta_a$ we have:

$$\tilde{J}_\pm = (1 \mp \tilde{q})^2, \ |\xi_\pm| > 1, \ |\tilde{q}| \le 1 \quad (14)$$

But, as we will find in the next paragraph, that is correct only for $|\xi_\pm| \gg 1$ - it's not for the general case. For $|\tilde{q}| > 1$ integration should be performed over the all sphere surface that gives $\tilde{J}_+ = 0$ and $\tilde{J}_- = 4\tilde{q}$ for $\tilde{q} \ge 1$; $\tilde{J}_+ = -4\tilde{q}$ and $\tilde{J}_- = 0$ for $\tilde{q} \le -1$.

ION CURRENT, $\xi_\pm \ge 1$

The new results are presented below for the "fast" ions.

Potential flow with $|\xi_\pm| > 1$ has stationary points as it is shown in Fig. 1,2 and has closed ion paths. Let us consider the parameter space $(\xi_\pm, \eta_\pm)$ for negative ions and let $\alpha = 0$. When $|\xi_-| \in [1, |\xi_m|]$ the current is determined by formula (12), right side. But for $|\xi_-| > \xi_m$ we have to find the attachment azimuth which is not equal to the old value $\theta_{a-} = \arccos(1-2\tilde q)$ for $\tilde q > 0$ (positive net charge, negative ions).

Since, the coordinate for saddle ring is known, we can connect two conjugate points $(1, \theta_{a-})$ and $(1, \tilde\theta_{a-})$ with separatrix ring state (R_{S-}, θ_{S-}). Thus, all these points lie on the same ion path, see Fig. 2. Solving (1) for closed separatrix path and using the relation $\cos\theta_{a\pm} + \cos\tilde\theta_{\pm} = -2\tilde q$ gives:

$$\cos\theta_{a-} = -\tilde q + \sqrt{\tilde q^2 + 1 + 2\tilde q\cos\theta_{s-} - \tilde q_{s-}\sin^2\theta_{s-}}, \quad (15)$$

where $\tilde q_{S-} = (\Psi_1(R_{S-}) + \Psi_0(R_{S-}))/3\xi_-$, Ψ_0, Ψ_1 are the flow function for potential flow determined by (2). Extending this expression for the positive ions too, $\tilde q_{S\pm}$ is written as:

$$\tilde q_{s\pm} = \frac{2\xi_\pm \pm 1}{2\xi_\pm R_{s\pm}} = \frac{(2\xi_\pm \pm 1)^{2/3}(\xi_\pm \mp 1)^{1/3}}{2^{2/3}\xi_\pm}, \quad (16)$$

where $(R_{S\pm}, \theta_{S\pm})$ are the coordinates of the saddle ring. Integrating the radial electric field over the sphere and using the relation $\cos\theta_{S-} = -\tilde q/\tilde q_{S-}$ the negative ion current be as:

$$\tilde J_- = \frac{(\tilde q_{s-} + \tilde q)^2}{\tilde q_{s-}}, \quad (17)$$

At the point (ξ_m, η_m) we have $\tilde q_{S-} = \tilde q$ and formula (17) yields: $\tilde J_- = 4\tilde q$ that is the same as for all range $|\xi_-| < |\xi_m|$. From the other side, in the limit $|\xi_-| \gg 1$, current turns to the value $(1 + \tilde q)^2$, since $\tilde q_{S-} \to 1$. Finally, for all $\tilde q$ range, $|\xi_-| > 1$, $\alpha = 0$, negative ions current is:

$$\tilde J_- = \begin{cases} 0, & \tilde q < -\tilde q_{s-}; \\ \dfrac{(\tilde q_{s-} + \tilde q)^2}{\tilde q_{s-}}, & |\tilde q| < \tilde q_{s-}; \\ 4\tilde q, & \tilde q > \tilde q_{s-}; \end{cases} \quad (18)$$

,

where $\tilde q_{S-} = (1 + 2|\xi_-|)/2|\xi_-|R_{S-}$ and it is always positive. If $\tilde q \geq 1$ the current remains a constant for all $\xi_\pm$. For positive ions there are no closed tracks if $|\xi_+| > 1$, $\alpha = 0$. So, $\tilde J_+$ can be obtained by the simple integration over the sphere with limit azimuth $\cos\theta_{a+} = 1 - 2\tilde q$. It gives the left part of equation (17). Due to the symmetry, currents $\tilde J_\pm$ for $\alpha = \pi$ can be obtained using the following substitution in (18): $\tilde J_\pm \to -\tilde J_\mp$, $\xi_- \to -\xi_+$, $\tilde q \to -\tilde q$, $\tilde q_{s-} \to -\tilde q_{S+}$, and setting - in formula for $\tilde q_{S-}$:

$$\tilde J_+ = \begin{cases} -4\tilde q, & \tilde q < -\tilde q_{s+}; \\ \dfrac{(\tilde q_{s+} - \tilde q)^2}{\tilde q_{s+}}, & |\tilde q| < \tilde q_{s+}; \\ 0, & \tilde q > \tilde q_{s+}; \end{cases} \quad (19)$$

where $\tilde q_{S+} = (1 + 2\xi_+)/2\xi_+ R_{S+}$, and $\tilde J_-$ is determined by the right side of (20). Near the point $|\xi_\pm| = 1$ we can find that $\tilde q_{S\pm} \approx (1.5)^{2/3}(\delta\xi)^{1/3} \ll 1$, where $\delta\xi = |\xi_\pm| - 1$. But if $|\xi_\pm| \gg 1$ then $\tilde q_{s\pm} \approx 1 - 1/3\xi_\pm^2 \sim 1$. At this limit formulas (19) and (18) became equivalent. This means that current does not depend on angle α if $|\xi_\pm| \gg 1$. That is right also for Stokes flow for any $|\xi_\pm| > 1$, then current will be expressed by the formula (19) with $\tilde q_{S+} = 1$.

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К ЗАРЯДКЕ СФЕРЫ В ЭГД ПОТОКЕ ГАЗА

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Исследовано взаимодействие проводящей сферы и электрогидродинамического (ЭГД) газового потока в присутствии внешнего электрического поля. Газ предполагается слабо ионизованным. Диффузией ионов пренебрегается, учитывается только их дрейф и перенос потоком газа. Аналитически и численно получены ионные токи на сферу для случая потенциального обтекания для любых значений управляющего параметра $\xi_\pm$ - отношение скорости дрейфа ионов в однородном электрическом поле к скорости газа вдали от поверхности сферы.

ЩОДО ЗАРЯДКИ СФЕРИ В ЕГД ПОТОЦІ ГАЗА

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Досліджено взаємодію провідної сфери і електрогідродинамічного (ЕГД) газового потоку в присутності зовнішнього електричного поля. Газ передбачається слабко іонізованим. Дифузія іонів нехтується, враховується тільки їхній дрейф і перенос потоком газу. Аналітично і чисельно отримані іонні струми на сферу для випадку потенційного обтікання для будь-яких значень керуючого параметра $\xi_\pm$ - відношення швидкості дрейфу іонів в однорідному електричному полі до швидкості газу удалині від поверхні сфери.