PECULIARITIES OF DISPERSION CHARACTERISTICS OF SINUSOIDALLY RIPPLED PLASMA WAVEGUIDES WITH SMALL RIPPLE DEPTH

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Analytical expressions for the dispersion equations of E-waves of flat and cylindrical sinusoidal rippled plasma waveguides with super conducting walls in a strong external magnetic field are received. Origin of forbidden bands, which are formed at crossings of the dispersion curves describing various own modes, is investigated. The width of formed forbidden bands and group speeds of own waves near to the forbidden bands are determined. Comparison of the theory with results of the previous researches in special cases gives good qualitative and quantitative conformity.
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For the first time use of plasma filling in periodical waveguides was offered in work [1] Further development idea of application of plasma filling and sinusoidal ripple has received in works [2-4]. In work [2] dispersion characteristics of axial-symmetric E-waves are investigated, dependence of forbidden band on ripple depth in a vicinity and far from a point of crossing of dispersion curves is determined. It is shown, that the forbidden band width is proportional to ripple depth. In works [3,4] excitation of rippled plasma structures by electron beams is investigated.

However in connection with the recent theoretical description of a Trivelpiece-Gould waves spectrum in periodic waveguides and introduction in a terminology of concept of a dense spectrum [5] mentioned above results become unfair. The matter is that in periodic wave guides instead of the determined dispersion curves the dot set having fractal properties [6,7] is organized. Thus the dot set represents the monotonous function having some constant value at equality last rational number (a devil ladder) [6,8].

In the present work the analysis of dispersion characteristics of electromagnetic E-waves of flat and cylindrical waveguides is carried out in view of finite, but satisfying Rayleigh hypothesis [9], ripple depth.

Let's receive the dispersion equation of a plasma waveguide with sinusoidally rippled walls. Dependence of distance between walls of a waveguide on longitudinal coordinate $X(z)$ we shall choose as $X(z) = R_0(1-\sin k_0 z)$, where $k_0 = \frac{2\pi}{D}$, $\alpha \ll 1$ - small parameter ($\alpha = \frac{\Delta R}{R_0}$), ΔR - ripple depth, $-\infty < z < \infty$, R_0, D, - average radius and the spatial period of a waveguide accordingly. Tensor of dielectric permeability ε_{ik} is considered to be

$$\varepsilon_{ik} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon_{II} \end{pmatrix}, \text{ where } \varepsilon_{II} = 1 - \frac{\omega_p^2}{\omega^2}.$$

Similarly [9] we count, that the wave guide is filled with homogeneous plasma and placed in a strong longitudinal magnetic field $H_0 \uparrow\uparrow e_z$. We assume also, that ions of plasma are infinitely heavy and their contribution plasma fluctuations is neglected.

By virtue of spatial periodicity of structure on z axis we shall present required fields as infinite sums of partial waves (spatial harmonics)

Analytical expressions for the dispersion equations of symmetric E-waves of flat and cylindrical plasma waveguides with sinusoidally rippled super conducting walls in a strong magnetic field with small ripple depth can be received from infinite Hill's determinant [9].

These expressions are represented by following equations:

$$\begin{aligned} &Z(\kappa_0 R_0) Z(\kappa_{-n} R_0) = \\ &= \lambda^{2n} Z'(\kappa_0 R_0) Z'(\kappa_{-n} R_0) \kappa_0 \kappa_{-n} \times \\ &\times R_0^2 \left(1 + \frac{\varepsilon_{II} |h| k_0}{\kappa_n^2}\right)\left(1 + \frac{\varepsilon_{II} |k_{-n}| k_0}{\kappa_{-n}^2}\right) \end{aligned} \tag{1}$$

where $k_n = h + nk_0$ $\kappa_n = \sqrt{\varepsilon_{II}\left(\frac{\omega^2}{c^2} - k_n^2\right)}$,

$Z(x) = \{\cos(x); J_0(x)\}$; $\lambda = \{\alpha;\ \alpha/2\}$; $Z'(x) = \frac{dZ}{dx}$, the first member in braces corresponds to the flat rippled waveguide, the second - to the cylindrical one.

We'll investigate the dispersion characteristics of such waveguides using the equation (1). All calculations we shall carry out for a flat rippled waveguide. The received results, in view of the mentioned above replacement, may be easily transformed for the description of rippled cylindrical plasma waveguides.

DISPERSION OF ELECTROMAGNETIC E-WAVES ($\omega^2 \rangle\rangle \omega_p^2$)

Let's determine the frequency displacement caused by the presence of small ripple in a case, when the basic own mode ($n = 0$) is crossed with a counter own mode $n = -l$, $l \neq 0$. From the equation (1) for a flat rippled waveguide we have:

$$\begin{aligned} &\cos(\kappa_0 R_0)\cos(\kappa_{-l} R_0) = \\ &= \alpha^{2l} \sin(\kappa_0 R_0) \sin(\kappa_{-l} R_0) \kappa_0 \kappa_{-l} R_0 \times \\ &\times \left(1 + \frac{k_0 h}{\kappa_0^2}\right)\left(1 + \frac{|k_{-l}| k_0}{\kappa_{-l}^2}\right) \end{aligned} \tag{2}$$

In points of crossing of dispersion curves $h = \frac{lk_0}{2}$, decomposing the equation (2) in a line in a vicinity of points $\omega_0 + \Delta\omega$ and $\frac{lk_0}{2} + \Delta k$, $\left(\frac{\Delta\omega}{\omega_0}, \frac{\Delta k}{k_0} = \frac{h - \frac{lk_0}{2}}{k_0^2} \langle\langle 1\right)$, we shall receive the following equation connecting the amendments to frequency $\Delta\omega$ and wave number Δk:

$$(\Delta\omega)^2 - v_{гр}^2(\Delta k)^2 = \alpha^{2l}\omega_0^2 \frac{\left(1 + \frac{l}{2}\frac{k_0^2R_0^2}{\sigma_k^2}\right)^2}{\left(1 + \left(\frac{l}{2}\right)^2\frac{k_0^2R_0^2}{\sigma_k^2}\right)^2} \qquad (3)$$

where: $v_{гр} = \pm c\frac{l}{2}\frac{k_0R_0}{\sigma_k}\left(1 + \left(\frac{l}{2}\right)^2\frac{k_0^2R_0^2}{\sigma_k^2}\right)^{-\frac{1}{2}}$ - group velocities of own modes of a smooth гладкого waveguide at $\alpha \to 0$.

From the equation (3) follows, that in a point of crossing $h = \frac{lk_0}{2}$ of a zero mode ($n = 0$) with the counter mode $n = -l$ extending in an opposite direction, dispersion curves on a plane $\Delta\omega$, Δk ($\Delta\omega$ -an axis of abscissas, - Δk an axis of ordinates) are split. The received curves are described by the equation of a hyperbole with the beginning of coordinates in a point (ω_0, $\frac{lk_0}{2}$) and with the imaginary axis parallel to an axis of wave numbers h.

The distance between tops of hyperbolas (width of a band $\Delta\Omega_e$) is determined by expression:

$$\Delta\Omega_e \equiv 2\Delta\omega =$$
$$= \alpha^l\omega_0\left(1 + \frac{|l|}{2}\frac{k_0^2R_0^2}{2}\right)\left(1 + \left(\frac{l}{2}\right)^2\frac{k_0^2R_0^2}{\sigma_k^2}\right)^{-1}$$

The relative width of forbidden bands $\eta_l = \frac{\Delta\Omega_e}{\omega_0} \propto \alpha^l$ and with increasing of mode number decreases. Hyperbolas asymptotes are the straight lines $\Delta k = \pm v_{гр}\Delta\omega$ which correspond to the equations of tangents to the appropriate dispersion curve of a smooth wave guide with $\alpha \to 0$.

Proceeding from above-stated it is possible to draw a conclusion, that in a vicinity of points of crossing $h = \frac{lk_0}{2}$ of dispersion curves of rippled waveguide forbidden bands with width $\Delta\Omega_e$, i.e. frequency intervals in which the equation (3) has no valid solutions relative to Δk.

Value of group velocity in a vicinity of $h = \frac{lk_0}{2}$ follows from (3) and may be presented as:

$$w_{гр} = \pm v_{гр}^2\frac{\Delta k}{|\Delta\omega|} \qquad (4)$$

From expression (4) follows that in a vicinity of a point $h = \frac{lk_0}{2}$ group velocities may decrease to zero.

DISPERSION OF PLASMA E-WAVES ($\omega^2 \ll \omega_p^2$)

Let's consider dispersion properties of plasma E-waves of a flat rippled waveguide.

The displacement of frequency caused by presence of small ripple depth at crossing of the basic mode ($n = 0$) with a counter own mode $n = -l$ in points $h = \frac{lk_0}{2}$ we shall determine from the equation:

$$(\Delta\omega)^2 - v_{гр}^2(\Delta k)^2 = \alpha^{2l}\omega^2\frac{4}{l^2}\frac{\left(1 + \frac{2}{l}\frac{\sigma_k^2}{R_0^2k_0^2}\right)^2}{\left(1 + \left(\frac{2}{l}\right)^2\frac{\sigma_k^2}{R_0^2k_0^2}\right)} \qquad (5)$$

where: $v_{гр} = \pm|\varepsilon_{II}|\frac{\omega_0^3}{\omega_p^2}\frac{2}{lk_0}$ - group velocities of the own modes with $\alpha \to 0$.

The equation (5) as well as in case of an electromagnetic E-wave describes splitting dispersion curves and formation of the forbidden bands. The relative width of a forbidden band is determined by expression:

$$\eta_l = \frac{\Delta\Omega_e}{\omega_0} = \alpha^l\frac{4}{l}\frac{\left(1 + \frac{2}{l}\frac{\sigma_k^2}{R_0^2k_0^2}\right)}{\left(1 + \left(\frac{2}{l}\right)^2\frac{\sigma_k^2}{R_0^2k_0^2}\right)} \qquad (6)$$

From this expression follows, that the width of the forbidden band decreases like $\alpha^l\frac{1}{l}$ with growth of number l of a mode.

Value of group velocity in a vicinity of the forbidden band is determined by expression (4) in which it is necessary to use $v_{гр} = \pm|\varepsilon_{II}|\frac{\omega_0^3}{\omega_p^2}\frac{2}{lk_0}$. As well as in case of electromagnetic E-waves, group velocity for wave numbers $h = \frac{lk_0}{2}$ may decrease to zero.

In conclusion it is necessary to note the following.

The received expressions are fair for small ripple depths - $\alpha^2\kappa_n^2R_0^2\langle\langle 1$. Here follows the requirement on a waveguide ripple depth:

$$\alpha^2 \left\{ \frac{\left(\frac{\pi}{2}\right)^2 (2k+1)^2}{\lambda_k^2} \right\} \ll 1$$

At small ripple depth the quantity of modes for which consideration is fair, may be big enough and thus, decomposition of fields in infinite numbers is justified.

CONCLUSIONS

For the first time from infinite Hill's determinant analytical expressions for the dispersion equations of symmetric E-waves of flat and cylindrical plasma waveguides with sinusoidally rippled super conducting walls in a strong external magnetic field are received. It is shown, that the dispersion equation of a cylindrical plasma rippled waveguide may be received from the dispersion equation of a flat rippled plasma waveguide if in the last to replace $tg(\kappa_n R_0)$ on $\frac{J_1(\kappa_n R_0)}{J_0(\kappa_n R_0)}$ and to replace α on $\frac{\alpha}{2}$.

Formation of forbidden bands for electromagnetic ($\omega^2 \gg \omega_p^2$) and plasma ($\omega^2 \ll \omega_p^2$) waves of E-type which are formed at the crossing of basic mode dispersion curve with various own waves in flat and cylindrical rippled plasma waveguides is analytically investigated. It is shown, that near to any point of such crossing dispersion curves are split also their behaviour is described by the equation of a hyperbola-type in which the imaginary axis is parallel to an axis of wave numbers, and the distance between tops of a hyperbole determines width of a forbidden band.

The width of l-th forbidden band is determined. It appeared to be proportional α^l at modes crossing with identical radial wave numbers and proportional α at crossing of the basic and first longitudinal modes distinguished by radial wave numbers.

Values of group velocities of own waves in the vicinity of the forbidden band are calculated. It is shown, that near to points of crossing of own modes both with identical and with various wave numbers group velocity is small in comparison with light velocity and in the point of crossing decreases to zero. The received theoretical results in limiting cases will well be coordinated to results of other authors.

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ОСОБЛИВОСТІ ДИСПЕРСІЙНИХ ХАРАКТЕРИСТИК ГОФРОВАНИХ ПЛАЗМОВИХ ХВИЛЕВОДІВ З МАЛОЮ ГЛИБИНОЮ ГОФРА

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Отримано аналітичні вирази для дисперсійних рівнянь Е-хвиль плоского і циліндричного плазмових хвилеводів з сінусоідально гофрованими ідеально провідними стінками в сильному зовнішньому магнітному полі. Досліджено утворення смуг непрозорості, які утворюються при перетинанні дисперсійних кривих, що характеризують різноманітні власні моди. Визначено ширину смуг непрозорості, що утворюються, і групові швидкості власних хвиль поблизу смуг непрозорості. Порівняння теорії з результатами попередніх досліджень в окремих випадках дає гарну якісну і кількісну відповідність.

ОСОБЕННОСТИ ДИСПЕРСИОННЫХ ХАРАКТЕРИСТИК ГОФРИРОВАННЫХ ПЛАЗМЕННЫХ ВОЛНОВОДОВ С МАЛОЙ ГЛУБИНОЙ ГОФРА

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Получены аналитические выражения для дисперсионных уравнений Е-волн плоского и цилиндрического плазменных волноводов с синусоидально гофрированными идеально проводящими стенками в сильном внешнем магнитном поле. Исследовано образование полос непрозрачности, которые образуются при пересечении дисперсионных кривых, характеризующих различные собственные моды. Определены ширина

образующихся полос непрозрачности и групповые скорости собственных волн вблизи полос непрозрачности. Сравнение теории с результатами предыдущих исследований в частных случаях дает хорошее качественное и количественное соответствие.