# SYMMETRIC SURFACE ELECTROMAGNETIC WAVES AT FLAT INTERFACE PLASMA –VACUUM

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As it is known the surface electromagnetic waves (SEW) can propagate along surface, which separates two media with the permittivity of different signs [1]. A lot of works devoted to studies and applications of plane SEW. We have shown that new type of SEW can propagate along the boundary of free plasma with vacuum. The equiphase surfaces of these waves are the circular cylinders in contrast to planes in case of conventional surface waves. The properties of these waves were obtained. Comparison of properties of conventional and unconventional SEW have been analysed. PACS: 52.35.Hr

## **1. PROBLEM STATEMENT**

Probably, first devoted surface paper to electromagnetic waves was published almost 100 years ago [2]. These waves satisfy Maxwell's equations and may be excited on the boundary between two media with different permittivity. These modes are localized near the separating surface, i.e. their amplitudes are biggest at this surface and decays inside both of media. Vast deal of problems surface waves involved were solved and published (see [3]-[6] and references therein). Consideration in all these works concern with plane waves, i.e. the equiphase surfaces are the planes. Surface electromagnetic waves in solids of spherical and cylindrical shape have been investigated in [7].

We study the possibility of existence of solutions of Maxwell's equations with the equiphase surfaces giving circles at separating surface. In a sense, we are interested in such type of waves which are analogous to established circular waves on water surface. Mode of excitation is out of consideration in this report; one of possibilities was studied in [8].

Let us consider a plane interface (z = 0) between two half-spaces. Introduce a cylindrical coordinate system with Z-axis perpendicular to the separating plane. First domain z > 0 is a vacuum with permittivity  $\varepsilon_1 = 1$  and second is homogeneous free plasma with the frequencydependent permittivity  $\varepsilon_2 = \varepsilon (\omega) = 1 - (\omega_p / \omega)^2$ , where  $\omega_p^2 = 4\pi e^2 n_p / m_e$  is a plasma frequency. In cylindrical coordinate system  $(r, \varphi, z)$  the set of Maxwell's equations split in two subsystems. One of them consists of {  $E_r$ ,  $E_z$ ,  $H_{\varphi}$  }, another consists of {  $H_r$ ,  $H_z$ ,  $E_{\varphi}$  }. We will be interested in symmetric waves, i.e. all disturbances are independent of angle  $\varphi$ .

Let's consider the problem of describing wave propagation with nonzero components: an azimuth magnetic field  $H_{\varphi}$ , a radial electric field  $E_r$  and electric field  $E_z$ , which is perpendicular to the boundary. We will find solution in such form:  $A(r, z, t) = A(r) \exp(-\kappa_{1,2}z) \exp(-i\omega t)$ , where  $\omega$  is a

wave frequency,  $1/\kappa$  is a depth of field penetration in both media, indexes 1, 2 correspond to vacuum and plasma ( $\omega$ ,  $\kappa_{1,2}$  are real and positive).

## 2. RESULTS AND DISCUSSION

System of equations for such wave disturbances is as follow:

$$-\kappa_{1,2}H_{\varphi} = ik\varepsilon_{1,2}E_r, \qquad (1)$$

$$ikH_{\varphi} = -\kappa_{1,2}E_r - \frac{dE_z}{dr},\tag{2}$$

$$\frac{1}{r} \left( \frac{d}{dr} \left( r H_{\varphi} \right) \right) = -ik \varepsilon_{1,2} E_z, \qquad (3)$$

where  $k = \omega/c$ , c is a speed of light.

Expressions for the electric fields  $E_r$ ,  $E_z$  with  $H_{\varphi}$  are:

$$E_r = \pm i \left( \kappa_{1,2} / k \varepsilon_{1,2} \right) H_{\varphi} \tag{4}$$

$$\frac{dE_z}{dr} = -i\left(k + \kappa_{1,2}^2 / \varepsilon_{1,2}k\right) H_{\varphi} \,. \tag{5}$$

Substituting (4), (5) in (3) we obtain equation for  $H_{0}$ 

$$r^{2} \frac{d^{2} H_{\varphi}}{dr^{2}} + r \frac{d H_{\varphi}}{dr} + \left[ (K_{1,2}r)^{2} - 1 \right] H_{\varphi} = 0, \quad (6)$$

where  $K_{1,2}^2 = \kappa_{1,2}^2 + \varepsilon_{1,2}k^2$ . Equation (6) is a well-known Bessel's equation and its solution is any linear combination of independent Bessel functions of first order:  $H_{\varphi} = Z_1(K_{1,2}r)$ . This equation describes both divergent and convergent circular wave, according to choice of Bessel functions as a solution. Let us obey the boundary conditions consisting in a continuity of tangential component of electric and magnetic field of a wave on plasma - vacuum boundary. It gives dispersion equation for circular surface electromagnetic waves that can propagate along plasma - vacuum boundary:

$$K_{1} = K_{2} = K(\omega) = k\sqrt{\varepsilon(\omega)/(1+\varepsilon(\omega))}, \quad (7)$$

from which follows conditions of existence for these waves  $\omega < \omega_p / \sqrt{2}$  and expressions for the inverse depths of penetration:

$$\kappa_{1} = k / \sqrt{-(1+\epsilon(\omega))}, \ \kappa_{2} = -k\epsilon(\omega) / \sqrt{-(1+\epsilon(\omega))}.$$
(8)

The electric fields of such circular surface electromagnetic waves are:

$$E_r = \pm i \left( \kappa_{1,2} / k \varepsilon_{1,2} \right) Z_1 \left( K r \right), \tag{9}$$

$$E_{z} = i \left[ \left( k + \kappa_{1,2}^{2} / \varepsilon_{1,2} k \right) / K \right] Z_{0} \left( Kr \right).$$
(10)

Evidently, both the range of existence and dispersion law for circular surface electromagnetic waves are the same as for plane surface electromagnetic waves [1-5]. Difference from conventional plane surface waves consists in the spatial dependence of wave amplitude.

To concretize the task let's choice the type of solution of equation (6) in the form  $J_1(Kr)$ . It means that we have divergent circular surface electromagnetic waves. Its fields are

$$H_{\varphi}\left(r,z,t\right) = J_{1}\left(Kr\right) \cdot \Theta\left(z,t\right),\tag{11}$$

$$E_r(r,z,t) = \pm i \left( \kappa_{1,2} / k \varepsilon_{1,2} \right) J_1(Kr) \cdot \Theta(z,t), \quad (12)$$

$$E_{z}(r,z,t) = i \left[ \left( k + \kappa_{1,1}^{2} / k \varepsilon_{1,2} \right) / K \right] J_{0}(Kr) \cdot \Theta(z,t), \quad (13)$$

where  $\Theta(z,t) = \exp(-\kappa_{1,2}z - i\omega t)$ . At large distance from origin (for Kr >> 1) we may use asymptotic forms for Bessel's functions  $J_0$  and  $J_1$ :

$$J_0(Kr) \approx \sqrt{2/\pi Kr} \cdot \cos(Kr - \pi/4) J_1(Kr) \approx \sqrt{2/\pi Kr} \cdot \cos(Kr - \pi/4 - \pi/2), \quad (14)$$

It can be seen that the divergent circular surface electromagnetic waves becomes quasi-plane surface electromagnetic waves (TM-polarization) with harmonic dependence on radial coordinate and slowly (~ $\sqrt{2/\pi Kr}$ ) decreasing amplitude that propagate in radial direction.

The time averaged (per period  $T = 2\pi/\omega$ ) Poynting vector for the wave with components {  $H_{\omega}$ ,  $E_r$ ,  $E_z$  }, is

$$S_r = \frac{c}{8\pi\omega} \left| H_{\varphi} \right| \left| E_z \right|. \tag{15}$$

If the wave components  $\{H_{\varphi}, E_r, E_z\}$  are set by expressions (11)-(13), then the values of time averaged Poynting vector near to the separating surface (z = 0) are:

$$S_{r1} = K \cdot J_0 (Kr) J_1 (Kr) / 8\pi$$
  

$$S_{r2} = K \cdot J_0 (Kr) J_1 (Kr) / 8\pi \varepsilon (\omega)$$
(16)

We can see from (16) that directions of energy flows in vacuum and plasma are the opposite at any distance from origin:

$$S_{r1}/S_{r2} = \varepsilon(\omega) < -1 \tag{17}$$

Dependence of normalized averaged per period Poynting vector (in vacuum) versus normalized radius is shown in

the Figure. There are the circular regions of alternation of sign of Poynting vector, that is typical for plane SEW [4].



Normalized averaged per period Poynting vector (in vacuum) versus normalized radius.

Experimental studies of circular surface electromagnetic waves is presented in [9]. These waves were exited in thin metal films. In this experiment the optical probe of a scanning near-field microscope operated as transmitting aerial. It should be noted that our results are in good qualititative agreement with [9].

## CONCLUSIONS

Starting from Maxwell's equations we have obtained the solutions in the form of divergent (or convergent) circular surface electromagnetic waves. Equiphase surfaces of these waves are circles. These waves have dispersion law similar to well-known plane surface electromagnetic waves. At large distance from centre of excitation these waves become quasi-plane.

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## СИММЕТРИЧНЫЕ ПОВЕРХНОСТНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ НА ПЛОСКОЙ ГРАНИЦЕ ПЛАЗМА - ВАКУУМ

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Известно, что поверхностные электромагнитные волны (ПЭВ), могут распространяться по поверхности, которая разделяет две среды с диэлектрическими постоянными различных знаков [1]. Много работ посвящено изучению плоских ПЭВ. Мы показали, что ПЭВ нового типа могут распространяться по границе свободной плазмы с вакуумом. Эквифазными поверхностями у этих волн есть круговые цилиндры, а не плоскости, как в случае обычных поверхностных волн. Получены дисперсия, распределение полей и вектор Пойнтинга для этих волн. Проведено сравнение свойств обычных и рассмотренных в данной работе ПЭВ.

## СИМЕТРИЧНІ ПОВЕРХНЕВІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ НА ПЛОСКІЙ МЕЖІ ПЛАЗМА - ВАКУУМ

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Відомо, що поверхневі електромагнітні хвилі (ПЕХ), можуть поширюватись уздовж поверхні, що розділяє два середовища з діелектричними сталими різних знаків [1]. Багато робіт присвячено вивченню плоских ПЕХ. Ми показали, що ПЕХ нового типу можуть поширюватись уздовж межі вільної плазми з вакуумом. Еквіфазними поверхнями у цих хвиль є кругові циліндри, а не площини, як у випадку звичних поверхневих хвиль. Отримано дисперсію, розподіл полів та вектор Пойнтинга для цих хвиль. Проведено порівняння властивостей звичайних та вивчених в даній роботі ПЕХ.