SPONTANEOUS GENERATION OF BETA-LIMITING MHD MODES IN TOKAMAKS

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1. Neoclassical tearing modes (NTMs) are suggested to be one of the main obstacles to achieve acceptable values of beta (ratio of plasma pressure to magnetic field pressure) in fusion reactors of tokamak type [1]. The existing theory of NTMs [2] is based on the notion that these modes are triggered by other types of MHD (magnetohydrodynamic) activity such as ELMs (Edge Localized Modes), sawteeth and fishbones. Meanwhile, the observational data from ASDEX Upgrade [3] and TFTR [4] show that NTMs can be generated spontaneously, i.e. in the absence of any MHD activity. To explain spontaneous generation of NTMs one should appeal to a linear instability excited when the parameter beta of ideally stable plasma exceeds a threshold value. However, the existing linear theory [5] does not predict such an instability.

The need to have a non-ideal instability excited when beta exceeds a threshold value is felt also in explanation of observational data on NTMs from a series of tokamaks with hot-electron plasma (plasma with electron temperature essentially larger than the ion one) like T-10 [6], COMPASS-D [7] and TCV [8]. The fact is that, after the revision of the traditional NTM theory [9] confirmed by [10], it became clear that the polarization current effect in such a plasma is destabilizing. Therefore, the revised theory does not predict a threshold beta for NTM onset in these devices.

Certainly, in order to interpret these data from the devices with hot-electron plasma one can turn to the transport threshold models of NTMs (see [11] and references therein). However, these models deal with a rather obscure coefficients of anomalous perpendicular transport. Therefore, though within the scope of the transport models it is possible to appeal to such an interpretation, it seems to be doubtful that all totality of the data from different devices can be satisfactorily explained.

Thus, a rather broad series of experiments using the notions of NTM theory needs a β-limiting linear instability.

In principle, preceding theoretical investigations of linear modes in tokamaks include certain indications in favor of existence of beta-limiting instabilities. Thus, it was shown in [12] that for $q \simeq 1$ an instability distorting the magnetic surfaces of tokamak-type toroidal systems can be excited if, qualitatively (see Eq. (3.29) of [12])

$$\beta_p > \beta_p^{(0)} \equiv s^2 L_p^2 / r_s^2. \tag{1}$$

Here q is the safety factor, β_p is the poloidal beta, s is the shear, L_p is the characteristic scale of the plasma pressure gradient, r_s is the radial coordinate of the rational magnetic surface where the mode is localized. According to [12], for excitation of this instability the presence of temperature gradient is necessary.

The local approximation was used in [12], which is insufficient to show the existence of the eigenmodes. This defect of [12] has been corrected in [13, 14]. The eigenmodes found in [13, 14] have been called the beta-induced temperature gradient (BTG) eigenmodes.

It was assumed in [12 - 14] that the BTG modes are excited due to toroidal acoustic resonance, i.e. for the condition $\omega_* \simeq v_{Ti}/qR$ where ω_* is the characteristic diamagnetic drift frequency, v_{Ti} is the ion thermal velocity, R is the torus major radius. Such a resonance is effective only for sufficiently high poloidal and toroidal mode numbers, $m = nq = L_p r_s/(qR\rho_i) > 1$, where ρ_i is the ion Larmor radius. Experimental observation of BTG modes on JET was reported in [15].

The analysis of [12 - 14] was based on the approximation that the characteristic radial scale of the mode is larger than ρ_i or ρ_s, $k_x\rho_i \ll 1$, $k_x\rho_s \ll 1$, where ρ_s is the ion Larmor radius calculated for the electron temperature, k_x is the perpendicular projection of the wave vector (the variable x is defined by $x = r - r_s$). Nevertheless, in order to reveal the eigenmodes the authors of [13, 14] have been forced to allow for the formally small terms of the order of $(k_x\rho_i)^2$ and $(k_x\rho_s)^2$.

At the same time, the MHD-like modes, including those with $k_x\rho_i \geq 1$, $k_x\rho_s \geq 1$, are subject of the theory of semicollisional modes [5, 16 - 18]. One of the main mathematical results of this theory is a rather complicated general dispersion relation (see, for details, Eq. (24.33) of [5] and Eq. (9) of the present paper). This dispersion relation contains a dimensionless parameter ν which in the case

$T_{0i} = T_{0e} = T_0$ (T_{0i} and T_{0e} are the equilibrium ion and electron temperatures, respectively) is given by

$$\nu^2 = \frac{1}{4} - \frac{(\omega - \omega_{*e})(\omega - \omega_{*i})}{2k_y^2\rho_i^2\omega_A^2}. \quad (2)$$

Here ω is the mode frequency, $\omega_A = sv_A/(qR)$ is the Alfven frequency, v_A is the Alfven velocity, ω_{*e} and ω_{*i} are the electron and ion diamagnetic drift frequencies, respectively, $k_y = m/r_s$ is the poloidal projection of the wave vector. Up to now, the analysis of the dispersion relation (24.33) of [5] was performed only for the particular case $\left|\nu^2 - 1/4\right| \ll 1$ In this case it describes the semicollisional internal kink and tearing modes [17] and the semicollisional ballooning modes [18]. All these modes do not belong to the class of the β-limiting modes.

The goal of the present paper is to discover a new linear instability excited for a beta value larger than a critical one and to show that this critical beta is of the same order as that necessary for explanation of the observational data from [3, 4, 6-8].

2. Turning to dispersion relation (24.33) of [5], one can see that it is satisfied for

$$\nu = 0. \quad (3)$$

Then one has from (2) and (3)

$$(\omega - \omega_{*e})(\omega - \omega_{*i}) - 2\omega_{*e}^2\beta_p^{(0)}/\beta_p = 0. \quad (4)$$

We call the modes described by (4) the "sub-Larmor" modes.

It follows from (4) that in the case of sufficiently low β_p, $\beta_p \ll \beta_p^{(0)}$, the mode frequencies prove to be essentially larger than both the electron and ion diamagnetic drift frequencies. Therefore, in this case the modes can not be excited by the electron or ion diamagnetic drift effects. However, with increasing the β_p the mode frequency (4) decrease and prove to be of the order of ω_{*e} or ω_{*i} for the condition (1). For such β_p the dissipative electron/ion drift effects can excite the sub-Larmor modes.

3. Now we consider the case of vanishing ion temperature. Similar to section 24.1 of [5], we start from the current continuity equation written in the Fourier space k_x. We take

$$\boldsymbol{\nabla}_\perp \cdot \mathbf{j}_\perp = -\frac{i\omega}{4\pi}k_x^2\varepsilon_\perp\phi. \quad (5)$$

Here ϕ is the electrostatic potential, $\varepsilon_\perp = c^2 f/v_A^2$ is the perpendicular plasma permittivity, c is the speed of light, $f = f(\omega)$ is the toroidal renormalization of perpendicular inertia [5], $\mathbf{j}_\perp$ is the perturbed electric current density across the equilibrium magnetic field $\mathbf{B}_0$, $\boldsymbol{\nabla}_\perp$ is the perpendicular (with respect to $\mathbf{B}_0$) gradient. We use the parallel Ohm's law (cf. (22.1) of [5])

$$E_\parallel - \frac{T_{0e}}{e_e n_0}\left(\nabla_\parallel \widetilde{n} + \frac{B_x}{B_0}n_0'\right) = \frac{j_\parallel}{\sigma}. \quad (6)$$

Here $E_\parallel = -\nabla_\parallel\phi + i\omega A_\parallel/c$ is the perturbed parallel electric field, $A_\parallel = 4\pi j_\parallel/\left(ck_x^2\right)$ is the parallel projection of the perturbed vector potential, $j_\parallel$ is the perturbed parallel electric current density, σ is the plasma electric conductivity, $B_x = ik_y A_\parallel$ is the x-projection of the perturbed magnetic field, k_y is the poloidal projection of the wave vector, e_e is the electron electric charge, n_0 is the equilibrium plasma density, the prime is the radial derivative, $\widetilde{n}$ is the perturbed plasma number density, $\nabla_\parallel$ is the parallel gradient. The perturbed plasma number density $\widetilde{n}$ is assumed to satisfy the electron continuity equation

$$-i\omega\widetilde{n} + V_{Ex}n_0' + \nabla_\parallel j_\parallel/e_e = 0, \quad (7)$$

where $V_{Ex} = -ick_y\phi/B_0$ is the x-projection of the perturbed cross-field velocity.

Far from the resonant point $r = r_s$ we use the "constant ψ approximation", where $\psi \equiv -A_\parallel$. In the Fourier space this means that for $k_x \to 0$

$$\phi \sim 1 - \Delta'/k_x, \quad (8)$$

where Δ' is the standard tearing mode theory matching parameter.

As a result, following the approach explained in chapter 24 of [5], we arrive at the dispersion relation

$$\frac{\Gamma^2\left(-\frac{1}{4} + \frac{\nu}{2}\right)\Gamma^2(-\nu)\,Q(\nu)}{\Gamma^2\left(-\frac{1}{4} - \frac{\nu}{2}\right)\Gamma^2(\nu)\,Q(-\nu)} = \left(\frac{4\kappa}{\widehat{\beta}^2}\right)^\nu. \quad (9)$$

Here Γ is the gamma function,

$$Q(\nu) = 1 + \frac{\kappa^{-1/2}}{8r_s\Delta'}\frac{\Gamma^2\left(-\frac{1}{4} - \frac{\nu}{2}\right)}{\Gamma^2\left(-\frac{5}{2} - \frac{\nu}{2}\right)}\left(\nu^2 - \frac{1}{4}\right), \quad (10)$$

the value ν is given by (cf. (2))

$$\nu^2 = \frac{1}{4} - \frac{\omega(\omega - \omega_{*e})}{k_y^2\rho_s^2\omega_A^2}, \quad (11)$$

$\kappa = fk_y^2\rho_s^2/(1 - \omega_{*e}/\omega)$, $\widehat{\beta} = (-i\omega\gamma_R)^{1/2}/(k_y\rho_s\omega_A)$, $\gamma_R = c^2k_y^2/(4\pi\sigma)$ is the characteristic resistive decay rate.

One can see that the dispersion relation (9) is satisfied for the condition (3). It follows from (11) that in this case the mode frequency is determined by the dispersion relation similar to (4):

$$\omega(\omega - \omega_{*e}) = k_y^2\rho_s^2\omega_A^2/4. \quad (12)$$

Equation (12) yields

$$\omega = \omega_{\pm} = -\frac{\omega_{*e}}{2}\left[1 \pm \left(1 + \frac{2s^2 L_n^2}{\beta_p r_s^2}\right)^{1/2}\right], \quad (13)$$

One can see that, for the condition (1) the mode propagating in the electron drift direction has frequency of the order of electron diamagnetic drift frequency.

4. It is possible that nonlinear development of the "sub-Larmor" modes leads to NTMs. It is then of interest to estimate the linear growth rate γ of the modes considered and to elucidate the collisionality dependence of the beta threshold.

One can suggest that the growth rate γ of the mode of type (13) is determined by

$$\gamma = \gamma_e + \gamma_i, \quad (14)$$

where γ_e is the electron growth rate and γ_i is the ion decay rate given by, respectively,

$$\gamma_e \simeq f_e\left(\frac{\nu_e}{\epsilon\omega_{*e}}\right)|\omega_{*e}|, \quad (15)$$

$$\gamma_i \simeq -\frac{\beta_p^{(0)}}{\beta_p}\nu_i. \quad (16)$$

Here $f_e\left[\nu_e/\left(\epsilon\omega_{*e}\right)\right]$ is a small dimensionless parameter whose explicit form can be found turning to [19], ν_e and ν_i are the electron and ion collision frequencies, respectively. Then one can see that for finite ν_i the condition (1) is insufficient for excitation of the "sub-Larmor modes". In this case, instead of (1), one should use the estimate

$$\beta_p > \beta_p^{\mathrm{crit}}, \quad (17)$$

where

$$\beta_p^{\mathrm{crit}} = \begin{cases} \beta_p^{(0)}, & \nu_i < \gamma_e, \\ \beta_p^{(0)}\nu_i/\gamma_e, & \nu_i > \gamma_e. \end{cases} \quad (18)$$

To determine β_{crit} one was forced to appeal to the polarization current threshold model [2] (see also [20]) or to the transport threshold model. In this context, Eq. (18) is an alternative to these models in determining the β_{crit}.

The estimates, following from (18), prove to be compatible with the experimental data from [3, 4, 6-8].

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