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THE OVERVIEW OF MANUFACTURED SYSTEMS MODELS

Introduction

Questions of planning and management of production of industrial products identify key tasks for many theoretical disciplines. A large number of works, which devoted to the design of control systems of industrial processes, use the mathematical apparatus of the theory of operations research (Arrow KJ, Karlin S., Bir S. Pervozvanskii Shkurba AA VV Bessonov VA Biegel J. ., Prytkin BV), theory of optimization (M. Intriligator, Kempf K., Jakimovich SB, system dynamic (Fort rester J.), queuing theory (Gross D, Harris S.) theory of inventory management (Ryzhikov YI), the theory of planning and control production (Modigliani F., Holn C., Buslenko NP Dudorin VI, Emelyanov SV, Mitrofanov SP , Sokolitsyn SA, IM Razumov, Balashevich VA), the statistical theory of dynamical systems (AA Vlasov, Kazakov, IE, Krasovsky AA), the statistical theory of manufacturing systems (Azar -kov NA Demutsky VP Pignasty OM, Loktev II, Petrov Boris Tikhomirov IA, stroke-owls VD, Armbruster D., Ringhofer C.). Problems of a common methodology modeling of complex production and technical systems devoted to the works of domestic and eign scientists Arrow K.J., Solow RM, Wilson A. Vlasov VA, VM Glushkov, E. Goldratt, Zhang VG, LV Kantorovich, Letenko VA, Razumov IM Sokolitsyna SA, G. Haken, Shananina AA, Shkurba VV .. The application DES-, TQ- b Fluid-models in the design high-performance systems of control of production lines addressed in the works of Cassandras C., Heymann M., Hopp WJ, Eekelen J., Ramadge P., Roset B., Wardi Y., Wonham W., Buslenko NP, Bragin KA, Lysenko Yu, Rumentseva NV. We used mathematical models for decision administrations, design and technological decisions in conditions of uncertainty are devoted in the works of Buzacott JA, Harrison J., Shanthikumar JG, Bessonov VA Dudorina VI, Yermolyeva YM, Zaruba In .I., Zarubin BC, Ivanilova IP, Leontyev VV Lot's AV, Novikov DA, Prangishvili IV, Redkina AK, AK Tikhomirov, Tikhonov AN

Model of the strategic development of the enterprise is determined by the demand for manufactured products, directly. Industrial development has access during the production cycle to a limited set of technological resources, produces per unit of time, a limited count of products, determined the production capacity (effectiveness using technological equipment) [1]. Competitiveness of companies is largely characterized by the amount of output per unit of time, the duration of the production cycle and the volume of work in production (WIP: work in process or in-process inventory) [1,2]. There are taken like basic parameters of the model in the most works. Objects of labor, are in progress production, distributed along the process route. This distribution is determined by the dynamic of incoming items of work in the first process step and the output of finished products with the latest, non-uniformity of processing time and resource limits for each process step [1,3,4,5,6,7]. Optimization of parameters of control systems of production lines has led to the emergence of two interrelated problems of planning and production management. The direct problem consists in the over-estimation of the duration of the production cycle and production capacity of the system depending on the amount distributed along the route technological items of work [8, c.4591]. The inverse problem is to determine the required number of items of labor in progress its distribution along the process route for the production of finished products with an intensity in a predetermined time. To solve these tasks, use different models of controlled production process`, the main characteristics of some of them are listed below.

Discrete-event model of the production process (DES-model) [1,9,10].

Currently, in the design of control systems of production lines are widely used discrete-event model of the controlled process (Ankenman BE, Bekki JM, Fowler J., 2010) [9], based on a detailed simulation of the transfer of technological resources on the subject of work for each piece of equipment the production line. Uzsoy R. (2011) [10], Kacar N. (2012) used the DES-model for description of the production line for information about the status of objects of labor along the technological route. The error of models is identified factors associated with the choice of the time scale planning. Using of DES- models planning production with a detailed graphics loading capacity utilization in the time allowed us to obtain the dependence of finished products from the distribution function objects of labor at technology operations (Lu S., [11]). Dispatching rules (Shkurba VV, [7]), describing the management strategy to regulate the appearance of discrete beginning and end of the treatment of the subject of labor at each processing operation. More difficult planning algorithms in the construction of dispatching rules optimize production cycle and the rate of motion of the objects of labor along the technology route (Lasserre J. [12]). Exacting corresponding DES-model real manufacturing process, can be arbitrarily high by detailing-description of the technological operation and increasing the number of repetitions of numerical experiment [13], it is theoretically limited accuracy of the computational scheme. The cost of machine time required for calculations (4 GHz processor frequency) million iterations of DES-model (Berg R., Lefeber E., [13]) for a party of 10 thousand details moving on technological route, consisting of 20 manufacturing operations range from a few hours to a few days [14]. At the same time, the required range of planning for systems control the parameters of the production line of modern production enterprises should not exceed a few minutes [15, p.7]. The next problem discrete simulation is due to the fact that pro-process transfer of technological resources on the subject of work as a result of techno logical operation is stochastic [8, c.4589]. Random is as a time of executing of technological operations and the amount of resources transferred to the item of labor. The duration calculation exceeds the required period of planning, stochastic process, transfer of technological resources on the subject of work and the lack of a final functional relationship between the parameters of the production line production line is not possible to effectively to use DES-model as a management tool parameters of modern industrial production line. DES-model which use to describe production systems, for example, to simulate the production lines for the production of semiconductor products [16], are "relatively slow" [8, c.4589], which does not allow to use them effectively to build systems of managing, and planning production.

Models of queuing theory (TQ-model) [1,5,17, 18] are widely used to describe the production lines in a steady state. For higher-performance computing is achieved by the transition from a discrete description in reference to the continuous work items described using average features. In terms of computing resources model of queuing theory [17] for stationary modes of operation of production lines are more efficient than DES-models that allow you to present analytic functional relationship between the stream parameters of the model. Dost exactly well represented TQ-model production lines in the works Buzacott JA, Shanthikumar JG, Chao X., Hopp WJ [18], Lefeber E. [19], who focused their attention to the mutual relationship between the duration of the production cycle of manufacturing a kit of parts and the number of items of work in interoperable storage devices. The line of products waiting to be processed, is a not finished work (WIP) in the production system. Reasonably accurate estimate of output parameters could only be obtained for the steady state operation of the production line, consisting of a small number of process operations. The constant presence of process in the operation of the production line requires utilization of the more advanced non-stationary models, resulting in a significant withdrawn time accounts and to the absence of evident functional relationship between the parameters of the production line, [14]. In describing the transition of unsteady processes TQ-models lose their advantages over the DES-models. Using of TQ-models for transients leads to excessive complexity of the task. A significant limitation of their use is that modern production lines consist of a large number of technological operations. As a rule, used cross-sectional models of queuing theory. Two-moments models that take into account this very dispersion of processing parameters objects of labor, to describe the production lines practically do not occur due to the complexity of their construction. Another limitation to the use of TQ-model is the fact that the basic formula derived for steady state ($t \to \infty$), which automatically implies the fulfillment of equation $\lambda < \mu$ (2), where λ - the intensity of the receipt of objects of labor for processing; μ - the intensity of the treatment of the subjects of labor.

In fact, for most of the production lines with the final production cycle of inequality $\lambda \ge \mu$ that prevents the use of well-developed apparatus of queuing theory for steady processes. The solution of these problems leads to high dimension of the problem, cumbersome calculations, requires considerable computing resources.

Models of fluid production processes (Fluid-model) [6,20,21].

A wide class of description controlled manufacturing processes include the transfer equation. This class is effectively used to describe the time-dependent processes, mainly deterministic manufacturing processes. Traditionally differ two approaches to the representation of the flow of technological objects of labor along the road. The first approach is the aggregation of items of work over the states [21] and in the following construction of the transport equations, the second, more common in the representation of the flow of products in a continuous fluid (Fuid-model, Forrester J., 1961, [6]), which are used to describe the transport equation. Conceptually discrete Fluid-model are m-th work place in the network cater for mass in the form of m-th storage tank and the valve between the (m-1) th and m-th accumulate governing with the intensity μ_{m-1} of the movement of objects of labor between them (Kefeli, Uzsoy, Fathi, Kay, 2011) [1]:

$$\frac{dq_m}{dt} = \mu_{m-1} - \mu_m \,, \; m = 1..M \,, \; \mu_0 = \lambda \,, \qquad (1)$$

where q_m - work in progress (WIP) backlog before the m-th technological equipment (queue length), μ_m - the rate of processing of items of work on m-th process equipment, $\mu_0 = \lambda$ – the intensity of the receipt of the objects of labor in the first process step. Sampling of model due to convenience of relationship of equations of model with the specific parameters of stream site technological route of production line:-fabrication backlog in the storage q_m, processing rate of the subject of labor in the current μ_m and previous section μ_{m-1}. Equation (1) Fluid-models are continuous in time and discrete in the space, describe N the spatial parts of the stream of objects of labor. A set of ordinary differential equations (1) represents the time evolution of the queue length of the objects of the labor, can be represented in the form of equations Forrester (1961) [6], simulate the flow of items of work on technological route with discretely located manufacturing equipment. Stages of production disregarded as a change of continuous variables describing the state of the objects of the work in the technological space of states. Unlike a real fluid, states do not describe the physical space and indicate the degree of finishing parts or stage of production. As a variable that determines the position equipment in the calculation of the production lines often use variable ζ (m) represents the path that passes the subject of work as a result of treatment with an initial machining operation to the current technological operations at the general length of the process route D (m) [22]. At the same time as the change, which determines the rate of processing of objects of labor along the process route is taken velocity (m / h) movement of objects of labor on technological route production line (for conveyor lines - the velocity of the conveyor). The system of equations of the model (1) is represented as:

$$\frac{dq(t,\zeta_m)}{dt} = \mu(t,\zeta_{m-1}) - \mu(t,\zeta_m) \,,$$
$$m = 1..M \,, \; \mu(0) = \lambda \,. \qquad (2)$$

If the count of technological operations in the technological route M >> 1 ($M \approx 10^2$ [14], $M \approx 250$ [8], $M \approx 300$ [23, s.445]) for building systems of control of industrial processes use Fluid-model (1), (2) becomes ineffective . Fluid-models lose advantages over TQ-models. However, the fact of having a large number of technological operations M >> 1 allows Implemented twist limit as $(\zeta_m - \Delta\zeta_{m-1}) = \Delta\zeta << D$ to the representation of the system of equations (2) in the form of non-stationary continuum (continuous coordinate space) model of line [8, c.4591]:

$$\mu(t,\zeta_m) = \mu(t,\zeta_{m-1}) + \left.\frac{\partial\mu(t,\zeta)}{\partial\zeta}\right|_{\zeta=\zeta_m} \Delta\zeta_m + 0\left(\Delta\zeta_m^{\ 2}\right) \,,$$

$$\left.\frac{\partial\rho(t,\zeta)}{\partial t}\right|_{\zeta=\zeta_m} = -\left.\frac{\partial\mu(t,\zeta)}{\partial\zeta}\right|_{\zeta=\zeta_m} + 0(\Delta\zeta_m), \; m = 1..M \,, \qquad (3)$$

where $\rho(t,\zeta_m) = {q(t,\zeta_m)}/{\Delta\zeta_m}$ [1.21] - average density of objects of the labor-process backlog $q(t,\zeta_m)$ m-th technological operation for the area of technological route $]\zeta_{m-1},\zeta_m]$, $\mu(0) = \lambda$. The function $\mu(t,\zeta)$ is set, determines the rate of work of individual sections of the production line. The fact that the production processes are not fixed and stochastic makes it difficult for using Fluid-models in the systems of control sharpen-governmental lines require their further improvement. Another feature of the model is that the equation (3) must be supplemented by the equation of state, defining function $\mu(t,\zeta)$. The foundation for the construction of the equation of state is a furnish detailed interaction of objects of labor with technological equipment. This led to the need to use to build the equation of state DES-models with all of the above-described disadvantages. The use of empirical dependence defining function $\mu(t,\zeta)$ for time-dependent transient it turned-were futile because of the complexity of construction and lack of precision between numerical and practical results [6].

Along with the calculation of the quantitative distribution of the objects of labor along the technological route at a given in the time the intensity of incoming orders $\mu(0) = \lambda$ and the rate of production of finished products $\mu(D) = \mu_M$ of interest to the task determination for each technological process of stock level of different types of raw and materials needed to ensure the smooth running of the production line. This task of production planning in the classical formulation is formulated by Modigliani and F. Hohn F. (1955) [24] defines a template for intensive rate resources required for executing technical operations. The model of production proposed by Modigliani F. and Holn F., provided at discrete points in time, coordination the various parts of the production with raw material suppliers and consumers of the finished product. Total consumption of resources for technological operations sharpen-term lines associated with discrete moments in the time separated by periods of planning. It is possible to use simplified restrictions that give imprint on the behavior of resources in the aggregate model of optimization. Proposal approach is dominant in the scientific literature over the ten years [15]. However, having ignored the limitations when aggregating narrows the scope of the models (Johnson LA, Montgomery DC) [25], (Voß S., Woodruff DL).

Models using the function of wait [26].

When setting targets production planning raises the question of the extent of time of planning. Despite the fact that the production planning is carried out at discrete moments of time BPE to time, in fact, plans are generated in continuous time in accordance with the received orders. In this regard, Schneeweiss S. (2003) [27] suggested as the equation of state of the production system to use the function of wait periods of incoming orders and the parameters in the model is planned of production at discrete points in time. Under steady-state conditions, the expected duration of the production cycle is a nonlinear function of the use of resources (Buzacott JA, Shanthikumar JG [26], Hopp WJ [18]).

For the construction of nonstationary wait functions proposed to use discrete Fluid-models. Discrete Fluid-model of the managed process of "one product – one technological resource" is presented in the form of:

$$I_i = I_{i-1} + R_i - D_i = I_{i-1} + X_i - D_i, \; X_i = R_i, \quad (4)$$

where I_i is the number of the finished product for a period of planning Δt_i, $t_i = t_1 \ldots t_T$, R_i is the number of the material entered in the period Δt_i, D_i and X_i – the demand for products and production volume for the period Δt_i. The balance equation (4) corresponds to the simplest type of function of wait (cycle time is much less than the planning period). It is assumed that the incoming material to be processed in the period Δt_i available for use in the finish of the period. Because of the short duration of the production cycle T_d, $T_d << \Delta t_i$ the material is recycled for a period Δt_i and **work in progress (WIP)** can be neglected. When the duration T_d exceeds the number of scheduling periods, quantity of products for the period of planning I_i becomes dependent on the number of **work in progress**. Volume production X_i for the period Δt_i. related to the number of material R_{i-L}, enrolled at a time Δt_{i-L}, $L=1,2,3....$

$$I_i = I_{i-1} + X_i - D_i = I_{i-1} + R_{i-L} - D_i, \; X_i = R_{i-L}. \quad (5)$$

Balance Equation (5) widely used in the material requirements planning MRP-industrial systems (Vollmann, TE, 2005) [15], (Vob S., Woodruff DL, 2003) [28]. Although there are models (Hackman ST, Leachman RC, 1989) [29], in which the terms of delivery of technological resources are equal to the fractional number of periods of planning, overall, both for the theory and for industrial practice, is the assumption that the time of delivery is the whole the number of periods of planning. Most of the models impose restrictions on the type $X_i \le C_i$ of the maximum output X_i for the period of planning, where C_i – the maximum power output. At the end of the period t_i production system has a level of work in progress:

$$W_t = \sum_{n=i-L+1}^{i} R_n - \sum_{n=i+1}^{i+L} X_n \quad (6)$$

Material received in the period t_i stays in the system during time-interval $\Delta t_i = (t_i - t_{i-L})$. The common point of view on the use in the discrete Fluid- models MRP-systems of capacity constraints led to a task of linear programming:

$$\sum_{i=1}^{N}(h_i \cdot I_i + \sigma_i \cdot R_i) \to \min, \; I_i = I_{i-1} + R_{i-L} - D_i,$$
$$R_{i-L} \le C_i, \; R_i \ge 0, \; I_i \ge 0, \quad (7)$$

where h_i – the cost of the unit where the work in progress, σ_i – cost per unit of production of resources used at the time t_i.

Hopp W.J., Spearman M.L. [18], based on a detailed DES -model of the interaction the individual items of work with equipment, presented during the interval of the planning Δt_i the dependence of productivity on the production line, the intensity of the receipts of the needed for processing technological resources. Thus Liu J., Li C., Yang F., Wang H., Uzsoy R. [10] point to the need to use to solve the problem of large resources of computer time. For in-depth detailed study of sidered problem requires higher processor (Kacar N.). Application of these models give good agreement between the theoretical and practical information in the description of the quasi-stationary production processes. However, their ability to describe the non-linear relationship between the rate of motion of the objects of labor on technological route and duration of the production cycle at the intensive using technological resources is doubted [14].

The main problem in determining the extent of the time $\Delta t_i = (t_i - t_{i-L})$ is that the system of planning and management of production $T_d >> \Delta t_i$ requires an assessment of the consequences the impact of decisions on the status of the parameters of the production system. When using fixed intervals $\Delta t = (t_i - t_{i-L})$ are ignored effects fustic within a period of planning. Achieving maximum capacity generator interoperable backlog within the period of planning resulted in the setting of production line, which limited the volume of production X_i for the period Δt_i.

Model-driven production processes using clearing function [30,31].

The presence of repetitive tasks of production planning and control, for solution which used different models or their combinations, led to the idea of creating a unified theory of optimization of production systems inline with the way the organization of production, for the construction of which Graves SC (1986) [30], Karmarkar US (1989) [31] proposed to use as the basic parameters of the state of the capacity $[\chi]_{CL}$ of the production system, the volume of **work in progress** W and the length of the production cycle T_d. To describe the behavior of the system parameters, Karmarkar US entered the equation of state $[\chi]_{CL} = \Phi(W)$ that specifies the relationship between capacity and volume of work in progress, which called clearing-function [31]. Clearing-function can be defined for a group of machines, equipment, production lines, one or more plants, including in the single production process. Clearing function $[\chi]_{CL} = \Phi(W) = const$ puts a fixed limit output production, assuming instantaneous build-cardinality of production, $[\chi]_{CL} = \Phi(W) = a \cdot W$, $a = const$ [30] suggests a fixed time to produce at full capacity, which in the presence of restriction-you start planning production for the period Δt_i, known as the combined clearing- function (Karmarkar US, 1989) [31]. An important class is the nonlinear clearing function used to build single-product models:

a) TQ-model M / M / 1 queue for the steady state [1]

$$[\chi]_{CL} = \Phi(W) = \frac{\mu \cdot W}{1+W} \, , \qquad (8)$$

b) the fundamental diagram of model of traffic for the stationary state

$$[\chi]_{CL} = \Phi(W) = \mu \cdot W - W^2 \, , \qquad (9)$$

c) Model G/M/1 queue for the steady state (Mehdi J.,1991) [1], (Berg R., 2004) [13, c.7]:

$$W = \frac{c_a^2 + c_s^2}{2} \cdot \frac{\rho^2}{1-\rho} + \rho, \; \rho = \frac{\lambda}{\mu} < 1. \qquad (10)$$

Where c_a^2 and c_s^2 represent the standard deviation of the admission requirements on products and time of processing, μ – the rate of processing of objects of labor λ – in-flow of objects of labor intensity in the first process step [1]. Model G / M / 1 queue for the steady state (10) is a development for model (8), the movement of objects of labor on technological route to the serial LAYOUT -technological equipment. Berg R. pointed out that the steady state model (10) is provided under the condition $\rho < 1$ [13, c.6]. When $\lambda \to \mu$ inter-operating backlogs infinitely large ($W \to \infty$), and if $\lambda > \mu$ equation (10) can not be used as it is assumed in its derivation $\rho = (\lambda / \mu) < 1$. The solution (10) with respect to $\rho < 1$, described in [1]:

$$\rho = \frac{\sqrt{(W+1)^2 + 4W(c^2-1)} - (W+1)}{2(c^2-1)}$$

at $c = \dfrac{c_a^2 + c_s^2}{2} > 1$. (11)

If $c \to 1$ the model M / M / 1 queue (8) is the limiting case of the model (10):

$$\lim_{c \to 1} \rho = \frac{W}{(W+1)} \, , \; \lim_{c \to 1} [\chi]_{CL} = \frac{\mu \cdot W}{1+W} . \qquad (12)$$

Asmundsson JM (2006) [32] proposed distributed AC-functions (allocated clearing function) to simulate multi-product lines. AC-function are assuming that produced aggregate product, aggregating a technological resources for the production of individual products. An alternative approach is the representation of clearing-function as a sum of clearing-functions of individual products. Experimental data indicate that satisfactory results using AC-functions for products similar in nature consumption of resources. Intensity in consumption of resources at the same time expressed in units of processing time [1]. However, if the model of transport resources for each nomenclature objects of labor are complex, of the using of AC-functions does not allow to describe the manufacturing processes. Selçuk B., Fransoo J.C., Gok A.G. (2007) approximated clearing function piecewise linear function [1], which allow to use for optimization of the parameters production line unit for linear programming. Clearing-function can be obtained as analytically and as numerically using the TQ-models, DES-models, Fluid-models of production systems or determined empirically. Selçuk, B., Fransoo J.C., Gok A.G. (2007) [1] presented a methodic for constructing transitional clearing-functions analytically. Due to the fact that the operational information about these products and on the status of work in progress at the plant is closed, in most studies in structure clearing-function instead of the empirical data used TQ- and DES-model. The exceptions are the work Haeussler S., Missbauer H. (2012) [1], which is for the structure clearing-function applied derived from the production line digital media. Kacar N. (2012) used to build clearing-functions optimized parameters of line. Computational experiments related to the calculation of the parameters of the production lines of the company Intel, showed a good approximation of the calculated and experimental data for the established processes [1].

Despite the fact that clearing function are the best tool for definition of instant communication between the capacity of the production system and the volume of work in progress, the presence of a limited number of the parameters in the equation of state, does not allow effectively simulate the time because of changes of parameters of the production process due to factors processing products (Armbruster D, Kempf K., 2012) [1]. Attempts to create a time-dependent clearing-functions limited special refinements theoretical and experimental studies (Fonteijn J., Wienke M., 2012) [1]. As a refinement Lefeber E. (2008) [20] introduces clearing-function $[\chi]_{CL} = \Phi(W(t-\tau_0))$ effectively time of processing $\tau_0 = \sum_{m=0}^{M} \mu_m^{-1}$ (basic processing time) of objects of labor on technological operations. [13] Missbauer H (2009) attempted to expand the use of clearing-function to transient production processes. This drew attention to the significant dependence of the capacity of the production system, from initial distribution of objects of labor on technological route and the need to ensure the conditions for the transition of the production system from one steady state to another. Production processes are stochasticity [21,33], but despite this the construction of clearing-functions virtually no attention no paid to the study of the stability of stream parameters of production lines, there are no estimates of the time decay of random perturbation of stream parameters and assess their absolutely values. Research -line Intel's semiconductor manufacturing products, conducted Armbruster D., Kempf KG (2012) [1] showed that the daily stochastics perturbation factors of production parameters are the decay time from 1-2 days to a week, which requires the availability of insurance reserves and 20% of the units of the normative quantity. The assumption that the transition process is quasi-stationary significant limitation for the wide application of the equation of state (clearing-function) in an analytical form, built mainly using TQ- models.

Models of production processes, using the equation in part-derivatives. PDE-model [8].

In modern literature can to identify the main three types of models and their combination for the output equation of state determining the relationship of parameters of stream of production lines. This queuing model (TQ-model), discrete-event model (DES-model) and Fluid-model [1]. Each type of model has its advantages, but none of them are not suitable fully for modeling a steady and transient operation of production systems [13, p.2]. Existing TQ-models describe the flow lines in the steady state [17]. Using them in the description of transient process to excessive complexity and high costs of computer time. DES-models are used to describe the production lines in the transient and steady-state conditions, but are discrete and need much machine time. Fluid- models are oriented on the small number of intervals partitioning process route and inear stationary solutions within a given interval. The requirement for increasing the accuracy of the model increases the quantity of generalization, and to complicate the model due to the increase in dimension of the system of differential equations (1) (Kefeli A., Uzsoy R., 2011).

In the last decade in the design of industrial production lines, using models describing the behavior of the production system with the help of partial differential equations (PDE-model) [1,9,13,20,21]. Introduced class of models combines the advantages of TQ-models, DES- models and Fluid-models, much extension possibilities of designing control systems production lines. PDE-models generally are continuous, can be successfully used in the description of steady state and transient modes of work of the production lines, do not require a lot of computer time [1].

A key issue in the construction of PDE-model production lines is the choice of the coordinate system. A common approach is to use as a variable, which determines the place of processing of the subject of work in the technology route, the cost S (UAH.) transferred technology resources on the subject of work (Dabaghyan AV, 2008) [1] (Fedyukin VK 2004) $S \in [0, S_d]$ (S_d (UAH) - the self-cost of manufacturing production), effective time of processing of the subject of work τ_m (h), $\tau_m \in [0, \tau_M]$ (Eekelen J., 2006), (Ramadge P., Wonham W.) [1] ($\tau_M = \sum_{m=1}^{M} \Delta\tau_m$ (h) -general effective the time of processing of the subject of work, $\Delta\tau_m$ is the average time of processing of the subject of work in the m-th step of process) or the degree of incompleteness of manufacturing product x (Armbruster D., Ringhofer S., Berg V., Lefeber E., 2004) [1] $x \in [0,1]$. The degree of incompleteness manufacturing product x is the position of the subject of work in the technological route, which can be represented as a ratio of the average time $\Delta\tau_m$ to its total processing time [13, c.16]. For the object of labor, treated at the m-th operation, can be recorded $x = \frac{\tau_m}{\tau_M} = \left(\sum_{k=1}^{m} \Delta\tau_k \Big/ \sum_{k=1}^{M} \Delta\tau_k \right)$. Every time of processing $\tau_m = \sum_{k=1}^{m} \Delta\tau_k$ correspondence the value of resources $S_m = S(\tau_m)$ transferred on object of labor and total time τ_M - self-cost $S_d = S(\tau_M)$. Thus, the degree of incompleteness of manufacture of the product x can be determined after a time of processing τ or cost-migrated cost on object of labor $S = S(\tau)$. It is suitable for modeling industrial production line with a generalized technological resource use dimensionless variable that determines the position of the subject of work in process flow [3]. Dimensionless variable $x = \tau_m / \tau_M$ used in the case where the model of production line does not consider the when re-nose for the structure of the labor resources (Armbruster D., Ringhofer C) [1]. For the model of xxx line, which takes into account in the result of technological operations consumption several interrelated technological resources on the using which are the restrictions, the using of dimensionless variable x -is difficult. If you enter a function of the density of objects of labor $\rho(t,x)$ in the state x at time t, the total number of objects of work, which are in various stages of readiness is the value (Armbruster D., Ringhofer S.) [1,20,21]

$$W(t) = \int_0^1 \rho(t,x)dx \ , \ x \in [0,1] . \qquad (13)$$

Since the processing of objects of labor is stochastic, then in the results executing of the operation, the subject of work may be in a non-particular state [8, c.4544]. This allowed to record the average density of the objects of labor $\rho(t,x)$ (pcs.) and the flow of objects of labor $F(t,x)$ (pcs. / h) on technological route through the distribution function of the objects of labor $f(t,r,x)$ over the states $r = \Delta\tau_m^{-1}$ (Armbruster D., Ringhofer S., 2005) [1]:

$$\rho(t,x) = \int_0^\infty f(t,r,x)dr \ , \ F(t,x) = \int_0^\infty \frac{1}{r} f(t,r,x)dr \ .(14)$$

The position of the object of work in the space of states characterized by a point with co-ordinates $(q_1, q_2, .., q_j, .., q_n)$ that determine the quantitative value of the parameters of the object of labor. The state space is used in the construction of models of multi-threaded lines, consuming during the production of multiple resources. Using in the one-dimensional description of the dimensionless variable x [1,3 for the depth study state of changing of state of the subject of labor is difficult. On the contrary, the using of as a variable parameter of the model S, which characterizes technological position of the subject of labor through a cost allows you to apply an elaborate mathematical apparatus of production functions [21], which allows summate resources by adding their values. However, despite the opening prospects through the using of the value changing in the state representation of the subject of work, the vast majority of authors (Armbruster D., Ringhofer S. (2005), Berg R., Lefeber E., Rooda J. (2008) [13] Wienke M., Fonteijn J., (2012), Kempf K., (2012) [16]), use PDE-model production lines with no restrictions on the consumption of technological resources, introduce for describing the state of the object of labor status variables (r,x).

In the PDE-model flow of objects of labor $F(t,x) = \rho(t,x) \cdot v(t,x)$ (capacity of the production line) is represented as the product of the density of objects of labor $\rho(t,x)$ and rate $v(t,x)$ their movement [33]. Assuming that the defective objects of labor don't exist (not sources and sinks), the movement of objects of labor on technological route satisfies the conservation law:

$$\frac{dQ}{dt} = \lambda - \mu \ , \ Q(t) = W(t) = \int_0^1 \rho(t,x)dx \ , \quad (15)$$

where λ - the intensity of the flow of objects of work in the first process operation, μ - output production[1]. Equation (15) is an integral form of the conservation law, the number of objects of work in the process of processing, can be presented in the differential form [1]:

$$\frac{\partial\rho(t,x)}{\partial t} + \frac{\partial F(t,x)}{\partial S} = 0, \ F(t,x) = \rho(t,x) \cdot v(t,x) \ . \ (16)$$

The boundary condition $F(t,0) = \lambda(t)$ specifies the **flow** blanks in the first operation. Profile of work in progress at the initial time is determined by the condition $\rho(0,S) = \rho_0(S)$ that characterizes the distribution of blanks on the technological operations of production lines. For a free flow line $\rho(0,S) = 0$. Equation (16) provides the connection in the time distribution $\rho(t,x)$ density of objects of labor and rate of motion $F(t,x)$ (capacity of the production line) for each point x on technological route [21]. Inequality in the distribution of work in progress along the route due to different effective time of handling objects of work on each transaction. Uneven in the performance equipment along technological route determines the dynamics of changes in the density of objects of labor that substantially affects the capacity of the line. Models of production process, which contained in the equation (16) take into account the influence of internal factors on throughput capacity and restrictions determined by the maximum-manufacturer of the equipment and interoperable storage capacity. This allowed the PDE-models compete with DES-models, the advantage of which is that they allow analytic form of solution and does not require significant computation inflammatory resources. The difficulty of building a PDE-model is determined by the balance of the equation of the form (16) are not closed [13,21,33]. To close the equation (16) is supplemented by the equation of state. If the equation of state is given clearing linear function $\Phi(W) = \int_0^1 F(t,x)dx = c\int_0^1 \rho(t,x)dx = c \cdot W$, (Constant Proportion, Graves SC, 1986 [30]), the system of equations PDE-model:

$$\frac{\partial\rho(t,x)}{\partial t} + \frac{\partial F(t,x)}{\partial S} = 0, \ v(t,x) = c = const$$
$$F(t,x) = \rho(t,x) \cdot c \qquad (17)$$

admits an analytic solution. If the velocity of movement of the objects of labor along the techno-logical route is constant $v(t,x) = c$, the flow of money for items of work, is stepping in time t = 0 to the first process step of production line (17) and has a solution:

$$c \cdot \rho(t,x) = H(c \cdot t - x) = \begin{cases} 0, & t < x/c, \\ 1, & t \geq x/c, \end{cases} \quad \lambda(t) = H(t). \tag{18}$$

Built-in model value $\tau = c^{-1}$ determines the length of the delay between the time of receipt of raw materials to the first operation and the time of release of the finished product. Constant speed of movement of objects of labor along technological route exists a constant time delay τ. Communication of flow of objects of work is presented linearly dependency. Next PDE-model (Lighthill-Whitham) [1] using the equation of state in the form of a non-linear dependence of the flux density $F(\rho)$ of the objects of labor ρ :

$$\frac{\partial \rho(t,x)}{\partial t} + \frac{\partial F(t,x)}{\partial x} = 0,$$

$$F(\rho) = \rho \cdot v(\rho) = \rho \cdot v_0 (1 - \frac{\rho}{R}), \; \frac{\rho}{R} \leq 1. \tag{19}$$

Widespread PDE-models of production lines containing stationary equations of state. The equation of state of production line for factory is presented by M / M / 1-model of queue with size $W(t)$, parameters λ, μ, W (15) and the duration of the production cycle $T_d = (1+W)/\mu$ for a stationary state determined dependency (D. Gross, C. Harris) [17] :

$$\lambda = \frac{\mu \cdot W}{1+W} \text{ where } T_d = \frac{1}{\mu - \lambda}, \; W = \frac{\lambda}{\mu - \lambda},$$

$$W(t) = \int_0^1 \rho(t,x) dx. \tag{20}$$

The velocity of the objects of labor $v(t,x)$ and local flow $F(t,x)$ for M / M / 1- model expressed in the intensity of output [13]

$$\frac{\partial \rho(t,x)}{\partial t} + \frac{\partial F(t,x)}{\partial S} = 0, \; v(t,x) \approx \frac{\mu}{1+W},$$

$$F(t,x) = \rho(t,x) \cdot v(t,x) \approx \frac{\mu \cdot \rho(t,x)}{1+W}. \tag{21}$$

The equation of state $F(t,x)$ in the integral form for a sustainable mode of work of production line (Armbruster D., Fonteijn J., Wienke M., 2012) has the form [1]

$$F(t,W) = \int_0^1 F(t,x) dx = \frac{\mu \cdot W}{1+W}. \tag{22}$$

M/M/1 model of clearing-function (6). Attempts to build sophisticated PDE-models of production lines, specifying the equation of state through the using of experimental measurements at the plant, a detailed simulation model or TQ-models [1]. Widely used in the study of production lines of models:

$$F(t,x) = \Phi(\rho) = \frac{\mu_0}{1+\rho(t,x)} \cdot \rho(t,x),$$

$$F(t,x) = \frac{\mu_0}{1+\rho(t,x)+k \cdot \rho(t,x)(1-x)}, \tag{23}$$

$$F(t,x) = k_1 \cdot \left(1 - e^{k_2 \cdot W}\right), \quad F(t,x) = \frac{k_1 \cdot W}{k_2 + W}, \tag{24}$$

$$F(t,x) = \frac{1-x}{\tau(t,x)\left(\int_0^1 \rho(t,z) dz - x \cdot \tau_0\right)} \cdot \rho(t,x),$$

$$\tau(t,x) = \frac{1}{\mu}, \tag{25}$$

who offered D.Armbruster, K.Kempf (2012) (23), J.Asmundsson., R.Uzsoy (24), Ringhofer On (2012) (25). $\tau(t,x)$ - the time required to complete the production of items of work which are in the step x; k, k_1, k_2 -technological factors; M-characteristic of the maximum storage capacity of generator.

Detailed analysis of the operation of the semiconductor of production line with xxx PDE-model (solid line) and DES-model (shading) performed Perdaen D. (2008), Lefeber E. (2010). [1] The characteristic behavior of the output stream objects of labor, calculated using PDE- model and DES-model (Lefeber E., 2010). The principal disadvantage of methods using of the DES-models is the extremely large number of calculations for complex production systems such as semiconductor production lines [14]. It is in this area of using PDE-models offer the highest prospects for the design of control systems of production lines. These methods are able to optimally combine the precision of DES-models when using of much less productive processors [13]. The using in PDE- models clearing-function exhibit significant promise at an early stage of development. A growing body of publications related to the development and refinement of the equation with state for PDE-models indicates that further development of the approach with the using of clearing-function is not effective. We need to develop statistical methods which to build a many moment PDE-models for transients, which for closing is used the time-dependent equation based on the mechanism of interaction of objects of labor between themselves and the equipment.

Kinetic models of controlled production processes [3,21,34].

In a series of work of Armbruster D, Ringhofer S., Lefeber E., Kempf K. [1,9,16] presented kinetic model production lines. Armbruster D., Ringhofer S. (2004) introduce the distribution function $f(x,v,t)$ of the objects of labor conditions characterizing the number of details in the state x in the moment of time t. A typical approach with which defined to the evolution of the distribution function of the objects of labor at the states, is in the derivation of closed equations for the moments of the distribution function. Build of PDE- models using kinetic theory contains a hierarchical set of equations.

This allows you to go beyond the limits of applicability of the quasistatic models. The proposal method by Bogolyubov, based on the selection of a small parameter, **allows** you to trim the number of equations at the right level. With the using of the kinetic ap da written equations for the first moments of the distribution function of the objects of labor $f(x,v,t)$ (Armbruster D., Ringhofer C) [35, s.819]

$$\frac{\partial \rho(t,x)}{\partial t}+\frac{\partial \rho(t,x)\cdot v(t,x)}{\partial x}=0,$$
$$\frac{\partial v(t,x)}{\partial t}+v(t,x)\cdot\frac{\partial v(t,x)}{\partial x}=0, \qquad (26)$$

$$\rho(t,0)\cdot v(t,0)=\lambda(t)\,,v(t,1)=\frac{\mu}{1+W(t)} \qquad (27)$$

$$\frac{dv(t,0)}{dt}=-\sigma\left(v(t,0)-\frac{\mu}{1+W(t)}\right),\text{ when } \lambda<\mu\,,$$

$$v(t,0)=\frac{\mu}{0{,}5+W(t)},\text{ when } \lambda\geq\mu\,. \qquad (28)$$

with boundary conditions (27) M / M / 1 model, resulting in steady resistant mode, where σ -experimental quantity. Integrating the first equation (26) with respect to x allows us to write the equation:

$$\frac{dW(t)}{dt}=\rho(t,0)\cdot v(t,0)-\rho(t,1)\cdot v(t,1)=$$
$$=\lambda(t)-\lambda\left(t-\frac{1}{c}\right), \qquad (29)$$

Which was received by Lefeber E. (2008) [1] from the very general considerations. It is known from practical studies that output from processing of the first product of occurs through some time of delay relative to the arrival time of the party processing [21]. In the transition from $\rho(t,x)$ to aggregate variables of the Fluid-model $W(t)$ (15) the effect of unevenly-dimensional distribution of the objects of labor along the process flow [5] and the availability of storage capacity constraints are not taken into account. The calculation results of streaming parameters of the production line, obtained with the using of model (26) - (28) are closer to the experimental data than the results of calculations using the M / M / 1 model (8), G / M / 1 -model (1.10), continuous Fluid-model (15) (Fonteijn J., Missbauer H.), although, according to Armbruster D. (2012), a detailed study of approximations associated by equations (26) - (28), in this moment not done. Not clear there was a question for which production systems which model is the most successful. Most governmental problems in the construction of kinetic models of production systems, lies in the fact that the kinetic equation is the integral-differential, the solution of which is a difficult mathematical problem [34]. In view of the complexity of the law of impact of equipment on the object of work, the kinetic equation can be not written in a precise form for the specific production processes. Even with simple assumptions about the nature of the impact of the equipment on the subject of labor can not obtain the exact analytical solutions. In this regard, of particular importance of build effective methods for the approximate solution of the kinetic equation of the production line.

Conclusions

This review of the basic models, which are used in the design systems of control of production lines. We consider the application of models and restrictions which prevent their effective use for the design of systems of control. Paying much attention to new types of models and kinetic models, containing equations in the partial differential equation (PDE-models). The analysis of the use of models for the simplest cases, the function-conditioning manufacturing production lines. The validity of the application is determined comparative analysis of results obtained using DES-model and PDE-model that studied. It is shown how to construct of the PDE-models statistical methods used to describe large systems. Wherein the general nature of the statistical regularities does not depend on the manner in which describes the behavior of a single object of labor. Using a statistical approach allows to obtain closed a many-balance equations (transport equation) not from phenomenological considerations, but based on the laws of motion of the individual items of work on technological route are certain production technology. Development and using of PDF-models requires addressing: of questions: The output of non-stationary equations of state, based on a detailed processing technology object of labor given hardware circuit. 2. Construction of multi-moment closed balance models for steady state and transient unsteady modes of operation of the production line. 3. Building a two-level models control parameters of the production line for steady-state and transient conditions taking into account the parameters of the equipment, scheme of arrangement of its priorities and movement of objects of labor.

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Тубичко К. В., Заруба В. Я., Пігнастий О. М., Ходусов В. Д. Огляд моделей промислових систем

У статті наведено огляд основних моделей виробничих систем. Виконано порівняльний аналіз різних типів моделей і показано області їх застосування. Надана коротка характеристика основних параметрів моделей. Детально розглянуто потокові моделі з використанням рівнянь в частинних похідних. Проведена їх класифікація залежно від виду рівняння стану. Розглянуто одномоментний і двохмоментний опис виробничого процесу.

Ключові слова: PDE-модель, виробнича лінія, масове виробництво, незавершене виробництво, система управління, балансові рівняння виробничої лінії, рівняння стану, дискретно-подієва модель, теорія масового обслуговування, модель рідини, Clearing-функція, квазістатичний процес, перехідний процес, стохастичний процес.

Тубычко Е. В., Заруба В. Я., Пигнастый О. М., Ходусов В. Д. Обзор моделей промышленных систем

В статье приведен обзор основных моделей производственных систем. Выполнен сравнительный анализ разных типов моделей и показаны области их применения. Дана краткая характеристика основных параметров моделей. Детально рассмотрены потоковые модели с использованием уравнений в частных производных. Проведена их классификация в зависимости от вида уравнения состояния. Рассмотрено одномоментное и двухмоментное описание производственного процесса.

Ключевые слова: PDE-модель, производственная линия, массовое производство, незавершенное производство, система управления, балансовые уравнения производственной линии, уравнение состояния, дискретно-событийная модель, теория массового обслуживания, модель жидкости, Clearing-функция, квазистатический процесс, переходный процесс, стохастический процесс.

Tubychko K. V., Zaruba V. Y., Pignasty O. M., Khodusov V. D. The Overview of Manufactured Systems Models

The article provides an overview of the main models of manufacturing systems. Here a comparative analysis of different types of models and shows their applications. In this article A brief description of the main model parameters. The detailed analysis of the streaming models with the using of partial differential equations. In this article is their classification depending on the equation of state. We examined the cross-sectional and two-moment description of the production process.

Keywords: PDE-model, production line, mass production, work in progress, management system, balance equations of the production line, equation of state, discrete-event model, queuing theory, model fluid, Clearing-function, quasistatic process, transient process, stochastic process.

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