SPACE-CHARGE LIMITING CURRENT OF CHARGED-PARTICLE BEAM IN COAXIAL DRIFT TUBE WITH DIELECTRIC INSERTS IN STRONG AXIAL MAGNETIC FIELD

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We obtain an analytical estimate of space-charge limiting (SCL) current of relativistic charged-particle beam propagating in infinitely long grounded coaxial drift tube with dielectric inserts in the strong magnetic field approximation. The received analytical estimate is compared with numerical calculations of SCL current.

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INTRODUCTION

Recently, there has been a some activity in studying space-charge limiting (SCL) current of a relativistic charged-particle beam in infinite coaxial geometry with dielectric inserts lining the outer and inner conductors of the drift tube [1–12].

In our paper in the approximation of strong magnetic field, we consider the charged-particle beam propagating in the grounded infinitely long coaxial drift tube with the dielectric insert of permittivity ε_1 lining the inner conductor and that of permittivity ε_2 lining the outer conductor. We find an analytical estimate of the SCL current for such a beam and compare it with results of numerical calculations.

MAIN PART

In approximation of the strong magnetic field the scalar potential created by the charged-particle beam in the infinitely long grounded coaxial drift tube with dielectric inserts lining the inner and outer conductors is described by the Poisson equation [1–3]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)=\begin{cases}0, & r_1\le r<r_{d1}, r_{d1}\le r<r_i\\ & r_o\le r<r_{d2},\\ & r_{d2}\le r\le r_2;\\ -\dfrac{4\pi I_0}{c\beta_{\parallel}(r_o^2-r_i^2)}, & r_i\le r<r_o,\end{cases} \tag{1}$$

with the boundary conditions

$$\begin{gathered}\varphi(r_1)=0=\varphi(r_2)=0,\\ \varphi(r_i-0)=\varphi(r_i+0),\quad \varphi(r_o-0)=\varphi(r_o+0),\\ \left.\frac{\partial\varphi}{\partial r}\right|_{r_i-0}=\left.\frac{\partial\varphi}{\partial r}\right|_{r_i+0},\ \left.\frac{\partial\varphi}{\partial r}\right|_{r_o-0}=\left.\frac{\partial\varphi}{\partial r}\right|_{r_o+0},\\ \varphi(r_{d1}-0)=\varphi(r_{d1}+0),\ \varepsilon_1\left.\frac{\partial\varphi}{\partial r}\right|_{r_{d1}-0}=\left.\frac{\partial\varphi}{\partial r}\right|_{r_{d1}+0},\\ \varphi(r_{d2}-0)=\varphi(r_{d2}+0),\ \left.\frac{\partial\varphi}{\partial r}\right|_{r_{d2}-0}=\varepsilon_2\left.\frac{\partial\varphi}{\partial r}\right|_{r_{d2}+0},\end{gathered} \tag{2}$$

where r_1, r_2, r_i, r_o, r_{d1}, and r_{d2} are the radii of the inner and outer tube coaxial conductors, inner and outer radii of charged-particle beam, outer radius of inner dielectric insert, and inner radius of outer dielectric insert, respectively; I_0 is the injection current;

$$\beta_{\parallel}=\sqrt{1-\frac{\gamma_0^2}{\gamma_{\parallel0}^2}\frac{1}{(\gamma-q\varphi/m_qc^2)^2}}, \tag{3}$$

is the longitudinal dimensionless velocity of the beam in the units of speed of light c in vacuum; γ_0 and $\gamma_{\parallel0}=\gamma_0(1+\gamma_0^2\beta_{\perp0}^2)^{-1/2}$ are the initial relativistic factor and dimensionless longitudinal beam kinetic energy; $\beta_{\perp0}$ is the transversal dimensionless initial velocity of the beam; m_q and q are the mass and charge of charged-particles, respectively.

We can found analytically the solution to Eq. (1) with boundary conditions (2) under the assumption that the longitudinal beam velocities, $\beta_{\parallel}$, are equal to their respective injection values, $\beta_{\parallel0}$, i.e. in the approximation of constant longitudinal velocities [10]

$$\varphi(r)=\begin{cases}\varphi_1(r), & r_1\le r<r_{d1},\\ \varphi_2(r), & r_{d1}\le r<r_i,\\ \varphi_3(r), & r_i\le r<r_o,\\ \varphi_4(r), & r_o\le r<r_{d2},\\ \varphi_5(r) & r_{d2}\le r\le r_2,\end{cases} \tag{4}$$

where

$$\varphi_1(r)=\frac{I_0\ln(r/r_1)G_{d2}}{\varepsilon_1 c\beta_{\parallel0}Z},$$

$$\varphi_2(r)=\frac{I_0G_{d2}}{c\beta_{\parallel0}Z}\left(\ln(r/r_1)-\frac{\varepsilon_1-1}{\varepsilon_1}\ln(r_{d1}/r_1)\right),$$

$$\varphi_3(r)=\frac{I_0}{c\beta_{\parallel0}}\left[\frac{G_{d2}}{Z}\left(\ln(r/r_1)-\frac{\varepsilon_1-1}{\varepsilon_1}\ln(r_{d1}/r_1)\right)-\frac{r^2-r_i^2}{r_o^2-r_i^2}+\frac{2r_i^2\ln(r/r_1)}{r_o^2-r_i^2}\right],$$

$$\varphi_4(r)=\frac{I_0(G_{d2}-2Z)}{c\beta_{\parallel0}Z}\left(\ln(r/r_2)+\frac{\varepsilon_2-1}{\varepsilon_2}\ln(r_2/r_{d2})\right),$$

$$\varphi_5(r)=\frac{I_0(G_{d2}-2Z)\ln(r/r_2)}{\varepsilon_2 c\beta_{\parallel0}Z}.$$

Here

$$Z = \ln(r_2 / r_1) - \frac{\varepsilon_1 - 1}{\varepsilon_1} \ln(r_{d1} / r_1) - \frac{\varepsilon_2 - 1}{\varepsilon_2} \ln(r_2 / r_{d2}),$$

$$G_{d2} = G - 2\frac{\varepsilon_2 - 1}{\varepsilon_2} \ln(r_2 / r_{d2}),$$

$$G = 1 + 2\ln(r_2 / r_o) + \frac{2r_i^2}{r_o^2 - r_i^2} \ln(r_i / r_o).$$

We can easily find the radial position

$$r_{\text{ext}} = r_i \left(1 + G_{d2} \frac{r_o^2 - r_i^2}{2r_i^2 Z}\right)^{1/2}, \qquad (5)$$

at which the dimensionless potential $f(r) = q\varphi(r) / (m_q c^2)$ has the extremal value $f_{\text{ext}} \equiv q\varphi(r_{\text{ext}}) / (m_q c^2)$.

In Fig. 1 dimensionless distributions of normalized scalar potential $f(r)$ are shown for different values of permittivities of the inner, ε_1, and outer, ε_2, dielectric linings. Results of (linear) analytical calculations and nonlinear numerical simulations presented in Fig. 1 show that greater values of the permittivity of dielectric insert lead to the reduction of extremal value of the dimensionless scalar potential $f(r_{\text{ext}})$. Also one can see that for symmetric geometry this influence is more significant than for an asymmetric one.

We can easily obtain the analytical estimate of SCL current:

$$I_{\text{lim}}^{\text{coax}} = I_A \frac{\gamma_0 (\gamma_{\parallel 0}^{2/3} - 1)^{3/2}}{\gamma_{\parallel 0} W_d}, \qquad (6)$$

where $I_A = m_q c^3 / q$ is the Alfven current ($I_A \approx 17.05$ kA for the electrons), $W_d = \frac{G_{d2}}{Z}\left[\ln(r_{\text{ext}} / r_1) - \frac{1}{2}\right] + \frac{2r_i^2}{r_o^2 - r_i^2} \ln(r_{\text{ext}} / r_i)$.

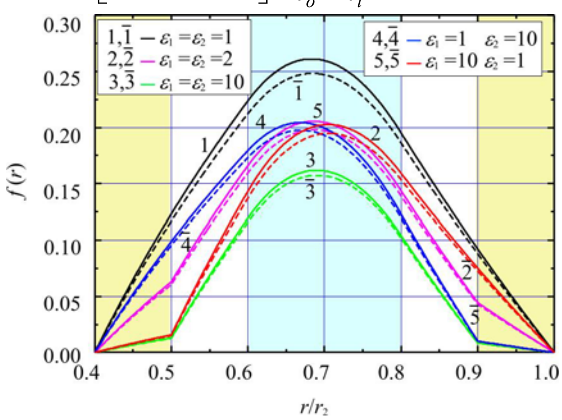

Fig. 1. Normalized scalar potential $f(r)$ received by numerical solution for electron beam ($q = -|e|$, $m_q = m_e$): $r_1 = 1$ cm, $r_{d1} = 1.25$ cm, $r_i = 1.5$ cm, $r_o = 2$ cm, $r_{d2} = 2.25$ cm, $r_2 = 2.5$ cm, $\gamma_0 = \gamma_{\parallel 0} = 2$. Solid lines are numerical solution (nonlinear modelling), dashed lines are analytical results (linear approximation)

In Fig. 2 we plot the SCL current, $I_{\text{lim}}^{\text{coax}} / I_A$, of charged-particle beam propagating in a coaxial drift

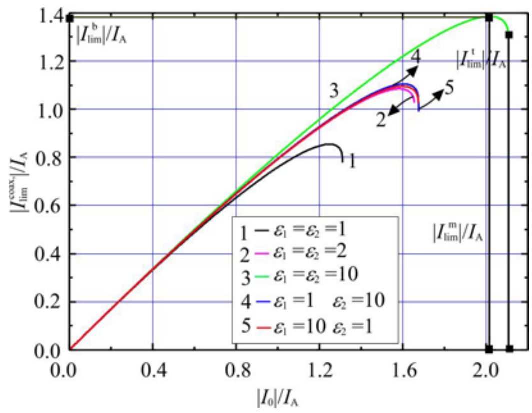

Fig. 2. Dependence of SCL current $|I_{\text{lim}}^{coax}| / I_A$ [10,12] on normalized injection current $|I_0| / I_A$: $r_1 = 1$ cm, $r_{d1} = 1.25$ cm, $r_i = 1.5$ cm, $r_o = 2$ cm, $r_{d2} = 2.25$ cm, $r_2 = 2.5$ cm, $\gamma_0 = \gamma_{\parallel 0} = 2$

tube as a function of the injection current. Also, we show three proposed definitions of the SCL current, [12], given below:

1. $|I_{\text{lim}}^b| = \underset{I_0}{\text{ext}} \left\{ \frac{2|I_0|}{\beta_{\parallel 0}(r_o^2 - r_i^2)} \int_{r_i}^{r_o} \beta_{\parallel}(r) dr \right\}$, ($\rho \equiv I_0 / v_{\parallel} = const$);
2. $|I_{\text{lim}}^t|$ is maximal injection current $|I_0|$, for which solution $f(r)$ of nonlinear equation (1) still exists;
3. $|I_{\text{lim}}^m|$ is injection current, for which maximum of right hand side in the definition 1 just above is obtained.

One can see that dielectric inserts raise the SCL current values, symmetric cases being more influential.

CONCLUSIONS

Thus, we presented the analytical estimate of space-charge limiting current of a charged-particle beam propagating in an infinitely long drift tube with the dielectric inserts lining the inner or outer conductor in the strong magnetic field approximation. Accomplished numerical modelling of the SCL current in such a system shows a good agreement with the analytical estimate.

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ПРЕДЕЛЬНЫЙ ТОК ПУЧКА ЗАРЯЖЕННЫХ ЧАСТИЦ В КОАКСИАЛЬНОЙ КАМЕРЕ ДРЕЙФА С ДИЭЛЕКТРИЧЕСКИМИ ВСТАВКАМИ В ПРИБЛИЖЕНИИ СИЛЬНОГО МАГНИТНОГО ПОЛЯ

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Получена аналитическая оценка предельного тока релятивистского пучка заряженных частиц в бесконечно длинной заземленной коаксиальной камере дрейфа с диэлектрическими вставками в приближении сильного магнитного поля. Проведено сравнение полученной аналитической оценки с численными расчетами предельного тока.

ГРАНИЧНИЙ СТРУМ ПУЧКА ЗАРЯДЖЕНИХ ЧАСТИНОК У КОАКСІАЛЬНІЙ КАМЕРІ ДРЕЙФУ З ДІЕЛЕКТРИЧНИМИ ВСТАВКАМИ В НАБЛИЖЕННІ СИЛЬНОГО МАГНІТНОГО ПОЛЯ

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Знайдено аналітичну оцінку граничного струму релятивістського пучка заряджених частинок у заземленій коаксіальній камері дрейфу нескінченої довжини з діелектричними вставками в наближенні сильного магнітного поля. Проведено порівняльний аналіз отриманої аналітичної оцінки з чисельними розрахунками граничного струму.