EIGEN DIPOLAR ELECTROMAGNETIC WAVES OF COAXIAL PLASMA-METALL WAVEGUIDE STRUCTURE WITH AZIMUTH MAGNETIC FIELD

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This report is devoted to the investigation of the propagation peculiarities of the eigen dipolar electromagnetic waves in coaxial plasma-metal waveguide with non-uniform azimuth magnetic field. The dependence of the dispersion properties, spatial attenuation coefficient, radial wave field structure, phase and group velocities on the effective collision rate, the value of the direct current that flows along the central conductor of the waveguide structure and waveguide geometric parameters were considered. It was shown that mentioned parameters can be used to control the dispersion and attenuation properties of the studied waves.

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INTRODUCTION

Till now, the coaxial plasma-metal waveguide structures are the object of intensive both theoretical and experimental studies. This is stipulated by the fact that such waveguide structures are widely used in the devices of plasma electronics [1] and also as the discharge chambers for plasma-technological processes [2, 3]. In the previous researches the basic attention was paid to the eigen electromagnetic waves with azimuth wavenumber $m = 0$ [4] due to its wide usage in different applications. But it is necessary to mention, that the dipolar waves with $m = \pm 1$ are also often used for various technological applications [5]. Electrodynamic properties of such dipolar waves essentially differ from the symmetric waves with $m = 0$ [4] and needs for the further study.

1. BASIC EQUATIONS

The studied coaxial waveguide structure consists of the central metal conductor of radius R_1, that is placed at the axis of waveguide system. This conductor is enclosed by the cylindrical plasma layer with outer radius R_2. The vacuum region $(R_2 < r < R_3)$ separates the plasma layer from outer waveguide metal wall with radius R_3. The radial non-uniform azimuth magnetic field $H_0(r)$ is created by the direct current J_z that flows along the central metal conductor. Cylindrical plasma layer was considered in the hydrodynamic approach as cold slightly dissipative medium with constant effective collision rate ν ($\nu/\omega < 1$, where ω is wave frequency). It was also supposed that plasma density vary slightly along the plasma column on the distances of wavelength order. Permittivity tensor of cold magnetized collisional plasma can be written as:

$$\begin{pmatrix} \varepsilon_1(r) & 0 & -i\varepsilon_2(r) \\ 0 & \varepsilon_3(r) & 0 \\ i\varepsilon_2(r) & 0 & \varepsilon_1(r) \end{pmatrix},$$

here $\varepsilon_1 = 1 - \dfrac{\omega_p^2(r)\omega'}{\omega[(\omega')^2 - \omega_c^2(r)]}$, $\varepsilon_2 = \dfrac{|\omega_c(r)|\omega_p^2(r)}{\omega[(\omega')^2 - \omega_c^2(r)]}$, $\varepsilon_3 = 1 - \dfrac{\omega_p^2(r)}{\omega\omega'}$, $\omega' = \omega + i\nu$, $\omega_p(r)$ and $\omega_c(r)$ are electron plasma and cyclotron frequencies, respectively. It is necessary to mention that these frequencies depend on radial position r.

The solution of the Maxwell equations in the cylindrical coordinates that govern the considered wave propagation can be found in the form:

$$E, H = E(r), H(r)\exp\left(i[k_3 z + m\varphi - \omega t]\right), \qquad (1)$$

where k_3 is complex axial wave number, m is azimuth wave number.

In the plasma region $(R_1 < r < R_2)$ the system of ordinary differential equations that describe the radial distribution of tangential wave field components can be written as follows:

$$\begin{cases} \dfrac{dE_\varphi}{dr} = -\dfrac{E_\varphi}{r} + F_1 H_\varphi - F_2 E_z + F_3 H_z \\ \dfrac{dH_\varphi}{dr} = -F_4 E_\varphi - F_5 H_\varphi + F_6 E_z - F_2 H_z \\ \dfrac{dE_z}{dr} = -F_7 H_\varphi - F_8 E_z - F_1 H_z \\ \dfrac{dH_z}{dr} = -F_9 E_\varphi + F_4 E_z \end{cases}, \qquad (2)$$

here $F_1 = i\dfrac{m}{r}\dfrac{k_3}{k\varepsilon_1}$, $F_2 = \dfrac{m}{r}\dfrac{\varepsilon_2}{\varepsilon_1}$, $F_3 = i\left(k - \dfrac{m^2}{r^2}\dfrac{1}{k\varepsilon_1}\right)$, $F_4 = i\dfrac{m}{r}\dfrac{k_3}{k}$, $F_5 = \dfrac{1}{r} - k_3\dfrac{\varepsilon_2}{\varepsilon_1}$, $F_6 = \dfrac{i}{k}\left(\dfrac{m^2}{r^2} + k^2\dfrac{\varepsilon_2^2 - \varepsilon_1^2}{\varepsilon_1}\right)$, $F_7 = \dfrac{i}{k\varepsilon_1}\left(k_3^2 - k^2\varepsilon_1\right)$, $F_8 = k_3\dfrac{\varepsilon_2}{\varepsilon_1}$, $F_9 = \dfrac{i}{k}\left(k_3^2 - k^2\varepsilon_3\right)$,

$k = \omega/c$ is the vacuum wave number. To obtain the solutions of this system for arbitrary problem parameters one must used special numerical methods.

In the vacuum region $(R_2 < r < R_3)$ the corresponding system of Maxwell equations can be solved analytically [5]. So, wave field components can be expressed in terms of linear combination of modified Bessel functions. Constants that are present in these expressions can be obtained with the help of boundary conditions consisting in the continuity of tangential wave field components at plasma – vacuum interface:

$$\begin{cases} C_1 &= A_1 H_\varphi^p(R_2) - A_2 E_z^p(R_2) - A_3 H_z^p(R_2) \\ C_2 &= -A_4 H_\varphi^p(R_2) + A_5 E_z^p(R_2) + A_6 H_z^p(R_2) \\ C_3 &= -A_1 E_\varphi^p(R_2) + A_3 E_z^p(R_2) - A_2 H_z^p(R_2) \\ C_4 &= A_4 E_\varphi^p(R_2) - A_6 E_z^p(R_2) + A_5 H_z^p(R_2) \end{cases}, \quad (3)$$

here $A_1 = i\dfrac{\Delta\kappa_v}{k}K_m(\Delta)$, $A_2 = \Delta K_m'(\Delta)$, $A_3 = i\dfrac{mk_3}{k}K_m(\Delta)$, $A_4 = i\dfrac{\Delta\kappa_v}{k}I_m(\Delta)$, $A_5 = \Delta I_m'(\Delta)$, $A_6 = i\dfrac{mk_3}{k}I_m(\Delta)$, $\Delta = \kappa_v R_2$, $\kappa_v^2 = k_3^2 - k^2$ and $E_z^p(R_2)$, $H_z^p(R_2)$, $E_\varphi^p(R_2)$, $H_\varphi^p(R_2)$ are the values of wave field components at plasma – vacuum interface ($r = R_2$), obtained by the numerical solution of the equations (2), prime denotes the derivative with respect to the argument.

The analogue of the dispersion equation can be obtained from the boundary conditions for $E_z(r)$ and $E_\varphi(r)$ wave field components at the waveguide metal wall $r = R_3$. These conditions lead to the dispersion equation in the following form:

$$\begin{cases} C_1 I_m(\kappa_v R_3) + C_2 K_m(\kappa_v R_3) &= 0 \\ C_3 I_m'(\kappa_v R_3) + C_4 K_m'(\kappa_v R_3) &= 0 \end{cases}. \quad (4)$$

2. MAIN RESULTS

The main attention in this report was focused on the dipolar wave with $m = 1$ due to its wide usage in different practical applications [1-4]. It is necessary to mention that dipolar wave possess all six wave field components. So the solution of the problem became rather hard and bulky.

The influence of direct current value and waveguide geometric parameters on the dispersion properties of the waves considered was studied for the case of collisionless plasma. In the case considered the dispersion equation (4) possesses five solutions (curves 1-5) that are shown on the Fig. 1.

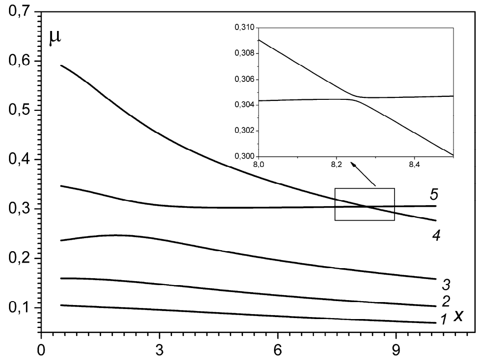

Fig. 1. The solutions of the dispersion equation $\mu = \omega/\omega_p$ on the dimensionless wave number $x = k_3 R_2$. Problem parameters are equal: $R_1/R_2 = 0.1$, $R_2\omega/c = 0.5$, $R_3/R_2 = 1.5$, $j = 2.0$

These solutions correspond to the eigen modes that can exist in the considered waveguide structure under the given conditions. Mentioned modes differ mainly in the radial wave field structure in the plasma region. The decrease of the eigen wave frequency under the fixed wavenumber value leads to the decrease of the scale length of spatial wave field oscillations of the eigen modes in radial direction. Each of these modes essentially differs in the dependence of phase and group velocities on the wavenumber.

The value of the normalized direct current ($j = eJ_z/(2mc^3)$) substantially affects the dipolar mode dispersion. The dependence of the normalize frequency μ on the normalized axial wavenumber x for different normalized direct current values j is shown on the Fig. 2.

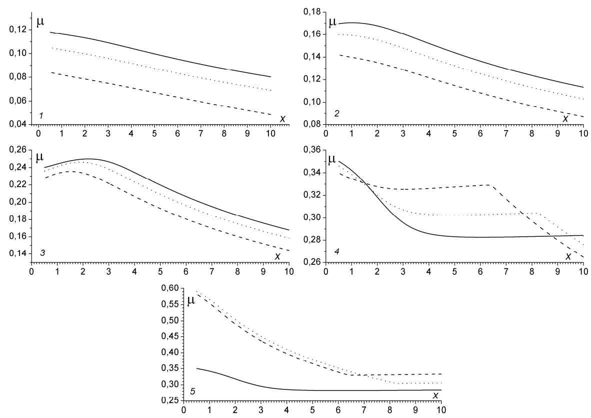

Fig. 2. The dependence of dimensionless frequency μ on the dimensionless wave number x. Numbers just near the axes origin correspond to the solution numbers in accordance to the Fig 1. Dash lines corresponds to the j value 1.6, dot lines – $j = 2.0$, solid lines – $j = 2.4$. Other parameters are the same as for the Fig. 1

It is shown that different solutions of the dispersion equation have different dependency type on the normalized direct current value j. Thus, the increase j value from $j = 1.6$ up to $j = 2.4$ leads to the increase of the phase velocities of the first three solutions of the dispersion equation (see subplots 1-3 on the Fig. 2). The next two solutions have different dependence on the considered parameter j. The increase of the direct current leads to the decrease of wave phase velocities for the solutions 4 and 5 in the region of middle wavenumbers (see subplots 4, 5 on the Fig. 2).

The influence of geometric parameters of the waveguide structure on the dipolar wave dispersion was studied as well. The influence of vacuum gap thickness on the dipolar wave properties is presented on the fig. 3. The value of the distance from cylindrical plasma layer to the outer waveguide metal wall strongly influences on the wave dispersion. The parameter $\eta = R_3/R_2$, that characterizes this distance, has the main influence on the dispersion in the range of small and moderate values ($\eta < 1.5$). When parameter η grows up to rather large values ($\eta > 2$) it has negligible influence on the dispersion. It was shown that the waves that correspond to the third and fourth solutions of the dispersion equation (see subplots 3 and 4 on the Fig. 3) greatly react to the parameter η variation.

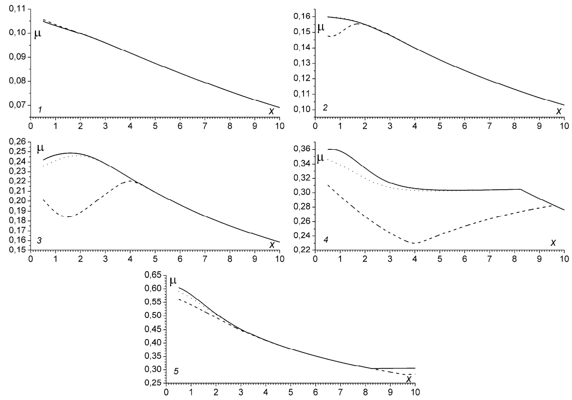

Fig. 3. The dependence of dimensionless frequency μ on the dimensionless wave number x. Numbers just near the axes origin correspond to the solution numbers in accordance to the Fig 1. Dash lines corresponds to the $\eta = 1.1$, dot lines – $\eta = 1.5$, solid lines – $\eta = 2.0$. Other parameters are the same as for the Fig. 1

The influence of the effective electron collision frequency ν on the spatial attenuation coefficient $\alpha = \mathrm{Im}(k_3)R_1$ was also studied. It was obtained that the increase of the effective collision rate value ν leads to the increase of the wave attenuation coefficient. It is necessary to mention that collisions have different influence on different solutions of the dispersion equation (4). Thus, solutions of the dispersion equations presented on the Fig. 1 have difference in value and in direction of group velocities in different wavenumber regions. As was obtained earlier in [6] $\alpha \sim \nu / \omega / (d\mu / dx)$, so attenuation coefficient α has the same sign as group velocity. It was obtained that the third solution has rather wide range of wavenumbers where attenuation coefficient is positive, so this solution can be used for the gas discharge sustaining.

CONCLUSIONS

It was shown that five eigen dipolar waves can propagate in the considered waveguide structure. It was studied the influence of the value of direct current, the effective collision frequency and geometric parameters of waveguide system on the dispersion properties and attenuation coefficient of each considered eigen waves. It was shown the existence of the wave that can be used for gas discharge sustaining.

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СОБСТВЕННЫЕ ДИПОЛЬНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ КОАКСИАЛЬНОЙ ПЛАЗМЕННО-МЕТАЛЛИЧЕСКОЙ СТРУКТУРЫ С АЗИМУТАЛЬНЫМ МАГНИТНЫМ ПОЛЕМ

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Исследованы особенности собственных дипольных электромагнитных волн, распространяющихся в коаксиальном плазменно-металлическом волноводе с неоднородным азимутальным магнитным полем. Рассмотрена зависимость дисперсионных свойств, коэффициента пространственного затухания волны, радиального распределения поля волны, фазовой и групповой скоростей от эффективной частоты столкновений электронов, от величины постоянного тока, протекающего вдоль центрального проводника, а также от геометрических параметров волновода. Исследована эффективность управления дисперсией и затуханием изучаемых волн с помощью указанных параметров.

ВЛАСНІ ДИПОЛЬНІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ КОАКСІАЛЬНОЇ ПЛАЗМОВО-МЕТАЛЕВОЇ СТРУКТУРИ З АЗИМУТАЛЬНИМ МАГНІТНИМ ПОЛЕМ

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Досліджено особливості власних дипольних електромагнітних хвиль, що розповсюджуються в коаксіальном плазмово-металевому хвилеводі з неоднорідним азимутальним магнітним полем. Розглянуто залежність дисперсійних властивостей, коефіцієнта просторового загасання хвилі, радіального розподілу поля хвилі, фазової та групової швидкостей залежно від ефективної частоти зіткнень електронів, від значення сталого електричного струму, що протікає уздовж центрального провідника, та від геометричних параметрів хвилеводу. Досліджено ефективність управління дисперсією та просторовим загасанням хвиль, що вивчаються, за допомогою вказаних параметрів.