NONLINEAR ANALYSIS OF MM WAVES EXCITATION BY HIGH–CURRENT REB IN DIELECTRIC RESONATOR

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A nonlinear self-consistent theory of excitation of millimeter wave lengths electromagnetic fields by high current relativistic azimuthally-modulated electron beam in cylindrical resonator with a dielectric rod is constructed. For generation of high frequency waves is used an electron beam. Nonlinear numerical analysis is carried out.

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INTRODUCTION

The problem of increasing the frequency of an excited electromagnetic waves is a current and active development task due to the fact that different physical and technological applications require the availability of resources millimeter and submillimeter diagnostics. For example, using of these radiation sources allow diagnostics of plasma with a density of the order of $10^{11}...10^{13}$ cm^{-3}.

Effective use of such sources as the dielectric waveguide structures, excited by relativistic electron bunches, demonstrated experimentally in [1]. In similar structures the main mechanism of generation is transition radiation and Cherenkov radiation. Excitation of high frequency in [1] was achieved by using structures with the transverse dimensions of the order of a few hundreds microns. In [2] it was suggested that the azimuthally modulated electron beam to excite oscillation modes with a high index number. It is possible to use the structure with the transverse dimensions of the order of a few centimeters.

In the present paper self-consistent theory of the electromagnetic field excitation by azimuthally modulated electron beam in a dielectric resonator is constructed. We have demonstrated the possibility of the mode selection by the special choice of the geometry of the electron beam. In contrast to earlier papers on this subject [3], in this theory an analytical expression for the potential field, which is not taken into account previously, is obtained.

STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

The structure under investigation is a cylindrical metal resonator with dielectric rod placed inside. An excitation source of the resonator is a multi-stream electron beam, which is a set of cylindrical beams, located equidistant in azimuth, and extending along the axis of the resonator near the surface of a dielectric rod. The side walls of the resonator are assumed to be a closed with metal grids, transparent for the charged particles and nontransparent for the excited electromagnetic fields.

For the construction of the analytical theory of the excitation of oscillation modes with high azimuthal number, we start from the technique developed in [4], where the general nonlinear theory of the excitation of the dielectric resonators is built. We represent the excited electric and magnetic field as a sum of solenoidal and potential parts

$$\mathbf{E} = \mathbf{E^t} + \mathbf{E^l},\ \mathbf{H} = \mathbf{H^t}, \qquad (1)$$

where $\mathbf{E^t}$ and $\mathbf{H^t}$ are the solenoidal components of electromagnetic field, and $\mathbf{E^l}$ is the potential electric field.

Solenoidal components of an excited electromagnetic field will seek in the form of decomposition by eigen solenoidal fields of empty dielectric resonator:

$$\mathbf{E^t} = \sum_s A_s(t)\mathbf{E}_s(\mathbf{r}),\ \mathbf{H^t} = -i\sum_s B_s(t)\mathbf{H}_s(\mathbf{r}), \qquad (2)$$

where $\mathbf{E}_s$, $\mathbf{H}_s$ are eigen solenoidal fields of resonator without an electron beam, which satisfy the equations:

$$\mathrm{rot}\mathbf{H}_s = -i(\omega_s / c)\varepsilon\mathbf{E}_s, \quad \mathrm{rot}\mathbf{E}_s = i(\omega_s / c)\mu\mathbf{H}_s, \qquad (3)$$

where $\omega_s \equiv \omega_{mnl}$ are the eigenfrequencies of resonator; $s \equiv n, m, \ell$ numerate, respectively, radial, azimuthal and axial indexes.

The current density of multi-stream electron beam, with taking into account the geometry of the problem, can be written as:

$$\mathbf{j}_e = \sum_{k=1}^{N}\sum_{p\in V_R}\frac{1}{r}\mathbf{v}_p q_p \delta(r - r_p(t))\delta(z - z_p(t))\times \\ \times\delta(\varphi - \varphi_p(t) - (k-1)2\pi/N), \qquad (4)$$

where q_p is the charge of the macroparticle; r_p, φ_p, z_p and $\mathbf{v}_p$ are its time-dependent coordinates and velocity. The summation in (4) is carried out over the particles of N beams, being in the resonator volume V_R.

By using the orthonormality conditions of the functions $\mathbf{E}_s$ and $\mathbf{H}_s$ [4]:

$$\int_{V_R} dV\varepsilon\mathbf{E}_s\mathbf{E}^*_{s'} = \int_{V_R} dV\mu\mathbf{H}_s\mathbf{H}^*_{s'} = 4\pi P_s\delta_{ss'}, \qquad (5)$$

for calculation the expansion coefficients $A_s(t)$ and $B_s(t)$ one can obtain the second–order differential equations:

$$\frac{d^2A_s}{dt^2}+\omega_s^2A_s=-\frac{dR_s}{dt},\quad \frac{d^2B_s}{dt^2}+\omega_s^2B_s=-\omega_sR_s,$$
(6)

where $R_s=\frac{1}{P_s}\int \mathbf{j_e}\cdot\mathbf{E}_s^*(\mathbf{r}_p)dV$.

Having solved Maxwell equations together with the boundary conditions, for the axial components of the electric and magnetic solenoidal fields we obtain:

$$E_{sz}=e_{zm}(r)\cos k_\ell z e^{im\varphi},H_{sz}=h_{zm}(r)\sin k_\ell z e^{im\varphi},\qquad(7)$$

where $e_{zm}(r)$ and $h_{zm}(r)$ are the functions, which describes the radial structure of the electromagnetic fields, in the dielectric – I ($r\le a$) and in the vacuum – II ($a\le r<b$). In present paper expressions for this functions are omitted, and presented in full view in [4].

Substituting the expression for the current density of the beam (4) to (6), and using found eigenfunctions of electric field components (7), we obtain the expression for $R_s=\mathrm{Re}\,R_s+i\,\mathrm{Im}\,R_s$, which are in the right-hand sides of the equations (6)

$$\mathrm{Re}\,R_{n,jN,\ell}=\frac{N}{P_{n,jN,\ell}}\sum_{p\in V_R}q_p\Big(\Big[v_{pz}e_{z,jN}(r_p)\cos k_\ell z_p+ v_{pr}e_{r,jN}(r_p)\sin k_\ell z_p\Big]\cos jN\varphi_p- v_{p\varphi}e_{\varphi,jN}(r_p)\sin k_\ell z_p\sin jN\varphi_p\Big),\qquad(8)$$

$$\mathrm{Im}\,R_{n,jN,\ell}=\frac{-N}{P_{n,jN,\ell}}\sum_{p\in V_R}q_p\Big(\Big[v_{pz}e_{z,jN}(r_p)\cos k_\ell z_p+ v_{pr}e_{r,jN}(r_p)\sin k_\ell z_p\Big]\sin jN\varphi_p+ v_{p\varphi}e_{\varphi,jN}(r_p)\sin k_\ell z_p\cos jN\varphi_p\Big),\qquad(9)$$

here $j=0,\pm1,...$ And besides, $R_{n,m,\ell}=0$ for $m\ne jN$.

Finally, for the solenoidal components of electromagnetic field we obtain

$$E_z^t=2\sum_{j,n,\ell}\varepsilon_j\left|A_{n,jN,\ell}\right|e_{z,jN}(r)\cos k_\ell z\cos(jN\varphi+\alpha_s),$$
$$H_z^t=2\sum_{j,n,\ell}\varepsilon_j\left|B_{n,jN,\ell}\right|h_{z,jN}(r)\sin k_\ell z\sin(jN\varphi+\beta_s);\qquad(10)$$

$$\mathrm{tg}\,\alpha_{n,jN,\ell}=\frac{\mathrm{Im}\,A_{n,jN,\ell}}{\mathrm{Re}\,A_{n,jN,\ell}},\ \mathrm{tg}\,\beta_{n,jN,\ell}=\frac{\mathrm{Im}\,B_{n,jN,\ell}}{\mathrm{Re}\,B_{n,jN,\ell}}.\qquad(11)$$

The potential electric field, which can be presented in the form $\mathbf{E}^l=-\nabla\Phi$, satisfy the Poisson equation:

$$\frac{1}{\varepsilon r}\frac{\partial}{\partial r}\left(r\varepsilon\frac{\partial\Phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{\partial^2\Phi}{\partial z^2}=-\frac{4\pi}{\varepsilon}\rho\qquad(12)$$

with the boundary conditions consisting in that the potential Φ on the resonator metal walls becomes zero and continuity of the potential and radial component electric induction vector. Eq. (12) is solved by the method of eigenfunction expansion. The eigenvalues and eigenfunctions are solutions of the corresponding Sturm-Liouville problem. The final expression for the potential Φ can be written as:

$$\Phi=\sum_{j=0}\sum_{n,l}\sum_{p}\frac{8N\varepsilon_jq_p}{(\kappa_{n,jN}^2+k_l^2)L\left\|R_n\right\|^2}R_n(\kappa_{n,jN}r)\sin k_lz\times$$
$$\times R_n(\kappa_{n,jN}r_p)\sin k_lz_p\cos jN(\varphi-\varphi_p).$$
(13)

Eigenvalues κ are determined from the equation

$$J_m(\kappa a)Z_m^{'}(\kappa a)=\varepsilon Z_m(\kappa a)J_m^{'}(\kappa a).$$
(14)

The orthogonality of the eigenfunctions $R_m(r)$, which define dependence of the potential versus radius, and their norms $\left\|R_m\right\|^2$ defines as follows:

$$\int_0^b rdr\varepsilon(r)R_m(r)R_{m'}(r)=\left\|R_m\right\|^2\delta_{mm'},\qquad(15)$$

where $R^I(r)=J_m(\kappa a)\frac{Z_m(\kappa r)}{Z_m(\kappa a)}$, $R^{II}(r)=J_m(\kappa a)$, and $Z_m(\kappa r)\equiv J_m(\kappa r)-J_m(\kappa b)Y_m(\kappa r)/Y_m(\kappa b)$.

NUMERICAL INVESTIGATION

The main goal of numerical investigation of excitation by multistream electron beam in a dielectric resonator was to analyze the possibility of excitation oscillations in the millimeter wavelength structure with transverse dimensions on the order more of the excited oscillations.The parameters of the resonator under study were: radius of the dielectric rod $a=4.0$ cm, radius of the resonator $b=7.5$ cm, resonator length $L=0.83$ cm, permittivity of the rod $\varepsilon=2.04$. The beam parameters are: quantity of the beams $N=36$, beam current and energy was, respectively, $1.5\,kA$ and $300\,$kV. Analysis of an excited field in the resonator has shown that the main mechanism of generation is monotron mechanism [5]. The main contribution to the energy make eigenmode with the frequency 34.117 GHz for which the monotron mechanism efficiency is the best, under the chosen parameters, and also the greatest coefficient of coupling of mode with the beam (demonstrated on Fig. 1).

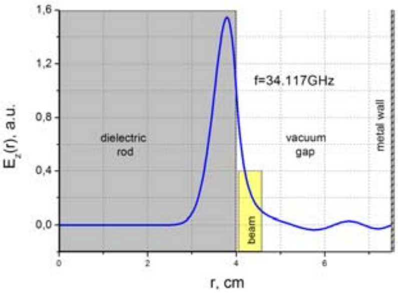

Fig. 1. Radial distribution of "whispering gallery" mode

Fig. 2 shows dependence of the electromagnetic field energy, stored in the resonator, on time.

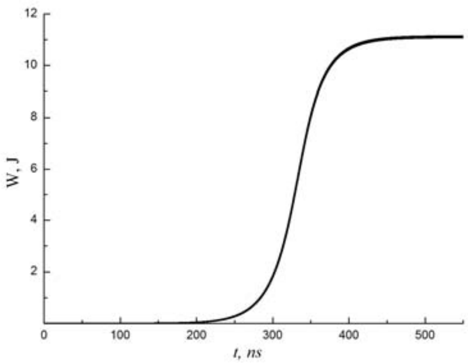

Fig. 2. Energy of electromagnetic field stored in the resonator

For a stationary regime of excitation is characterized essentially nonlinear dynamics of the beam. Fig. 3 hows the phase planes of the beam particles during the one oscillation period.

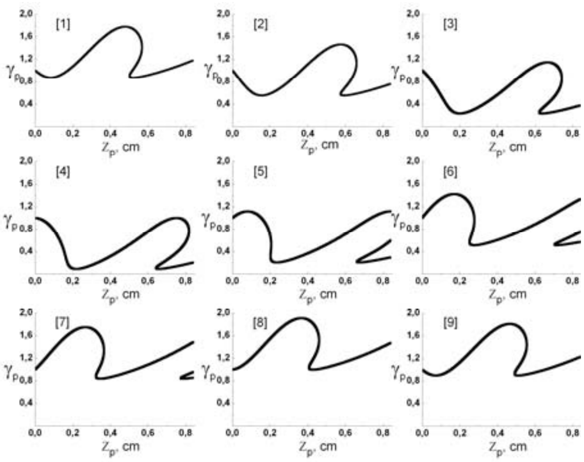

Fig.3. Phase planes for different moments of time corresponding to the energy saturation

Due to beam-excited field the modulation of the beam by density and the bunching of the beam is take place. Fig. 3 shows the formation of multystream flow and overturning of the forward front of the perturbed beam.

CONCLUSIONS

Analytical investigations and nonlinear numerical analysis of an excitation of the dielectric resonator by the high current relativistic azimuthally–modulated electron beam are carried out. Demonstrated the possibility of the mode selection and increasing the frequency of an excited oscillations through the modulation of the electron beam in azimuth.

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НЕЛИНЕЙНЫЙ АНАЛИЗ ВОЗБУЖДЕНИЯ МИЛЛИМЕТРОВЫХ ВОЛН СИЛЬНОТОЧНЫМ РЕЛЯТИВИСТСКИМ ЭЛЕКТРОННЫМ ПУЧКОМ В ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ

К.В. Галайдыч, Ю.Ф. Лонин, А.Г. Пономарев, Ю.В. Прокопенко, Г.В. Сотников

Построена нелинейная самосогласованная теория возбуждения электромагнитного излучения миллиметрового диапазона длин волн сильноточным релятивистским азимутально-модулированным электронным пучком в цилиндрическом резонаторе с диэлектрическим стержнем. Проведен нелинейный численный анализ.

НЕЛІНІЙНИЙ АНАЛІЗ ЗБУДЖЕННЯ МІЛІМЕТРОВИХ ХВИЛЬ СИЛЬНОСТРУМОВИМ РЕЛЯТИВІСТСЬКИМ ЕЛЕКТРОННИМ ПУЧКОМ У ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ

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Побудовано нелінійну самоузгоджену теорію збудження електромагнітного випромінювання міліметрового діапазону довжин хвиль сильнострумовим релятивістським азимутально-модульованим електронним пучком у циліндричному резонаторі із діелектричним стрижнем. Проведено нелінійний чисельний аналіз.