STATIONARY AND TRANSIENT BEAM DYNAMICS SIMULATION RESULTS COMPARISON FOR TRAVELLING WAVE ELECTRON LINAC WITH BEAM LOADING

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The methods of beam dynamic simulation taking into account the beam loading effect are discussed in this paper. It is one of main problems limiting the beam current. The simulation methods for stationary case were described in the paper [1]. The test simulations will discussed for transient mode and stationary case in this paper. The beam dynamics simulation will done using BEAMDULAC-BL and BEAMDULAC-BLNS. These codes were computed to study the beam dynamics in accelerators working on a traveling wave in stationary and transient cases respectively. The results of simulations were compared for both cases.

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INTRODUCTION

Charged particles linear accelerators are useful in experimental physics and in some areas of technology at present. The main advantages of linear accelerators are: high rate of energy gain, high limit beam intensity, simple beam extraction.

The beam space charge influence is the main factors limiting the beam current and accurate treatments of the beam own space charge field and its influence on the beam dynamics is one of the main problems in design of high current RF accelerators. Coulomb field, beam radiation and beam loading effect are the main factors of the own space charge. Typically, only one of the space charge field components takes into account for different types of accelerators. It is the Coulomb field for low energy linacs and radiation and beam loading for higher energies. But both factors should be treated in modern low and high energy high intensity linacs. The mathematical model should be developed for self consistent beam dynamics study taking into account both Coulomb field and beam loading influence in stationary case and transient mode. That is why three-dimensional self-consistent computer simulation of high current beam is very actually.

Let us describe the beam loading effect briefly. The beam dynamics in an accelerator should be studied self-consistently taking into account both external field and beam own space charge field. The RF field induced by the beam in the accelerating structure depends on the beam velocity as well as the current pulse shape and duration. The influence of the beam loading can decrease the external field amplitude and provide the irradiation in the wide eigen frequency modes. Therefore we should solve the motion equations simultaneously with Maxwell's equations for accurate simulation of beam dynamic.

The method of kinetic equation and the method of large particles are most useful methods for self-consistent problem solving. Maxwell's equation solving can be replaced by solving of the Poisson equation if we take into account only Coulomb part of the own beam field. This equation can be solved by means of the well-known large particles methods as particle in cell (PIC) or cloud in cell (CIC). There is no easy method of beam dynamics simulation that takes into account the beam loading effect.

The methods of beam dynamic simulations and three-dimensional code BEAMDULAC-BL were considered in [1-2]. The BEAMDULAC code is developing in MEPhI since 1999 [3] for high intensity beam dynamics simulation in linear accelerators and transport channels. The self-consistent beam dynamics can be studied using BEAMDULAC-BL code version taking into account the beam loading effect only for linacs, working on a traveling wave mode in a stationary case. Similar methods, algorithms and code should be designed to study the beam dynamic taking into account the beam loading effect for transient mode. The beam loading for transient mode was early considered by E.S. Masunov and mail equations were done [4].

The beam dynamics can be calculated for only one beam part that has the phase length equal to one period of the external RF field in the stationary case. It is necessary to calculate the dynamics for all beam particles for the transient case. We must to take into account all particles of short current pulse which are inside of the accelerating structure in the time moment. In this case, the analyzed beam can be represented in 2D or 3D phase space as a number of the large particles. These large particles would have the torus form (a ring with finite-size) with a rectangular cross-section for 2D simulation due to the axial symmetry of the task. The parallelepiped form large particles are conveniently use in 3D case.

Now it would be interesting to compare the simulation results for the stationary case and the transient mode.

Let us consider the algorithm of beam dynamics simulations taking into account the beam loading effect in accelerators, working on a traveling wave in the transient mode.

1. THE EQUATION OF MOTION IN SELF CONSISTENT FIELD AND SIMULATION METHODS FOR TRANSIENT MODE

The charge of any large particle is:

$$Q=J_{\text{pulse}}\cdot\tau_{\text{pulse}}/N, \tag{1}$$

where J_{pulse} – the pulse beam current, τ_{pulse} – the duration of the current pulse, N – the number of large particles.

The dynamics of every large particle should be simulated in the external field and in the own space charge field self-consistently. The initial particles distribution in the start of simulations is given with the help of especial algorithm in the 2D or 3D phase space. The initial particles distribution should takes into account the delay of each particle input into accelerating structure. Further on, the system will be defined self-consistently.

The beam which is traveling inside of the resonant structure decreases the amplitude and changes the phase of the external RF field. It also excites a number of wake fields for all resonant eigen frequencies of the structure. Let we consider for example waveguide section with $\beta_v>\beta_{gr}>0$, where β_v and β_{gr} are phase and group velocities of the wave, respectively. For simplicity we will consider only one (base) RF field harmonic with $\nu = 1$. The field acting in the beam cross section with coordinate z differ on value $\Delta\widetilde{E}^+ = \widetilde{E}(z,\tau_{k+1}) - \widetilde{E}(z,\tau_k)$ for the k-th and (k+1)-th beam bunches, i.e. during the time equals to pulse length T_b the field is changed to $\Delta\widetilde{E}^+$:

$$\Delta\widetilde{E}^+ = \frac{E_s^0(z_k)E_s^0(z_{k+1})T_b}{2P_s\left(v_{gr}^{-1}(z) - v_q^{-1}(z)\right)}\widetilde{I}_1(t), \qquad (2)$$

where v_q – particles velocity, E_s^0 – the amplitude of the accelerating field in the s-th bandwidth; P_s – the power of the s-th bandwidth, $\widetilde{I}_1$ – pulse beam current.

In the other hand, if we do not takes into account the attenuation of RF power in the walls and structure dispersion, the field will change at a fixed time t to the same value $\Delta\widetilde{E}^+$ on the length equal to:

$$\Delta z = \frac{v_{gr}v_q}{v_q - v_{gr}}T_b\,. \qquad (3)$$

Indeed, in accordance with $\tau_{gr}(z) - \tau_q(z) = T_b N_f$, where N_f – number of bunches, which are radiates into the structure and get part of the own space charge field in the coordinate z, the own field influence will increase with a displacement Δz in case when:

$$\tau_{gr}(z+\Delta z) - \tau_q(z+\Delta z) = T_b(N_f + 1)\,. \qquad (4)$$

and equation (3) can be easily rewritten.

Let we introduce the new variable:

$$\tau = t + \tau_{gr}(z) - \tau_q(z) = \sum\left[\Delta t_j + \Delta z_j\left(\frac{1}{v_{gr}} - \frac{1}{v_q^j}\right)\right], \qquad (5)$$

where j is the large particle number. According to the noted above

$$\frac{\Delta\widetilde{E}^+}{\Delta\tau} = -\frac{\nu v_{gr}}{\nu - v_{gr}}R_{sh}\widetilde{I}_1\,, \qquad (6)$$

where R_{sh} – series impedance of the structure in the base band width. When a large number of bunches are considered and $\Delta\tau \to 0$ we will have

$$\frac{\partial\widetilde{E}^+}{\partial\tau} + \frac{v v_{gr}}{v - v_{gr}}\frac{\partial\widetilde{E}^+}{\partial z} = -\frac{v v_{gr}}{v - v_{gr}}R_{sh}\widetilde{I}_1\,. \qquad (7)$$

It is easy to generalize this equation taking into account the field attenuation in the structure and the structure dispersion [4]. It should be remembered that for the fixed time t and for the length Δz the field value is additionally reduced by the small amount of $-\left(\alpha - \frac{1}{2R_{sh}}\frac{\Delta R_{sh}}{\Delta z}\right)\widetilde{E}^+$. Here α is the RF power attenuation. As the result we will have finally the equation of beam motion in the point of bunch placement taken into account the beam loading effect for the transient mode:

$$-\left(\frac{1}{v} - \frac{1}{v_{gr}}\right)\frac{\partial\widetilde{E}^+}{\partial\tau} + \frac{\partial\widetilde{E}^+}{\partial z} + \left(\alpha - \frac{1}{2R_{sh}}\frac{dR_{sh}}{dz}\right)\widetilde{E}^+ = -R_{sh}\widetilde{I}_1(z,t). \qquad (8)$$

For waveguide system with negative dispersion in the same way we can obtain:

$$-\left(\frac{1}{v} + \frac{1}{v_{gr}}\right)\frac{\partial\widetilde{E}^+}{\partial\tau} + \frac{\partial\widetilde{E}^+}{\partial z} + \left(\alpha - \frac{1}{2R_{sh}}\frac{dR_{sh}}{dz}\right)\widetilde{E}^+ = R_{sh}\widetilde{I}_1(z,t). \qquad (9)$$

There are no limitations on the amount of the group velocity in the derivation of non-stationary equations of excitation (8), (9). So they will be used just like for highly dispersed systems and for the weak dispersion of waveguide systems.

The solution of the equations (8), (9) should be done with the given initial and boundary conditions. For example, if the beam is injected with t=0 into the empty waveguide with length L, they will have two parts:

$$E^+(z,t=0) = 0 \text{ and } \left.\begin{aligned} E^+(z=0,t) = 0, v_{gr} > 0, \\ E^+(z=L,t) = 0, v_{gr} < 0. \end{aligned}\right\} \qquad (10)$$

Equation (10) can be generalized to take into account the reflection at ends of the waveguide. The field can be considered as the sum of direct $\widetilde{E}^+$ and backward $\widetilde{E}^-$ waves for reflection treatment. The backward wave does not interact with the beam but is produced by the reflection from the waveguide end (the wave source is stationary placed and v=0). Indeed we can to obtain the motion equation for the regular waveguide:

$$\frac{1}{v_{gr}}\frac{\partial\widetilde{E}^-}{\partial\tau} - \frac{\partial\widetilde{E}^-}{\partial z} + \alpha\widetilde{E}^- = 0 \qquad (11)$$

instead of Eq. (8) The boundary conditions can be written as:

$$\widetilde{E}_n^-(t,L) = \widetilde{\Gamma}_2\widetilde{E}_n^+(t,L),\ \widetilde{E}_{n+1}^+(t,L) = \widetilde{\Gamma}_1\widetilde{E}_n^-(t,0), \qquad (12)$$

where $\widetilde{\Gamma}_1$ and $\widetilde{\Gamma}_2$ are complex reflection coefficients, and $n = [t/\tau_{gr}]_c$ – the passes number of the wave front.

In the simplest case, when the waveguide has complete reflection from both ends $\widetilde{\Gamma}_1 \approx \widetilde{\Gamma}_2 \approx 1$ and $V_b >> V_{gr} > 0$. The solution to this problem allows us to formulate the physical limitations of the excitation equation of a long cavity:

$$-\frac{d\widetilde{E}}{dt} + i\left(\nu\omega - \omega_r^{'}\right)\widetilde{E} = \omega_r^{'}\frac{J_0 R_{sh}}{2QL}\widetilde{I}_\nu^{r,r'}\,, \qquad (13)$$

$$\widetilde{I}_\nu^{r,r'} = \frac{1}{L}\int_0^L I_\nu(z,t)e^{i\left(h_r - h_{r'}\right)z}dz\,. \qquad (13,a)$$

Let we assume that wave amplitudes $\widetilde{E}^+$ and $\widetilde{E}^-$ have negligible attenuation during the one pass time of the wave front $\tau_{gr}(L)$ and after each reflection $\widetilde{E}^+ = \widetilde{E}^-$. It is possible in case of low beam loading effect influence. We can summarize the equation for the forward and backward waves and do the longitudinal coordinate z averaging neglecting $\partial/\partial z$ derivative comparatively with $v_{gr}^{-1}\partial/\partial t$. Then we can obtain the equation for $\widetilde{E} = \widetilde{E}^+ + \widetilde{E}^-$:

$$\frac{d}{dt}\tilde{E} + \alpha v_{gr}\tilde{E} = -J_0 \frac{v_{gr}}{L}\int_0^L R_{sh} I_1(z,t)dz \qquad (14)$$

which has the same form as the equation (7) with $\omega = \omega_r^{/}$, $Q = \frac{\omega_r^{/}}{2\alpha v_{gr}}$ и $v_{gr} R_{sh} = \frac{\omega_r^{/} R_{shunt}}{2QL}$. Here $\omega_r^{/}$ is the real part of the complex resonant frequency ω, Q is the Q-factor and R_{shunt} is the series impedance for the base band width and J_0 is the average bunch current.

The transient mode field excited by the beam with $\tau_{pulse} < T_f$ can be calculated using Eq. (8) and (11) with the boundary conditions (12). Indeed the transient mode beam loading problem can be solved for the resonant system for different matching conditions at the waveguide section ends and without any limitations to the beam current value and the group velocity v_{gr}.

The time dependence in Eq. (8) disappears for the long pulse duration $\tau_{pulse} > T_f$ and we can obtain the equation:

$$\frac{d\tilde{E}^+}{dz} + \left(\alpha - \frac{1}{2R_{sh}}\frac{dR_{sh}}{dz}\right)\tilde{E}^+ = \mp R_{sh} I_1 \cdot \qquad (15)$$

The field stationary distribution and the current harmonics magnitudes along of the longitudinal coordinate z could be calculated using this equation. We can obtain the well-known waveguide excitation equation when the value of the series impedance does not depend versus z.

Until now, we consider the excitation equations taking into account only one (base) spatial field harmonic. The phase velocity of the base harmonic is close to the beam velocity. The consideration of other (non-synchronous) spatial harmonics can be performed for a periodic structure in a similar way. More general can be written for polyharmonic case as:

$$-\left(\frac{1}{\nu}-\frac{1}{\nu_{gr}}\right)\frac{d\tilde{E}^{(l)}}{dt} + \frac{d\tilde{E}^{(l)}}{dz} + \left(\alpha - \frac{1}{2R_{sh}}\frac{dR_{sh}}{dz}\right)\times$$
$$\times\tilde{E}^{(l)} = \mp R_{sh} I_1(z)\sum_{l_1=0}^{\infty} k_{l_1} e^{i\frac{2\pi}{D}(l_1-l)}, \qquad (16)$$

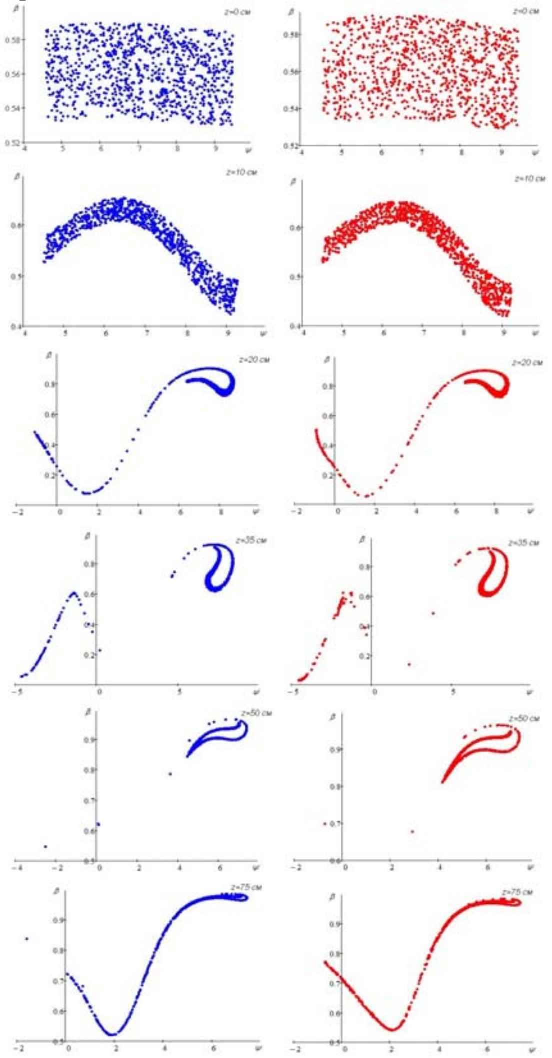
where $\tilde{E}^{(l)}$ is the complex amplitude of the synchronous field harmonic, $k_l = 1$, $k_{l_1} (l_1 \neq l)$ is the ratio of the nonsynchronous harmonic amplitude to the synchronous one $k_{l_1} = E^{(l_1)} / E^{(l)}$.

Thus the calculation of RF fields excited in the waveguide systems can be done for not relativistic or relativistic beams with the long duration of the current pulse $T_b >> T_f$ and in transient mode $T_b < T_f$ also. Same equations with zero right side (the homogeneous equation) describe the self-consistent beam dynamics taken into account external RF field and own space charge field in the stationary case. The self-consistent electromagnetic field can be founded from equations (7)-(9) using correct initial and boundary conditions. Equations (7)-(9) can be easily rewritten taking into account all own space charge RF field harmonics.

The algorithm of simulation for the transient mode is mainly similar to the algorithm developed for the stationary case [1-2]. The method of Coulomb field treatment used for BAMDULAC-BL and BAMDULAC-BLNS code was discussed in [3].

2. ELECTRON BEAM DYNAMICS SIMULATION

The results of beam dynamics simulation were compared with the measurement data obtained for the traveling wave electron linac U-28 of Radiation-Accelerating Centre of National Research Nuclear University "MEPhI". The main U-28 characteristics are given in Table. Three-dimensional code BEAMDULAC-BL has been used for beam dynamics simulation in U-28 for stationary case and new 3D code BEAMDULAC-BLNS was used for transient mode. It should be noted, that the comparison of simulation and measurement can be done only for beam current I<0.44 A and other results are interpolation.



Stationary mode Transient mode

Fig.1. Electron beam bunching in U-28 linac

Parameters of U-28 linac

Parameter	Value
Average output energy, MeV	10
Range output energy, MeV	2…12
Max pulse beam current, mA	440
Max average beam current, μA	170
Normalized energy spectrum $(\Delta W/W)_{min}$, %	3
Pulse duration, μs	0.5…2.5
Pulse repetition rate, 1/s	400

Beam bunching process simulation results are presented in Fig.1. It was shown, that beam loading influence is negligible small for beam with current $I \le 0.2$ A. The results of numerical simulation are in a good agreement with experimental one for $I<0.44$ A. It was shown that the results of the particle dynamics in stationary and transient case are in good agreement also.

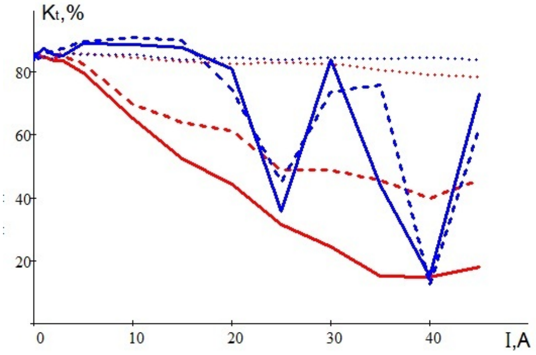


Fig.2. The current transmission coefficient versus of the initial pulse beam current for stationary (red lines) and transient (blue) modes

The current transmission coefficient and the output beam energy versus of the initial pulse beam current for stationary and transient modes are shown in Fig.2 and 3 respectively. Some tests of Coulomb field and beam loading influence were done to define which influence is more essentially. The simulation was done taking into account both beam loading and Coulomb field (solid lines), taking into account only the Coulomb field (points) and only beam loading (dot lines). It is clear from figures that the beam loading effect has the more essential influence to the dynamics for the long structure as it can be predicted analytically.

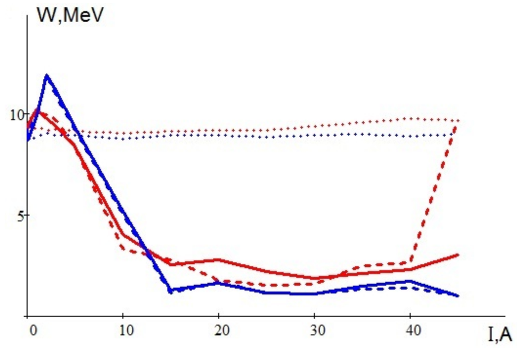


Fig.3. The output beam energy versus of the initial pulse beam current for stationary (red lines) and transient (blue) modes

CONCLUSIONS

The basic equations of beam motion in waveguide accelerating system were considered taking into account the beam loading effect. The beam loading can be studied for stationary beam mode and for transient mode also. Some results of beam dynamics simulation taking into account beam loading in both modes were presented and compared.

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СРАВНЕНИЕ РАСЧЕТОВ ДИНАМИКИ ПУЧКА В УСКОРИТЕЛЯХ НА БЕГУЩЕЙ ВОЛНЕ С УЧЕТОМ ЭФФЕКТОВ НАГРУЗКИ ТОКОМ В НЕСТАЦИОНАРНОМ И СТАЦИОНАРНОМ СЛУЧАЯХ

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Рассмотрен эффект нагрузки током, являющийся одной из основных проблем, ограничивающих ток пучка. В предыдущих работах были рассмотрены особенности расчета динамики пучка в ускорителях, работающих на бегущей волне с учетом эффектов нагрузки током в стационарном случае. В данной работе сравниваются результаты численного моделирования, произведенные в стационарном и в нестационарном случаях. Произведено моделирование нескольких структур при одинаковых начальных условиях с помощью программ BEAMDULAC-BL и BEAMDULAC-BLNS, позволяющих рассчитывать динамику пучков в ускорителях, работающих на бегущей волне в стационарном и нестационарном случаях соответственно.

ПОРІВНЯННЯ РОЗРАХУНКІВ ДИНАМІКИ ПУЧКА У ПРИСКОРЮВАЧІ НА БІГУЧІЙ ХВИЛІ З УРАХУВАННЯМ ЕФЕКТІВ НАВАНТАЖЕННЯ СТРУМОМ У НЕСТАЦІОНАРНОМУ І СТАЦІОНАРНОМУ ВИПАДКАХ

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Розглянуто ефект навантаження струмом, що є однією з основних проблем, що обмежують струм пучка. У попередніх роботах були розглянуті особливості розрахунку динаміки пучка в прискорювачах, що працюють на бігучій хвилі з урахуванням ефектів навантаження струмом у стаціонарному випадку. У цій роботі порівнюються результати чисельного моделювання, вироблені в стаціонарному і в нестаціонарному випадках. Вироблено моделювання декількох структур при однакових початкових умовах за допомогою програм BEAMDULAC-BL і BEAMDULAC-BLNS, що дозволяють розраховувати динаміку пучків у прискорювачах, що працюють на бігучій хвилі в стаціонарному і нестаціонарному випадках відповідно.