ABOUT THE INFLUENCE OF ASYMMETRY OF NUCLEAR MATTER ON THE UNITARITY OF SINGLET-TRIPLET SUPERFLUID STATES

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Unitarity of superfluid states in nuclear matter is studied in the case that the quasi-particle energy matrix holds scalar and vector components in isospin space at the same time, $\xi = \xi_0 + \vec{\xi}\vec{\tau}$. The vector component of ξ can be generated by the asymmetry of nuclear matter (different concentrations of neutrons and protons). It is shown that in considered case the superfluid states become non-unitary. The work is done using the Fermi liquid approach [2].

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1. INTRODUCTION

This work is done using a semiphenomenological method – Fermi liquid approach [1,2] which is a generalization on superfluid states of the Landau-Silin theory of normal Fermi liquid [3,4]. It is successfully applied to the study of the superfluidity phenomenon in different systems.

The aim of this paper is the determination of the influence of vector component of the quasi-particle energy matrix on unitarity of the order parameter of superfluid state in a two-component Fermi liquid (the definition of unitarity see below). One of examples of a two-component Fermi liquid is the nuclear matter. Nuclear matter is a model of an infinite system composed from two sorts of nucleons – neutrons and protons. Nuclear matter is called symmetrical in the case that the concentrations of neutrons and protons in the system are equal. If the concentrations are not equal this difference is described using the asymmetry parameter defined as

$$\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p). \qquad (1)$$

It is well known that nuclear matter can be in superfluid state. Those studies began from the work by N.N. Bogoliubov in 1958 [5] and the work by A. Bohr, B.R. Mottelson, D. Pines [6]. There is a series of papers beginning from [7] which deal with the superfluidity in nuclear matter using the Fermi liquid approach.

There is an equivalence in the description of superfluid order parameter which can be factorized in the Pauli matrices either in spin or in isospin space. Vector component of the quasi-particle energy ξ can be obtained, for example, with introduction of magnetic field or asymmetry of nuclear matter. However we will consider the states in isospin space only (the case of asymmetry). It is caused by the fact that a presence of vector part of quasi-particle energy in the spin space is conditioned by magnetic field. For nuclear matter the presence of magnetic field requires taking into account the neutron diamagnetism which would complicate the problem.

2. UNITARY SUPERFLUID STATES

Let us define the order parameter of the superfluid state as [1]

$$\Delta_{12} \equiv 2\frac{\partial \mathscr{E}(f,g)}{\partial g^{+}_{21}}, \qquad (2)$$

where $\mathscr{E}(f,g)$ is the energy functional and $f_{12} \equiv \mathrm{tr}\rho_0 a^{+}_2 a_1$, $g_{12} \equiv \mathrm{tr}\rho_0 a_2 a_1$ are the normal and anomalous density matrices respectively. Here ρ_0 is a nonequilibrium statistical operator and $a^{+}_{1,2}, a_{1,2}$ are the creation and annihilation operators respectively.

From the symmetry properties it follows

$$\widetilde{\Delta} = -\Delta, \qquad (3)$$

where the transposition means the permutation of the particles which compose the Cooper pair. The order parameter can be factorized in Pauli matrices, and because of (3) it has a form

$$\Delta = \left(\Delta_0 + \vec{\Delta}\vec{\tau}\right)\tau_2, \qquad (4)$$

which is predefined by the antisymmetry of the τ_2 matrix.

The problem of definition of order parameters consists in the construction of the self-consistent equation which determines the relation between the order parameters and anomalous density matrices. In order to create such an equation one should find an expression for the anomalous density matrix through

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the order parameter. This can be attained by solving the matrix equation for the value X [2]

$$X\Delta^{+}X-\xi X-X\widetilde{\xi}-\Delta=0. \qquad (5)$$

If X is known one can explicitly express the normal and anomalous density matrices [2]:

$$\begin{aligned} f&=Kn+X\left(1-\widetilde{n}\right)X^{+}K\\ g&=K\left(1-n\right)X-X\,\widetilde{n}\widetilde{K}. \end{aligned} \qquad (6)$$

The spectrum of elementary fermionic excitations for superfluid state is defined by the combination $\mathscr{E}=\xi-X\Delta^{+}$. This can be represented in the Pauli matrices space as $\mathscr{E}=-\zeta-\vec{y}\vec{\tau}$ and the problem of finding ζ in the common case comes to solving the bicubic equation [8]

$$\zeta^6-\zeta^4\varepsilon_0^2+\zeta^2\left(\varepsilon_0^2\vec{b}^{\,2}-\vec{b}^{\,2}\vec{b}^{\,2}+\vec{\varepsilon}^{\,2}\right)-\left(\vec{c}\vec{b}\right)^2=0, \quad (7)$$

following which $\vec{y}$ is expressed through ζ:

$$\vec{y}+\vec{\xi}=\frac{\zeta+\xi_0}{\zeta^2-\vec{b}^2}\left(\zeta\left(\vec{\xi}-\vec{b}\right)-i\left[\vec{\xi},\vec{b}\right]\right). \qquad (8)$$

Here we have introduced the designations

$$\begin{aligned} {\Delta^{+}}^{-1}\xi^{T}\Delta^{+}&\equiv\xi-2\,\vec{b}\vec{\tau},\\ \Delta\Delta^{+}\xi^2-2\,\xi b&=\varepsilon_0^2+2\,\vec{\varepsilon}\vec{\tau}-2\vec{\xi}\vec{b}. \end{aligned} \qquad (9)$$

In the common case the process of solving (7) and the results of it are quite cumbersome but in some special cases the equation and its solving substantially simplifies, in particular in the case of unitary states.

One calls *unitary* the superfluid states for which the combination $\Delta\Delta^{+}$ is a c-number. This means equality to zero of the value $\vec{c}$ in the definition:

$$\begin{aligned} \Delta\Delta^{+}&\equiv\Delta_0\Delta_0^{*}+\vec{\Delta}\vec{\Delta}^{*}+2\,\vec{c}\vec{\tau},\\ 2\vec{c}&=\Delta_0\vec{\Delta}^{*}+\Delta_0^{*}\vec{\Delta}+i\left[\vec{\Delta},\vec{\Delta}^{*}\right]. \end{aligned} \qquad (10)$$

One can see from the last formula that unitary states satisfy the properties

$$\Delta_0^{*}=-\lambda\Delta_0, \qquad \vec{\Delta}^{*}=\lambda\vec{\Delta}, \qquad (11)$$

where λ is a phase factor, $\lambda\lambda^{*}=1$.

Taking into account $\vec{c}=0$, the bicubic equation (7) for the unitary states turns into a biquadratic one

$$\zeta^4-\varepsilon_0^2\zeta^2+\omega_0^4=0, \qquad (12)$$

where

$$\begin{aligned} \omega_0^4&=\varepsilon_0^2b^2-b^2b^2+\vec{\varepsilon}^{\,2},\\ \varepsilon_0^2&=\xi_0^2+\eta^2+\Delta\Delta^{+}\\ \vec{\varepsilon}&=\xi_0\left(\vec{\xi}-\vec{b}\right)-i\left[\vec{\xi},\vec{b}\right]. \end{aligned} \qquad (13)$$

The quasi-particle energy has a structure $\xi=\xi_0+\vec{\xi}\vec{\tau}$ where the term $\vec{\xi}\vec{\tau}$ arises because of the asymmetry α of nuclear matter which is defined (1) as a function of a difference of densities of neutrons ρ_n and protons ρ_p in the system.

In the case $\vec{\xi}\neq0$ the co-ordinate system can be chosen in the way that

$$\vec{\xi}=(0,0,\xi_3). \qquad (14)$$

At the presence of asymmetry the term ξ_3 will be in the form

$$\xi_3=\kappa\alpha, \qquad (15)$$

where κ is an aspect ratio and α is the asymmetry parameter (1).

3. DETERMINATION OF THE ANOMALOUS DENSITY MATRIX

Let us now consider the influence of asymmetry on the unitarity of superfluid states. We can do this by finding the dependence of the anomalous density matrix on the order parameter. In order to obtain the self-consistent equations it is necessary to execute the procedure of diagonalization with taking into account the non-zero value of $\vec{\xi}$.

Let us obtain the expression for $\Delta\Delta^{+}$. From (12) and (13) it follows

$$\Delta\Delta^{+}=\frac{\left(\zeta^2-\xi_0^2\right)\left(\zeta^2-\eta^2\right)}{\zeta^2-b^2}. \qquad (16)$$

So, the solution of (12) can be presented in the form

$$\zeta^2=\frac{1}{4}\left(E_{+}+\sigma E_{-}\right)^2, \quad \sigma=\pm1, \qquad (17)$$

where

$$E_{\pm}=\sqrt{\varepsilon_0^2\pm2\omega_0^2}. \qquad (18)$$

For $K=K_0+\vec{K}\vec{\tau}$ we will obtain the following expressions:

$$\begin{cases} K_0=\frac{\zeta-\xi_0}{2\zeta}\left(1+\frac{\xi_0\left(\eta^2-b^2\right)\left(\zeta+\xi_0\right)}{\left(\zeta^2-\eta^2\right)\left(\zeta^2-b^2\right)+\left(\zeta^2-\xi_0^2\right)\left(\eta^2-b^2\right)}\right)\\ \vec{K}\vec{\tau}=\frac{\zeta^2-\xi_0^2}{2\zeta}\cdot\frac{\left(\zeta^2-\eta^2\right)b-\left(\zeta^2-b^2\right)\eta}{\left(\zeta^2-\eta^2\right)\left(\zeta^2-b^2\right)+\left(\zeta^2-\xi_0^2\right)\left(\eta^2-b^2\right)}. \end{cases} \qquad (19)$$

The matrix X can be obtained by calculating the combination

$$X=\frac{X\Delta^{+}}{\Delta\Delta^{+}}\cdot\Delta. \qquad (20)$$

Using (8) (for $X\Delta^{+}$) and (16) we finally get:

$$X=\frac{\left(\zeta+\eta\right)\left(\zeta-b\right)}{\left(\zeta-\xi_0\right)\left(\zeta^2-\eta^2\right)}\cdot\Delta. \qquad (21)$$

Now let us find the matrix expression for n:

$$n=\left\{1+\exp\left[\frac{1}{T}\left(\xi-X\Delta^{+}\right)\right]\right\}^{-1}. \qquad (22)$$

Using the common formula which is correct for an arbitrary function $f\left(\alpha+\vec{\beta}\vec{\tau}\right)$ [9]:

$$f\left(\alpha+\vec{\beta}\vec{\tau}\right)=\frac{f\left(\alpha+\beta\right)+f\left(\alpha-\beta\right)}{2}+\\ +\frac{f\left(\alpha+\beta\right)-f\left(\alpha-\beta\right)}{2}\frac{\vec{\beta}}{\beta}\vec{\tau}, \quad (23)$$

from the first formula of the system

$$\begin{cases} \zeta^2 + \vec{y}^{\,2} + 2\left(\vec{y}\vec{b} + \vec{\xi}\vec{b}\right) = \varepsilon_0^2, \\ \zeta\left(\vec{y} + \vec{b}\right) + i\left[\vec{y}, \vec{b}\right] = \vec{\varepsilon} \end{cases} \qquad (24)$$

and from (8) we have

$$\vec{y}^{\,2} = \varepsilon_0^2 - \zeta^2. \qquad (25)$$

Substituting here ζ^2 from (17), we will obtain

$$\begin{aligned} &\varepsilon_0^2 - \zeta^2 = \varepsilon_0^2 - \\ &- \frac{1}{4}\left(\varepsilon_0^2 + 2\omega_0^2 + \varepsilon_0^2 - 2\omega_0^2 - \sigma\sqrt{\varepsilon_0^4 - 4\omega_0^4}\right), \end{aligned} \qquad (26)$$

and then

$$y^2 = \frac{1}{4}\left(E_+ - \sigma E_-\right)^2, \quad \zeta^2 = \frac{1}{4}\left(E_+ + \sigma E_-\right)^2. \qquad (27)$$

So,

$$\zeta + y = \kappa E_+, \quad \zeta - y = \kappa\sigma E_- \quad (\kappa = \pm 1), \qquad (28)$$

$$\begin{aligned} n\left(-\zeta - \vec{y}\vec{\tau}\right) =& \frac{n\left(-\kappa E_+\right) + n\left(-\kappa E_-\right)}{2} + \\ &+ \frac{n\left(-\kappa E_+\right) - n\left(-\kappa E_-\right)}{2}\frac{\vec{y}}{y}\vec{\tau}. \end{aligned} \qquad (29)$$

Let us introduce the designations

$$n_\pm \equiv n\left(-\kappa E_\pm\right). \qquad (30)$$

For calculating $\widetilde{n}(-\mathbf{p})$ one could take advantage of the property

$$\tau_2 \widetilde{n}\left(\mathscr{E}(-\mathbf{p})\right)\tau_2 = \Delta^+ n\left(\mathscr{E}(\mathbf{p}) - 2b\left(\mathbf{p}\right)\right)\Delta^{+^{-1}}. \qquad (31)$$

In order to demonstrate this property let us pull the Pauli matrices inside n represented in the form $\tau_2 \widetilde{n}\left(-\mathbf{p}\right)\tau_2 = n\left(\tau_2 \widetilde{\mathscr{E}}\left(-\mathbf{p}\right)\tau_2\right)$. Now let us study just the argument of n. In using the properties $\widetilde{\Delta}\left(-\mathbf{p}\right) = -\Delta(\mathbf{p})$ and $\widetilde{X}\left(-\mathbf{p}\right) = -X(\mathbf{p})$, and (9) we obtain

$$\tau_2 \widetilde{\mathscr{E}}(-\mathbf{p})\tau_2 = \Delta^+(\mathscr{E}(\mathbf{p}) - 2b)\,\Delta^{+^{-1}}. \qquad (32)$$

The matrices Δ^+ and $\Delta^{+^{-1}}$ can be pulled out of brackets in the summed series from which we get (31).

So, we obtain for $n(\mathscr{E})$ and $n(\mathscr{E} - 2b)$ such formulas:

$$\begin{aligned} n(\mathscr{E}) &= \frac{n_+ + n_-}{2} + \frac{n_+ - n_-}{2}\cdot\vec{m}\vec{\tau} \equiv \\ &\equiv N_+ + N_- \cdot \vec{m}\vec{\tau}, \\ n(\mathscr{E} - 2b) &= \frac{n_+ + n_-}{2} + \frac{n_+ - n_-}{2}\cdot\vec{m}_1\vec{\tau} \equiv \\ &\equiv N_+ + N_- \cdot \vec{m}_1\vec{\tau}, \end{aligned} \qquad (33)$$

where

$$\vec{m} = \vec{y}/y, \quad \vec{m}_1 = \left(\vec{y} + 2\vec{b}\right)\Big/ y. \qquad (34)$$

From the formula (8) it follows $\left(\vec{y} + 2\vec{b}\right)^2 = \vec{y}^{\,2}$ which simplifies the solution of the problem. Then

$$\begin{aligned} \mathscr{E} &= -\zeta - y\cdot\vec{m}\vec{\tau}, \\ \mathscr{E} - 2b &= -\zeta - y\cdot\vec{m}_1\vec{\tau}. \end{aligned} \qquad (35)$$

Now we need to calculate

$$\begin{aligned} g =& K\left(1 - n\left(\mathscr{E}\right)\right)X - \\ &- X\,\Delta^+ n\left(\mathscr{E} - 2b\right)\,\Delta^{+^{-1}}\cdot\frac{1}{1 + X^+ X}. \end{aligned} \qquad (36)$$

It is suitable to calculate this expression with using (20) and (33), from which it follows

$$\begin{aligned} g = \frac{\zeta^2 - b^2}{R}\Bigg\{&\frac{\zeta^2 - b^2}{2\zeta}\left(1 - 2N_+\right) + \\ &+ \frac{1}{y}\frac{\left(\xi_0^2 - b^2\right)\left(\eta^2 - b^2\right)}{\zeta^2 - b^2}N_- - \\ &- \left(\frac{1}{2\zeta}\left(1 - 2N_+\right) + \frac{1}{y}N_-\right)\cdot \\ &\cdot\left(\xi_0\left(\vec{\xi} - \vec{b}\right) + i\left[\vec{\xi}, \vec{b}\right]\right)\vec{\tau}\Bigg\}\Delta. \end{aligned} \qquad (37)$$

Taking into account $\vec{\xi} = (0, 0, \xi_3)$, we obtain

$$\vec{b} = \frac{\xi_3}{D}\begin{pmatrix} \Delta_1\Delta_3 - i\Delta_0\Delta_2 \\ \Delta_2\Delta_3 + i\Delta_0\Delta_1 \\ \Delta_3^2 - \Delta_0^2 \end{pmatrix}, \quad \vec{b}^{\,2} = \xi_3^2\frac{\Delta_3^2 - \Delta_0^2}{\vec{\Delta}^2 - \Delta_0^2}. \qquad (38)$$

Substituting all the elements in (37) with the obtained expressions we will get the final formulas for the components of g_i:

$$\begin{aligned} g_0 &= \tfrac{1}{W}\left\{\Delta_0\left(\zeta^2 - \xi_3^2\right)\left(\zeta^2 D - \xi_3^2\chi\right)\tfrac{1 - 2N_+}{2\zeta} - \Delta_0\xi_3^2\left(\zeta^2 - \xi_0^2\right)\psi\tfrac{N_-}{y}\right\} \\ g_1 &= \tfrac{1}{W}\left\{\begin{array}{c} \left(\zeta^2 D - \xi_3^2\chi\right)\left(\zeta^2\Delta_1 + i\xi_3\xi_0\Delta_2\right)\tfrac{1 - 2N_+}{2\zeta} + \\ + \left(\xi_3\Delta_1\left[\left(\zeta^2 - \xi_3^2\right)\chi + \xi_0^2\psi\right] - i\xi_0\Delta_2\left(\xi_3^2\chi - \zeta^2 D\right)\right)\xi_3\tfrac{N_-}{y} \end{array}\right\} \\ g_2 &= \tfrac{1}{W}\left\{\begin{array}{c} \left(\zeta^2 D - \xi_3^2\chi\right)\left(\zeta^2\Delta_2 - i\xi_3\xi_0\Delta_1\right)\tfrac{1 - 2N_+}{2\zeta} + \\ + \left(\xi_3\Delta_2\left[\left(\zeta^2 - \xi_3^2\right)\chi + \xi_0^2\psi\right] + i\xi_0\Delta_1\left(\xi_3^2\chi - \zeta^2 D\right)\right)\xi_3\tfrac{N_-}{y} \end{array}\right\} \\ g_3 &= \tfrac{1}{W}\left\{\left(\zeta^2 D - \xi_3^2\chi\right)\left(\zeta^2 - \xi_3^2\right)\tfrac{1 - 2N_+}{2\zeta}\Delta_3 - \xi_3^2\left(\zeta^2 - \xi_0^2\right)\psi\tfrac{N_-}{y}\Delta_3\right\}, \end{aligned} \qquad (39)$$

where

$$\chi \equiv \Delta_3^2 - \Delta_0^2, \;\; \psi \equiv \Delta_1^2 + \Delta_2^2, \;\; D \equiv \chi + \psi, \\ W \equiv \left(\zeta^2 - \xi_3^2\right)^2 \chi - \left(\xi_3^2 \xi_0^2 - \zeta^4\right) \psi. \qquad (40)$$

4. RESULTS AND CONCLUSIONS

Let us examine the expressions (39). One can see that the order parameters obtained from them would not satisfy the properties of unitary states (11), which follows from the proportionality of the summands in these expressions both to real and to imaginary unities. From this it follows that for a two-component Fermi liquid the considered superfluid states defined in the form (4), where all Δ_i $(i = 0...\,3)$ are not equal to zero, cannot be unitary at $\vec{\xi} \neq 0$. Though the appearance of small asymmetry leads to a small declination from unitarity of superfluid states.

In the paper we have considered the possibility of existence of unitary superfluid states in asymmetrical nuclear matter. It is shown that asymmetry breaks the unitarity of superfluid states in isospin space.

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References

1. S.V. Peletminskii. Superfluidity in a model for a Fermi liquid // *Proceedings of the Steklov Institute of Mathematics.* 1992, v. 191, p. 193–211 (Russian version: *Труды математического института им. В.А. Стеклова.* 1989, т. 191, с. 174-191).
2. A.I. Akhiezer, V.V. Krasilnikov, S.V. Peletminskiy, A.A. Yatsenko. Research on superfluidity and superconductivity on the basis of the Fermi liquid concept // *Phys. Rep.* 1994, v. 245, p. 1-110.
3. L.D. Landau. Theory of a Fermi liquid // *Zh. Eksp. Teor. Fiz.* 1956, v. 30, p. 1058-1064 (in Russian).
4. V.P. Silin. To the theory of degenerated electron liquid // *Zh. Eksp. Teor. Fiz.* 1957, v. 33, p. 495-500 (in Russian).
5. N.N. Bogolyubov. To the problem of the condition of superfluidity of nuclear matter // *Doklady AS USSR.* 1958, v. 119, p. 52-55 (in Russian).
6. A. Bohr, B.R. Mottelson, D. Pines. Possible analogy between the excitation spectra of nuclei and those of the superconducting metallic state // *Phys. Rev.* 1958, v. 110, p. 936-938.
7. A.I. Akhiezer, A.A. Isayev, S.V. Peletminsky, A.P. Rekalo, and A.A. Yatsenko. Theory of superfluidity of nuclear matter based on the Fermi-liquid approach // *Journal of Experimental and Theoretical Physics.* 1997, v. 85, p. 1-12 (Russian version: *ЖЭТФ.* 1997, т. 112, с. 3-24).
8. S.V. Peletminskii, A.A. Yatsenko and S.N. Shulga. On the possibility of simultaneous spiral and superfluid ordering in a Fermi liquid // *Low Temperature Physics.* 2004, v. 30, p. 191-198 (Russian version: *ФНТ.* 2004, т. 30, с. 261-270).
9. L.D. Landau, E.M. Lifshitz. *Quantum Mechanics. Non-relativistic theory* (3 ed.). Oxford: “Pergamon”, 1991, 691 p.

О ВЛИЯНИИ АСИММЕТРИИ ЯДЕРНОЙ МАТЕРИИ НА УНИТАРНОСТЬ СИНГЛЕТ-ТРИПЛЕТНЫХ СВЕРХТЕКУЧИХ СОСТОЯНИЙ

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Рассмотрена возможность существования унитарных сверхтекучих состояний в ядерной материи в случае, когда в матрице энергии квазичастицы присутствуют одновременно как скалярные, так и векторные компоненты, $\xi = \xi_0 + \vec{\xi}\vec{\tau}$. Векторный компонент ξ может появиться благодаря асимметрии ядерной материи (различным концентрациям составляющих ее нейтронов и протонов). Показано, что в рассмотренном случае сверхтекучие состояния становятся неунитарными. Работа выполнена на основе фермижидкостного подхода [2].

ПРО ВПЛИВ АСИМЕТРІЇ ЯДЕРНОЇ МАТЕРІЇ НА УНІТАРНІСТЬ СИНГЛЕТ-ТРИПЛЕТНИХ НАДПЛИННИХ СТАНІВ

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Розглянуто можливість існування унітарних надплинних станів у ядерній матерії у випадку, коли в матриці енергії квазічастинки присутні як скалярні, так і векторні компоненти водночас, $\xi = \xi_0 + \vec{\xi}\vec{\tau}$, причому векторний компонент ξ може з'явитися завдяки асиметрії ядерної матерії (різним концентраціям нейтронів та протонів, з яких вона складається). Показано, що у розглянутому випадку надплинні стани стають неунітарними. Роботу виконано на основі фермірідинного підходу [2].