LOW-FREQUENCY GREEN FUNCTIONS ASYMPTOTICS IN UNIAXIAL AND BIAXIAL NEMATIC LIQUID CRYSTALS

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Uniaxial and biaxial nematic liquid crystals are examples of the complicated condensed matters possessing an internal microstructure shown on macroscopic level in the form of a number of the physical phenomena and processes. In this work the dynamic behavior of the studied condensed matters in alternative external field is investigated. On the basis of the nonlinear dynamic equations with sources for uniaxial and biaxial nematics the general analytical expressions of low-frequency Green functions asymptotics are obtained and the analysis of their features in the region of small wave vectors and frequencies is carried out.

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1. INTRODUCTION

Nowadays investigation of liquid crystalline matters is of great interest. These condensed states possess the property of liquid – fluidity and spatial anisotropy – property of solid state. The essential feature of liquid crystals is the presence of internal anisotropic ordered structure of mesoscopic or nanoscopic sizes, which is shown on macroscopic level in the form of a number of the physical phenomena and processes.

The purpose of the given work is the investigation of dynamics of uniaxial and biaxial nematic liquid crystals with the internal structure taken into account in the presence of alternative external field. The basis of investigation is the use of the conception of reduced description of multiparticle states, application and development of Hamiltonian mechanics for condensed matters with internal structure and Green functions formalism.

2. DYNAMICS OF UNIAXIAL NEMATIC LIQUID CRYSTALS IN ALTERNATIVE EXTERNAL FIELD

2.1. Nematics with rod-like molecules

Using Hamiltonian approach of [1, 2] we will obtain dynamic equations of uniaxial nematic with rod-like molecules in alternative external field. When field is turned on, Hamiltonian of a system $H(t) = H + V(t)$ consists of Hamiltonian of condensed matter H and interaction with external field $V(t)$ [3]:

$$H = \int d^3x \varepsilon \left(\underline{\zeta}_a(\mathbf{x}), n_i(\mathbf{x}), \nabla n_i(\mathbf{x}), l(\mathbf{x}) \right),$$
$$V(t) = \int d^3x \xi(\mathbf{x}, t)\, b\left(\underline{\zeta}_a(\mathbf{x}), n_i(\mathbf{x}), \nabla n_i(\mathbf{x}), l(\mathbf{x}) \right), \tag{1}$$

here $\xi(\mathbf{x}, t)$ is alternative external field, $b\left(\underline{\zeta}_a, n_i, \nabla n_i, l\right)$ is an arbitrary local physical quantity, which in the region of large times becomes function of reduced description parameters, $\underline{\zeta}_a \equiv \sigma, \pi_i, \rho$ are the densities of mass ρ, momentum π_i, entropy σ, ε is energy density, n_i is unit vector of spatial anisotropy, l is length of a molecule. Dynamic equations of uniaxial nematic with rod-like molecules in alternative external field are as follows [4]:

$$\begin{aligned} &\dot{\rho} = -\nabla_i \pi_i + \eta_\rho, \quad \dot{\pi}_i = -\nabla_k t_{ik} + \eta_{\pi_i}, \\ &\dot{\sigma} = -\nabla_k(\sigma v_k) + \eta_\sigma, \\ &\dot{l} = -v_s(\mathbf{x}) \nabla_s l(\mathbf{x}) \\ &\qquad + l(\mathbf{x}) n_i(x) n_j(x) \nabla_j v_i(\mathbf{x}) + \eta_l, \\ &\dot{n}_j = -v_s \nabla_s n_j + \delta^{\perp}_{ij}(\mathbf{n}) n_k \nabla_k v_i + \eta_{n_j}. \end{aligned} \tag{2}$$

Momentum flux density looks like

$$\begin{aligned} t_{ik} = &P\delta_{ik} + \tfrac{\partial\varepsilon}{\partial\pi_k}\pi_i + \tfrac{\partial\varepsilon}{\partial\nabla_k n_l}\nabla_i n_l \\ &+ \tfrac{\partial\varepsilon}{\partial l} n_i n_k l + n_k \delta^{\perp}_{il}(\mathbf{n}) \left(\tfrac{\partial\varepsilon}{\partial n_l} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j n_l} \right). \end{aligned} \tag{3}$$

Here $P \equiv -\varepsilon + \frac{\delta H}{\delta \underline{\zeta}_a}\underline{\zeta}_a + \frac{\partial\varepsilon}{\partial\nabla_l n_i}\nabla_l n_i$ is pressure, $v_i \equiv \pi_k/\rho$ is macroscopic velocity, $\delta^{\perp}_{ik}(\mathbf{n}) = \delta_{ik} - n_i n_k$ and $\eta_{\varphi_\alpha(\mathbf{x})}(\mathbf{x}) = \{\varphi_a(\mathbf{x}), V\}$ are sources caused by external field:

$$\begin{aligned} &\eta_\rho = -\rho\tfrac{\partial b}{\partial\pi_i}\nabla_i\xi, \quad \eta_{n_i} = n_k\delta^{\perp}_{ij}(\mathbf{n})\tfrac{\partial b}{\partial\pi_j}\nabla_k\xi, \\ &\eta_l = -n_i n_j l\tfrac{\partial b}{\partial\pi_i}\nabla_j\xi, \quad \eta_\sigma = -\sigma\tfrac{\partial b}{\partial\pi_i}\nabla_i\xi, \\ &\eta_{\pi_j} = -\underline{\zeta}_a\tfrac{\partial b}{\partial\underline{\zeta}_a}\nabla_j\xi - n_j n_k l\tfrac{\partial b}{\partial l}\nabla_k\xi + \\ &\qquad + \left(\tfrac{\partial b}{\partial n_i} - \nabla_\lambda\tfrac{\partial b}{\partial\nabla_\lambda n_i}\right)\delta^{\perp}_{ji}(\mathbf{n}) n_k\nabla_k\xi. \end{aligned} \tag{4}$$

From Eqs. (2), (4) on the basis of [3] the general expression of the low-frequency asymptotics of Green

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functions in Fourier representation is obtained:

$$G_{ab}^{(+)}(\mathbf{k},\omega) = -L_i^a(\mathbf{k},\omega)\frac{D_{ij}^{-1}(\mathbf{k},\omega)}{\rho}L_j^b(-\mathbf{k},-\omega) - -\rho\frac{\partial a}{\partial \pi_i}\frac{\partial b}{\partial \pi_i}, \tag{5}$$

where

$$L_i^a(\mathbf{k},\omega) = \underline{\zeta}_a\frac{\partial a}{\partial \underline{\zeta}_a}k_i + l\frac{\partial a}{\partial l}(\mathbf{kn})\,n_i + \omega\rho\frac{\partial a}{\partial \pi_i} + + \left(\frac{\partial a}{\partial n_j} - ik_\lambda\frac{\partial a}{\partial \nabla_\lambda n_j}\right)\delta_{ij}^{\perp}(\mathbf{n})(\mathbf{kn}). \tag{6}$$

Here the notation are used:

$$\begin{aligned} &D_{ij}^{-1}(\mathbf{k},\omega) = \tfrac{1}{\Delta}\Big(\delta_{ij}\left(\omega^4 - \omega^2k^2c^2 - \omega^2T^2\right) + \\ &+\omega^2c^2k_ik_j + \omega^2T_iT_j + c^2(\mathbf{k}\times\mathbf{T})_i(\mathbf{k}\times\mathbf{T})_j\Big), \\ &\Delta(\mathbf{k},\omega) = \det D_{ij}(\mathbf{k},\omega) = \\ &= \omega^6 + \omega^4 I_4(\mathbf{k}) + \omega^2 I_2(\mathbf{k}), \\ &T_i(\mathbf{k}) \equiv c\sqrt{\lambda}(\mathbf{kn})\,n_i, \quad \lambda = \tfrac{l^2}{\rho c^2}\tfrac{\partial^2\varepsilon}{\partial l^2} > 0, \end{aligned} \tag{7}$$

where ω is frequency, $\mathbf{k}$ is wave vector, c is sound velocity in usual liquid, I_2, I_4 are some functions of $\mathbf{k}, \mathbf{n}, \lambda$. Let's note, that low-frequency Green functions asymptotics with variables which have not been connected with broken symmetry behave in the regular way at $\omega \to 0, k_{||} \to 0, k_\perp \to 0$ and do not contain features on $\omega, k_{||}, k_\perp$. Here wave vector $\mathbf{k}$ is decomposed on longitudinal $\mathbf{k}_{||} = (\mathbf{kn})\,\mathbf{n}$ and transversal $\mathbf{k}_\perp = \frac{\mathbf{k}\times\mathbf{n}}{|\mathbf{k}\times\mathbf{n}|}|\mathbf{k}\times\mathbf{n}| = |\mathbf{k}\times\mathbf{n}|\mathbf{r}$ components concerning vector $\mathbf{n}$. Let's consider Green functions like $G_{n_in_j}\left(k_{||}, k_\perp, \omega\right)$ and $G_{an_j}\left(k_{||}, k_\perp, \omega\right)$, where $a \equiv \zeta_a, l$, so one or both quantities entering into Green function connected with broken symmetry according to rotations in the configuration space. For these Green functions we receive asymptotic expressions in the cases of different sequence of limiting transitions with small $k_{||}, k_\perp$ and ω:

$$\begin{aligned} &\lim_{\omega\to 0} G_{n_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}^2}{\omega^2}r_ir_j, \\ &\lim_{k_\perp\to 0} G_{n_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}^2}{\omega^2}\delta_{ij}^{\perp}(\mathbf{n}), \\ &\lim_{\omega\to 0} G_{\rho n_i}\left(k_{||}, k_\perp, \omega\right) = -\tfrac{1}{c^2}\tfrac{k_{||}}{k_\perp}r_i, \\ &\lim_{\omega\to 0} G_{\pi_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}}{\omega}r_ir_j, \\ &\lim_{k_\perp\to 0} G_{\pi_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}}{\omega}\delta_{ij}^{\perp}(\mathbf{n}), \\ &\lim_{\omega\to 0} G_{\mathrm{ln}_i}\left(k_{||}, k_\perp, \omega\right) = \tfrac{l}{\rho\lambda c^2}\tfrac{k_{||}}{k_\perp}r_i. \end{aligned} \tag{8}$$

It is seen, that obtained asymptotics (8) depend essentially on the order of convergence $\omega, k_{||}, k_\perp$ to zero and these limits are not permutative.

2.2. Nematics with disc-like molecules

Studying of dynamic behavior of uniaxial nematic with disc-like molecules in alternative external field we will carry out similar to earlier considered case of uniaxial nematic with rod-like molecules. Dynamic equations of uniaxial nematic with disc-like molecules in this field are as follows [4]

$$\begin{aligned} &\dot\rho = -\nabla_i\pi_i + \eta_\rho, \quad \dot\pi_i = -\nabla_k t_{ik} + \eta_{\pi_i}, \\ &\dot\sigma = -\nabla_k(\sigma v_k) + \eta_\sigma, \\ &\dot d = -v_s\nabla_s d - d\delta_{lk}^{\perp}(\mathbf{n})\,\nabla_k v_l + \eta_d, \\ :\ &\dot n_j = -v_s\nabla_s n_j - n_i\delta_{j\lambda}^{\perp}(\mathbf{n})\,\nabla_\lambda v_i + \eta_{n_j}, \\ &t_{ik} = P\delta_{ik} + \tfrac{\partial\varepsilon}{\partial\pi_k}\pi_i + \tfrac{\partial\varepsilon}{\partial\nabla_k n_l}\nabla_i n_l + \tfrac{\partial\varepsilon}{\partial d}d\delta_{lk}^{\perp}(\mathbf{n}) + \\ &\quad + n_k\delta_{il}^{\perp}(\mathbf{n})\left(\tfrac{\partial\varepsilon}{\partial n_l} - \nabla_j\tfrac{\partial\varepsilon}{\partial\nabla_j n_l}\right). \end{aligned} \tag{9}$$

Sources caused by external field look like:

$$\begin{aligned} &\eta_\rho = -\rho\tfrac{\partial b}{\partial\pi_i}\nabla_i\xi, \quad \eta_{n_j} = -n_k\delta_{ij}^{\perp}(\mathbf{n})\tfrac{\partial b}{\partial\pi_i}\nabla_k\xi, \\ &\eta_d = -d\delta_{ij}^{\perp}(\mathbf{n})\tfrac{\partial b}{\partial\pi_i}\nabla_j\xi, \quad \eta_\sigma = -\sigma\tfrac{\partial b}{\partial\pi_i}\nabla_i\xi, \\ &\eta_{\pi_j} = -\underline{\zeta}_a\tfrac{\partial b}{\partial\underline{\zeta}_a}\nabla_j\xi - d\delta_{ij}^{\perp}(\mathbf{n})\tfrac{\partial b}{\partial d}\nabla_i\xi - \\ &\quad - \left(\tfrac{\partial b}{\partial n_i} - \nabla_\lambda\tfrac{\partial b}{\partial\nabla_\lambda n_i}\right)\delta_{ji}^{\perp}(\mathbf{n})\,n_k\nabla_k\xi. \end{aligned} \tag{10}$$

Here d is molecule diameter. Proceeding similarly to the scheme of the previous case with the help of (9), (11) we obtain general view of low-frequency Green functions asymptotics in the form (5), where following notation are used:

$$\begin{aligned} &L_i^a(\mathbf{k},\omega) = \underline{\zeta}_a\tfrac{\partial a}{\partial\underline{\zeta}_a}k_i + d\tfrac{\partial a}{\partial d}\delta_{ij}^{\perp}(\mathbf{n})\,k_j + \\ &+\omega\rho\tfrac{\partial a}{\partial\pi_i} + \left(\tfrac{\partial a}{\partial n_j} - ik_\lambda\tfrac{\partial a}{\partial\nabla_\lambda n_j}\right)\delta_{ij}^{\perp}(\mathbf{n})\,n_kk_k, \\ &D_{ij}^{-1}(\mathbf{k},\omega) = \tfrac{1}{\Delta}\left(\delta_{ij}\left(\omega^4 - \omega^2k^2c^2 - \omega^2R^2\right) + \right. \\ &\left.+\omega^2c^2k_ik_j\right) + \tfrac{1}{\Delta}\Big(\omega^2R_iR_j + c^2(\mathbf{k}\times\mathbf{R})_i(\mathbf{k}\times\mathbf{R})_j\Big), \\ &\Delta(\mathbf{k},\omega) = \det D_{ij}(\mathbf{k},\omega) = \\ &= \omega^6 + \omega^4 I_4(\mathbf{k}) + \omega^2 I_2(\mathbf{k}), \\ &R_i(\mathbf{k}) \equiv c\sqrt{\lambda}\delta_{il}^{\perp}(\mathbf{n})\,k_l, \quad \lambda = \tfrac{d^2}{\rho c^2}\tfrac{\partial^2\varepsilon}{\partial d^2} > 0\,. \end{aligned} \tag{11}$$

Here I_2, I_4 are some functions of $\mathbf{k}, \mathbf{n}, \lambda$. Let's note, that low-frequency Green functions asymptotics with variables which have not been connected with broken symmetry behave in the regular way at $\omega \to 0, k_{||} \to 0, k_\perp \to 0$ and do not contain features on $\omega, k_{||}, k_\perp$. Let's now consider Green functions like $G_{n_in_j}\left(k_{||}, k_\perp, \omega\right)$ and $G_{an_j}\left(k_{||}, k_\perp, \omega\right)$, where $a \equiv \zeta_a, d$, so one or both quantities entering into Green function connected with broken symmetry according to rotations in the configuration space. For these Green functions we receive asymptotic expressions in the cases of different sequence of limiting transitions with small $k_{||}, k_\perp$ and ω:

$$\begin{aligned} &\lim_{\omega\to 0} G_{n_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}^2}{\omega^2}r_ir_j, \\ &\lim_{k_\perp\to 0} G_{n_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}^2}{\omega^2}\delta_{ij}^{\perp}(\mathbf{n}), \\ &\lim_{\omega\to 0} G_{\pi_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}}{\omega}r_ir_j, \\ &\lim_{k_\perp\to 0} G_{\pi_in_j}\left(k_{||}, k_\perp, \omega\right) = \tfrac{k_{||}}{\omega}\delta_{ij}^{\perp}(\mathbf{n}), \\ &\lim_{\omega\to 0} G_{dn_i}\left(k_{||}, k_\perp, \omega\right) = -\tfrac{d}{\rho\lambda c^2}\tfrac{k_{||}}{k_\perp}r_i. \end{aligned} \tag{12}$$

It is seen, that obtained asymptotics (13) depend essentially on the order of convergence $\omega, k_{||}, k_\perp$ to zero and these limits are not permutative.

3. DYNAMICS OF BIAXIAL NEMATIC LIQUID CRYSTALS IN ALTERNATIVE EXTERNAL FIELD

3.1. Nematics with ellipsoidal molecules

In the case of biaxial nematic with ellipsoidal molecules the set of thermodynamic variables contains additionally two unit and orthogonal vectors of spatial anisotropy $\mathbf{n}(\mathbf{x})$,$\mathbf{m}(\mathbf{x})$ and three conformational parameters $u(\mathbf{x}), v(\mathbf{x}), p(\mathbf{x})$describing sizes of long and short molecule axes and an angle between them. Acting further similarly to previously considered case of uniaxial nematics, we obtain dynamic equations of biaxial nematic with ellipsoidal molecules in alternative external field:

$$\begin{aligned}
&\dot{\rho} = -\nabla_i \pi_i + \eta_\rho, \quad \dot{\pi}_i = -\nabla_k t_{ik} + \eta_{\pi_i},\\
&\dot{\sigma} = -\nabla_k (\sigma v_k) + \eta_\sigma,\\
&\dot{n}_j(\mathbf{x}) = -v_s(\mathbf{x}) \nabla_s n_j(\mathbf{x})\\
&\qquad - F_{i\lambda j}(\mathbf{x}) \nabla_\lambda v_i(\mathbf{x}) + \eta_{n_j},\\
&\dot{m}_j(\mathbf{x}) = -v_s(\mathbf{x}) \nabla_s m_j(\mathbf{x})\\
&\qquad - G_{i\lambda j}(\mathbf{x}) \nabla_\lambda v_i(\mathbf{x}) + \eta_{m_j},\\
&\dot{u}(\mathbf{x}) = -v_i(\mathbf{x}) \nabla_i u(\mathbf{x}) - F_{ij}(\mathbf{x}) \nabla_j v_i(\mathbf{x}) + \eta_u,\\
&\dot{v}(\mathbf{x}) = -v_i(\mathbf{x}) \nabla_i v(\mathbf{x}) - G_{ij}(\mathbf{x}) \nabla_j v_i(\mathbf{x}) + \eta_v,\\
&\dot{p}(\mathbf{x}) = -v_s(\mathbf{x}) \nabla_s p(\mathbf{x}) - H_{ij}(\mathbf{x}) \nabla_i v_j(\mathbf{x}) + \eta_p,
\end{aligned}$$

$$\begin{aligned}
&t_{ik} = P\delta_{ik} + \tfrac{\partial\varepsilon}{\partial\pi_k}\pi_i + \tfrac{\partial\varepsilon}{\partial\nabla_k n_l}\nabla_i n_l + \tfrac{\partial\varepsilon}{\partial\nabla_k m_l}\nabla_i m_l +\\
&+\tfrac{\partial\varepsilon}{\partial u}F_{ik} + \tfrac{\partial\varepsilon}{\partial v}G_{ik} + \tfrac{\partial\varepsilon}{\partial p}H_{ik} +\\
&+F_{ikl}\left(\tfrac{\partial\varepsilon}{\partial n_l} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j n_l}\right) + G_{ikl}\left(\tfrac{\partial\varepsilon}{\partial m_l} - \nabla_j \tfrac{\partial\varepsilon}{\partial\nabla_j m_l}\right),
\end{aligned} \tag{13}$$

where $P \equiv -\varepsilon + \frac{\delta H}{\delta \underline{\zeta}_a}\underline{\zeta}_a + \frac{\partial\varepsilon}{\partial\nabla_l n_i}\nabla_l n_i + \frac{\partial\varepsilon}{\partial\nabla_l m_i}\nabla_l m_i$ is pressure and η are sources caused by external field:

$$\begin{aligned}
&\eta_\rho = -\rho\tfrac{\partial b}{\partial\pi_i}\nabla_i\xi, \quad \eta_\sigma = -\sigma\tfrac{\partial b}{\partial\pi_i}\nabla_i\xi,\\
&\eta_{n_j} = -F_{i\lambda j}\tfrac{\partial b}{\partial\pi_i}\nabla_\lambda\xi, \quad \eta_{m_j} = -G_{i\lambda j}\tfrac{\partial b}{\partial\pi_i}\nabla_\lambda\xi,\\
&\eta_p = -H_{ij}\tfrac{\partial b}{\partial\pi_i}\nabla_j\xi, \quad \eta_u = -F_{ij}\tfrac{\partial b}{\partial\pi_i}\nabla_j\xi,\\
&\eta_v = -G_{ij}\tfrac{\partial b}{\partial\pi_i}\nabla_j\xi,\\
&\eta_{\pi_j} = -\underline{\zeta}_a\tfrac{\partial b}{\partial\underline{\zeta}_a}\nabla_j\xi - H_{ij}\tfrac{\partial b}{\partial p}\nabla_j\xi - F_{ij}\tfrac{\partial b}{\partial u}\nabla_j\xi -\\
&-G_{ij}\tfrac{\partial b}{\partial v}\nabla_j\xi - F_{i\lambda j}\left(\tfrac{\partial b}{\partial n_j} - \nabla_k\tfrac{\partial b}{\partial\nabla_k n_j}\right)\nabla_\lambda\xi -\\
&-G_{i\lambda j}\left(\tfrac{\partial b}{\partial m_j} - \nabla_k\tfrac{\partial b}{\partial\nabla_k m_j}\right)\nabla_\lambda\xi.
\end{aligned} \tag{14}$$

Here F_{ij}, G_{ij}, H_{ij} and $F_{ijk}, G_{ijk}, H_{ijk}$ are some functions of $\mathbf{n}(\mathbf{x})$, $\mathbf{m}(\mathbf{x})$ and $u(\mathbf{x}), v(\mathbf{x}), p(\mathbf{x})$. As in the earlier considered case of uniaxial nematics, we obtain the general expression of the low-frequency asymptotics of Green functions in the form (5), where the following notation are used:

$$\begin{aligned}
&L_i^a(\mathbf{k},\omega) = \underline{\zeta}_a\tfrac{\partial a}{\partial\underline{\zeta}_a}k_i + \omega\rho\tfrac{\partial a}{\partial\pi_i} + \tfrac{\partial a}{\partial p}H_{ij}k_j +\\
&+\tfrac{\partial a}{\partial u}F_{ij}k + \tfrac{\partial a}{\partial v}G_{ij}k + \left(\tfrac{\partial a}{\partial n_j} - ik_\lambda\tfrac{\partial a}{\partial\nabla_\lambda n_j}\right)F_{ikj}k_k +\\
&+\left(\tfrac{\partial a}{\partial m_j} - ik_\lambda\tfrac{\partial a}{\partial\nabla_\lambda m_j}\right)G_{ikj}k,
\end{aligned}$$

$$\begin{aligned}
&D_{ij}^{-1} = \tfrac{1}{\Delta}\left(\omega^4\delta_{ij} - \omega^2c^2\left(\mathbf{k}^2\delta_{ij} - k_ik_j\right)\right) -\\
&-\tfrac{1}{\Delta}\omega^2\left(\left(\mathbf{H}^2\delta_{ij} - H_iH_j\right) + \left(\mathbf{F}^2\delta_{ij} - F_iF_j\right) +\right.\\
&\left.+\left(\mathbf{G}^2\delta_{ij} - G_iG_j\right)\right) + \tfrac{1}{\Delta}c^2\Big((\mathbf{k}\times\mathbf{H})_i(\mathbf{k}\times\mathbf{H})_j +\\
&+(\mathbf{k}\times\mathbf{F})_i(\mathbf{k}\times\mathbf{F})_j + (\mathbf{k}\times\mathbf{G})_i(\mathbf{k}\times\mathbf{G})_j\Big) +\\
&+\tfrac{1}{\Delta}\Big((\mathbf{H}\times\mathbf{G})_i(\mathbf{H}\times\mathbf{G})_j + (\mathbf{H}\times\mathbf{F})_i\times\\
&\times(\mathbf{H}\times\mathbf{F})_j + (\mathbf{F}\times\mathbf{G})_i(\mathbf{F}\times\mathbf{G})_j\Big),\\
&\Delta = \omega^6 + \omega^4I_4 + \omega^2I_2 + I_0,
\end{aligned} \tag{15}$$

where $I_a, a = 0,2,4$ are some functions of $\mathbf{k},\mathbf{F},\mathbf{G},\mathbf{H}$ and $\mathbf{F},\mathbf{G},\mathbf{H}$ are some functions of $\mathbf{n}(\mathbf{x})$,$\mathbf{m}(\mathbf{x})$,$\mathbf{k}$ and parameters $\lambda_\alpha, \alpha = 1,2,3$:

$$\lambda_1 \equiv \frac{u^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial u^2} > 0,$$
$$\lambda_2 \equiv \frac{v^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial v^2} > 0,$$
$$\lambda_3 \equiv \frac{p^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial p^2} > 0.$$

Let's note, that low-frequency Green functions asymptotics with variables which have not been connected with broken symmetry behave in the regular way at $\omega \to 0, k \to 0$ and do not contain features on $\mathbf{k},\omega$. Let's now consider Green functions like $G_{n_in_j}(k,\omega), G_{m_im_j}(k,\omega)$ and $G_{an_j}(k,\omega), G_{am_j}(k,\omega)$, where $a \equiv \zeta_a, p, u, v$, so one or both quantities entering into Green function connected with broken symmetry according to rotations in the configuration space. For these Green functions we receive asymptotic expressions in the cases of different sequence of limiting transitions with small ω, k and $\theta \to 0, \theta \to \pi/2$:

$$\begin{aligned}
&\lim_{\theta\to\pi/2} G_{uu}(k,\omega) = \tfrac{1}{\rho}\tfrac{k^2}{\omega^2}f_{2u}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{n_ln_k}(k,\omega) = \tfrac{1}{\rho}\tfrac{k^2}{\omega^2}f_{2n}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{m_lm_k}(k,\omega) = \tfrac{1}{\rho}\tfrac{k^2}{\omega^2}f_{2m}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{\pi_iu}(k,\omega) = u\tfrac{k}{\omega}f_{\pi_iu}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{\pi_in_l}(k,\omega) = \tfrac{k}{\omega}f_{\pi_in_l}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{\pi_im_l}(k,\omega) = \tfrac{k}{\omega}f_{\pi_im_l}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{un_l}(k,\omega) = \tfrac{u}{\rho}\tfrac{k^2}{\omega^2}f_{2un_l}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{um_l}(k,\omega) = \tfrac{u}{\rho}\tfrac{k^2}{\omega^2}f_{2um_l}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi),\\
&\lim_{\theta\to\pi/2} G_{n_lm_k}(k,\omega) =\\
&\qquad = \tfrac{1}{\rho}\tfrac{k^2}{\omega^2}f_{2n_lm_k}(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi).
\end{aligned} \tag{16}$$

Here in spherical system of coordinate polar angle θ and azimuthal angle φ are defined by equalities $\mathbf{em} = \sin\theta\cos\varphi$, $\mathbf{en} = \sin\theta\sin\varphi$, $\mathbf{el} = \cos\theta$, $\mathbf{l} = \mathbf{m}\times\mathbf{n}$ and set direction of wave vector $\mathbf{e} \equiv \mathbf{k}/k$ correspond to anisotropy axes, f_1 and f_2 are some functions of parameters $\lambda_1, \lambda_2, \lambda_3$, polar and azimuthal angles θ, φ. It is seen, that obtained asymptotics (17) depend essentially on the order of $\omega \to 0, k \to 0, \theta \to 0, \theta \to \pi/2$ and these limits are not permutative.

3.2. Nematics with discoidal molecules

Dynamic equations of biaxial nematic with discoidal molecules in alternative external field are as follows:

$$\begin{array}{l}\dot{\rho}=-\nabla_i\pi_i+\eta_\rho, \quad \dot{\pi}_i=-\nabla_k t_{ik}+\eta_{\pi_i},\\ \dot{\sigma}=-\nabla_k\left(\sigma v_k\right)+\eta_\sigma,\\ \dot{n}_j\left(\mathbf{x}\right)=-v_s\left(\mathbf{x}\right)\nabla_s n_j\left(\mathbf{x}\right)-f_{i\lambda j}\left(\mathbf{x}\right)\nabla_\lambda v_i\left(\mathbf{x}\right)+\eta_{n_j},\\ \dot{m}_j\left(\mathbf{x}\right)=-v_s\left(\mathbf{x}\right)\nabla_s m_j\left(\mathbf{x}\right)-\\ \quad -g_{i\lambda j}\left(\mathbf{x}\right)\nabla_\lambda v_i\left(\mathbf{x}\right)+\eta_{m_j},\\ \dot{q}\left(\mathbf{x}\right)=-v_i\left(\mathbf{x}\right)\nabla_i q\left(\mathbf{x}\right)-f_{ij}\left(\mathbf{x}\right)\nabla_j v_i\left(\mathbf{x}\right)+\eta_q,\\ \dot{t}\left(\mathbf{x}\right)=-v_i\left(\mathbf{x}\right)\nabla_i t\left(\mathbf{x}\right)-g_{ij}\left(\mathbf{x}\right)\nabla_j v_i\left(\mathbf{x}\right)+\eta_t,\\ \dot{p}\left(\mathbf{x}\right)=-v_s\left(\mathbf{x}\right)\nabla_s p\left(\mathbf{x}\right)-h_{ij}\left(\mathbf{x}\right)\nabla_i v_j\left(\mathbf{x}\right)+\eta_p,\\ t_{ik}=P\delta_{ik}+\frac{\partial\varepsilon}{\partial\pi_k}\pi_i+\frac{\partial\varepsilon}{\partial\nabla_k n_\lambda}\nabla_i n_\lambda+\frac{\partial\varepsilon}{\partial\nabla_k m_\lambda}\nabla_i m_\lambda+\\ \quad +\frac{\partial\varepsilon}{\partial q}f_{ik}+\frac{\partial\varepsilon}{\partial t}g_{ik}+\frac{\partial\varepsilon}{\partial p}h_{ik}+\\ \quad +f_{ik\lambda}\left(\frac{\partial\varepsilon}{\partial n_\lambda}-\nabla_j\frac{\partial\varepsilon}{\partial\nabla_j n_\lambda}\right)+\\ \quad +g_{ik\lambda}\left(\frac{\partial\varepsilon}{\partial m_\lambda}-\nabla_j\frac{\partial\varepsilon}{\partial\nabla_j m_\lambda}\right).\end{array} \tag{17}$$

Here $\mathbf{n}\left(\mathbf{x}\right)$, $\mathbf{m}\left(\mathbf{x}\right)$ are unit and orthogonal vectors of spatial anisotropy and $q\left(\mathbf{x}\right), t\left(\mathbf{x}\right), p\left(\mathbf{x}\right)$ are conformational parameters describing sizes of long and short molecule axes and an angle between them. Sources look like

$$\begin{array}{l}\eta_\rho=-\rho\frac{\partial b}{\partial\pi_i}\nabla_i\xi, \quad \eta_\sigma=-\sigma\frac{\partial b}{\partial\pi_i}\nabla_i\xi,\\ \eta_{n_j}=-f_{i\lambda j}\frac{\partial b}{\partial\pi_i}\nabla_\lambda\xi, \quad \eta_{m_j}=-g_{i\lambda j}\frac{\partial b}{\partial\pi_i}\nabla_\lambda\xi,\\ \eta_p=-h_{ij}\frac{\partial b}{\partial\pi_i}\nabla_j\xi, \quad \eta_q=-f_{ij}\frac{\partial b}{\partial\pi_i}\nabla_j\xi,\\ \eta_t=-g_{ij}\frac{\partial b}{\partial\pi_i}\nabla_j\xi,\\ \eta_{\pi_j}=-\underline{\zeta}_a\frac{\partial b}{\partial\underline{\zeta}_a}\nabla_j\xi-h_{ij}\frac{\partial b}{\partial p}\nabla_j\xi-f_{ij}\frac{\partial b}{\partial q}\nabla_j\xi-\\ -g_{ij}\frac{\partial b}{\partial t}\nabla_j\xi-f_{i\lambda j}\left(\frac{\partial b}{\partial n_j}-\nabla_k\frac{\partial b}{\partial\nabla_k n_j}\right)\nabla_\lambda\xi-\\ -g_{i\lambda j}\left(\frac{\partial b}{\partial m_j}-\nabla_k\frac{\partial b}{\partial\nabla_k m_j}\right)\nabla_\lambda\xi.\end{array} \tag{18}$$

Here f_{ij}, g_{ij}, h_{ij} and $f_{ijk}, g_{ijk}, h_{ijk}$ are some functions of $\mathbf{n}\left(\mathbf{x}\right)$,$\mathbf{m}\left(\mathbf{x}\right)$ and $q\left(\mathbf{x}\right), t\left(\mathbf{x}\right), p\left(\mathbf{x}\right)$. With the help of (18), (19) we find general view of low-frequency Green functions asymptotics in the form (5), where following notation are used:

$$\begin{array}{l}L_i^a\left(\mathbf{k},\omega\right)=\underline{\zeta}_a\frac{\partial a}{\partial\underline{\zeta}_a}k_i+\omega\rho\frac{\partial a}{\partial\pi_i}+\frac{\partial a}{\partial p}h_{ij}k_j+\\ +\frac{\partial a}{\partial u}f_{ij}k+\frac{\partial a}{\partial v}g_{ij}k+\left(\frac{\partial a}{\partial n_j}-ik_\lambda\frac{\partial a}{\partial\nabla_\lambda n_j}\right)f_{ikj}k_k+\\ +\left(\frac{\partial a}{\partial m_j}-ik_\lambda\frac{\partial a}{\partial\nabla_\lambda m_j}\right)g_{ikj}k,\\ \\ D_{ij}^{-1}=\frac{1}{\Delta}\left(\omega^4\delta_{ij}-\omega^2c^2\left(\mathbf{k}^2\delta_{ij}-k_ik_j\right)\right)-\\ -\frac{1}{\Delta}\omega^2\left(\left(\mathbf{h}^2\delta_{ij}-h_ih_j\right)+\left(\mathbf{f}^2\delta_{ij}-f_if_j\right)+\right.\\ \left.+\left(\mathbf{g}^2\delta_{ij}-g_ig_j\right)\right)+\frac{1}{\Delta}c^2\left(\left(\mathbf{k}\times\mathbf{h}\right)_i\left(\mathbf{k}\times\mathbf{h}\right)_j+\right.\\ \left.+\left(\mathbf{k}\times\mathbf{f}\right)_i\left(\mathbf{k}\times\mathbf{f}\right)_j+\left(\mathbf{k}\times\mathbf{g}\right)_i\left(\mathbf{k}\times\mathbf{g}\right)_j\right)+\\ +\frac{1}{\Delta}\left(\left(\mathbf{h}\times\mathbf{g}\right)_i\left(\mathbf{h}\times\mathbf{g}\right)_j+\left(\mathbf{h}\times\mathbf{f}\right)_i\times\right.\\ \left.\times\left(\mathbf{h}\times\mathbf{f}\right)_j+\left(\mathbf{f}\times\mathbf{g}\right)_i\left(\mathbf{f}\times\mathbf{g}\right)_j\right),\\ \\ \Delta=\omega^6+\omega^4I_4+\omega^2I_2+I_0,\end{array} \tag{19}$$

where $I_a, a=0,2,4$ are some functions of $\mathbf{k}$,$\mathbf{f},\mathbf{g},\mathbf{h}$ and $\mathbf{f},\mathbf{g},\mathbf{h}$ are some functions of $\mathbf{n}\left(\mathbf{x}\right)$,$\mathbf{m}\left(\mathbf{x}\right)$,$\mathbf{k}$and parameters $\lambda_\alpha, \alpha=1,2,3$:

$$\lambda_1\equiv\frac{q^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial q^2}>0,$$

$$\lambda_2\equiv\frac{t^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial t^2}>0,$$

$$\lambda_3\equiv\frac{p^2}{\rho c^2}\frac{\partial^2\varepsilon}{\partial p^2}>0.$$

Let's note, that low-frequency Green functions asymptotics with variables which have not been connected with broken symmetry behave in the regular way at $\omega\rightarrow 0, k\rightarrow 0$ and do not contain features on $\mathbf{k},\omega$. Let's now consider Green functions like $G_{n_in_j}\left(k,\omega\right), G_{m_im_j}\left(k,\omega\right)$ and $G_{an_j}\left(k,\omega\right), G_{am_j}\left(k,\omega\right)$, where $a\equiv\zeta_a, p, q, t$, so one or both quantities entering into Green function connected with broken symmetry according to rotations in the configuration space. For these Green functions we receive asymptotic expressions in the cases of different sequence of limiting transitions with small ω, k and $\theta\rightarrow 0$, $\theta\rightarrow\pi/2$:

$$\lim_{\theta\rightarrow\pi/2}G_{n_ln_k}\left(k,\omega\right)=\frac{1}{\rho}\frac{k^2}{\omega^2}g_{2n}\left(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi\right),$$

$$\lim_{\theta\rightarrow\pi/2}G_{m_lm_k}\left(k,\omega\right)=\frac{1}{\rho}\frac{k^2}{\omega^2}g_{2m}\left(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi\right),$$

$$\lim_{\theta\rightarrow\pi/2}G_{\pi_in_l}\left(k,\omega\right)=\frac{k}{\omega}g_{\pi_in_l}\left(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi\right),$$

$$\lim_{\theta\rightarrow\pi/2}G_{\pi_im_l}\left(k,\omega\right)=\frac{k}{\omega}g_{\pi_im_l}\left(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi\right),$$

$$\lim_{\theta\rightarrow\pi/2}G_{n_lm_k}\left(k,\omega\right)=\frac{1}{\rho}\frac{k^2}{\omega^2}g_{2n_lm_k}\left(\lambda_1,\lambda_2,\lambda_3;\theta,\varphi\right). \tag{20}$$

Here g_1 and g_2 are some functions of parameters $\lambda_1,\lambda_2,\lambda_3$, polar and azimuthal angles θ,φ. It is seen, that obtained asymptotics depend essentially on the order of $\omega\rightarrow 0, k\rightarrow 0, \theta\rightarrow 0, \theta\rightarrow\pi/2$ and these limits are not permutative.

4. CONCLUSIONS

On the basis of Hamiltonian approach the dynamic theory of uniaxial and biaxial nematic liquid crystals with molecules of different geometry in alternative external field is constructed. For all types of liquid crystals nonlinear dynamic equations with sources are derived, low-frequency Green functions asymptotics are found and their features are investigated. It is shown that in uniaxial and biaxial nematics low-frequency Green functions asymptotics have peculiarities of type $1/\omega, 1/\omega^2, 1/k_\perp$, which is consistent with Bogolyubov theorem. But unlike other condensed matters with broken symmetry (see, for example [5]) coefficients of these peculiarities depend essentially on the wave vector $\mathbf{k}$ and frequency ω.

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НИЗКОЧАСТОТНЫЕ АСИМПТОТИКИ ФУНКЦИЙ ГРИНА В ОДНООСНЫХ И ДВУХОСНЫХ НЕМАТИЧЕСКИХ ЖИДКИХ КРИСТАЛЛАХ

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Одноосные и двухосные нематические жидкие кристаллы являются примерами сложных конденсированных сред, обладающих внутренней микроструктурой, проявляющейся на макроскопическом уровне в виде ряда физических явлений и процессов. В работе исследовано динамическое поведение таких конденсированных сред во внешнем переменном поле. На основе нелинейных динамических уравнений с источниками для одноосных и двухосных нематиков получены общие аналитические выражения для низкочастотных асимптотик функций Грина, и проведен анализ их особенностей в области малых волновых векторов и частот.

НИЗЬКОЧАСТОТНІ АСИМПТОТІКИ ФУНКЦІЙ ГРІНА В ОДНОВІСНИХ ТА ДВОВІСНИХ НЕМАТИЧНИХ РІДКИХ КРИСТАЛАХ

В.Т. Мацкевич

Одновісні та двовісні нематичні рідкі кристали є прикладами складних конденсованих середовищ, що мають внутрішню мікроструктуру, яка виявляється на макроскопічному рівні у ряді фізичних явищ та процесів. В роботі досліджено динамічну поведінку таких конденсованих середовищ у зовнішньому змінному полі. На основі нелінійних динамічних рівнянь з джерелами для одновісних та двовісних нематиків отримано загальні аналітичні вирази для низькочастотних асимптотік функцій Гріна, та проведено аналіз їх особливостей в області малих хвильових векторів та частот.