

ON THE HYDRODYNAMICS OF POLARON GAS IN THE BOGOLYUBOV REDUCED DESCRIPTION METHOD

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On the basis of a generalization of the Chapman-Enskog method a new approach to derivation of hydrodynamic equations for weak density polaron gas has been elaborated taking into account the relaxation of temperature and velocity in the system. Non-locality of the collision integral of the used kinetic equation was taken into account also. Both circumstances lead to some modification of the standard transport theory.

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1. INTRODUCTION

In our paper [1] relaxation phenomena in spatially uniform low density polaron gas have been investigated. The consideration was conducted in standard model in which phonons form an ideal equilibrium gas with the temperature T_0 . Electron spin, phonon polarization and zone structure of electron spectrum are neglected; energy of electron is chosen as $\varepsilon_p = p^2/2m$.

In [1] kinetic equation for strong non-uniform states of polaron (low density polaron gas) in weak electric field has been obtained too and has the form

$$\frac{\partial f_p(x, t)}{\partial t} = -\frac{p_n}{m} \frac{\partial f_p(x, t)}{\partial x_n} + eE_n(x) \frac{\partial f_p(x, t)}{\partial p_n} + I_p(x, f(t)). \quad (1)$$

Here collision integral $I_p(x, f)$ is given by the formula

$$\begin{aligned} I_p(x, f) = & \frac{1}{4\pi^3 \hbar^2} \int_{-\infty}^0 d\tau \int d^3k g_k^2 \left\{ n_k f_{p-k\hbar} \left(x + k \frac{\hbar\tau}{2m} \right) - (1 + n_k) f_p \left(x - k \frac{\hbar\tau}{2m} \right) \right\} \times \\ & \times \cos \frac{\tau}{\hbar} (\varepsilon_{p-k\hbar} + \hbar\omega_k - \varepsilon_p) + \\ & + \frac{1}{4\pi^3 \hbar^2} \int_{-\infty}^0 d\tau \int d^3k g_k^2 \left\{ (1 + n_k) f_{p+k\hbar} \left(x - k \frac{\hbar\tau}{2m} \right) - n_k f_p \left(x + k \frac{\hbar\tau}{2m} \right) \right\} \times \\ & \times \cos \frac{\tau}{\hbar} (\varepsilon_p + \hbar\omega_k - \varepsilon_{p+k\hbar}), \end{aligned} \quad (2)$$

where $n_k = (e^{\frac{\hbar\omega_k}{T_0}} - 1)^{-1}$ is the Planck distribution for phonons, e is module of electron charge, $\hbar\omega_k$ is energy of a phonon. This result does not imply that gradients of the polaron distribution function $f_p(x, t)$ are small. In present paper on the basis of this kinetic equation and with the help of a generalization of the Chapman-Enskog method transport phenomena in polaron gas are discussed. In contrast to the standard theory we use kinetic equation with the collision integral which is expanded in a series in gradients of the polaron distribution function

$$+ \int d^3p' M_n(p, p') \frac{\partial f_{p'}(x)}{\partial x_n} + \dots \quad (3)$$

Also we do not assume that the distribution function $f_p(x, t)$ in hydrodynamic states in zero order in gradients approximation coincides with the local Maxwell distribution.

2. BASIC EQUATIONS

Transport phenomena in the polaron gas are discussed in the framework of its hydrodynamics. According to the kinetic equation laws of conservation (variation) of the gas mass, energy and momentum have the form

$$\frac{\partial \sigma}{\partial t} = -\frac{\partial \pi_n}{\partial x_n},$$

$$I_p(x, f) \equiv \int d^3p' M(p, p') f_{p'}(x) +$$

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$$\begin{aligned}\frac{\partial \varepsilon}{\partial t} &= -\frac{\partial q_n(f)}{\partial x_n} - \frac{e}{m} \pi_n E_n + R_0(f), \\ \frac{\partial \pi_l}{\partial t} &= -\frac{\partial t_{ln}(f)}{\partial x_n} - \frac{e}{m} \sigma E_l + R_l(f)\end{aligned}\quad (4)$$

where mass, energy and momentum densities are defined by the formulas

$$\begin{aligned}\sigma &= m \int d^3 p f_p, & \pi_l &= \int d^3 p p_l f_p, \\ \varepsilon &= \int d^3 p \varepsilon_p f_p.\end{aligned}\quad (5)$$

Flux densities of energy $q_n(x, f)$, momentum $t_{nl}(x, f)$ and sources $R_0(x, f), R_l(x, f)$ are given by expressions:

$$\begin{aligned}t_{ln}(x, f) &= \int d^3 p p_l \frac{\partial \varepsilon_p}{\partial p_n} f_p(x), \\ q_n(x, f) &= \int d^3 p \varepsilon_p \frac{\partial \varepsilon_p}{\partial p_n} f_p(x); \\ R_0(x, f) &= \int d^3 p \varepsilon_p I_p(x, f), \\ R_l(x, f) &= \int d^3 p p_l I_p(x, f).\end{aligned}\quad (6)$$

Basic hydrodynamic parameters, temperature $T(x, t)$, velocity $u_l(x, t)$ and number of particles density $n(x, t)$ of the system, are defined by usual formulas

$$\varepsilon = \frac{3}{2m} \sigma T + \frac{1}{2} \sigma u^2, \quad \pi_l = \sigma u_l, \quad \sigma = nm. \quad (7)$$

Equations (4) give hydrodynamic equations if their right sides are expressed through these variables $\xi_\mu(x, t) : \xi_0(x, t) = T(x, t), \xi_l(x, t) = u_l(x, t), \xi_4(x, t) = n(x, t)$. This is possible if after some time τ_0 the polaron distribution function has the structure

$$f_p(x, t) \xrightarrow[t \gg \tau_0]{} f_p(x, \xi(t)), \quad (8)$$

where $f_p(x, \xi)$ is a functional of the variables $\xi_\mu(x)$. This relation (called the functional hypothesis) is the basis of the Chapman-Enskog method which is a special case of the Bogolyubov reduced description method (see, for example, [2]). Time τ_0 in our consideration is assumed satisfying the condition $\tau_0 \ll \tau_T, \tau_u$ where τ_T, τ_u are relaxation times of the temperature and velocity of the polaron gas. As a result, the hydrodynamic equations can be written as

$$\frac{\partial \xi_\mu(x, t)}{\partial t} = L_\mu(x, f(\xi(t))), \quad (9)$$

where $L_\mu(x, f)$ are some functionals of $f_p(x)$. For the distribution function $f_p(x, \xi)$ from the kinetic equation (1) with (8) we obtain the equation

$$\begin{aligned}\sum_\mu \int d^3 x' \frac{\delta f_p(x, \xi)}{\delta \xi_\mu(x')} L_\mu(x', f(\xi)) &= -\frac{p_n}{m} \frac{\partial f_p(x, \xi)}{\partial x_n} + \\ &+ e E_n(x) \frac{\partial f_p(x, \xi)}{\partial p_n} + I_p(x, f(\xi)).\end{aligned}\quad (10)$$

Definitions (6) of the parameters $\xi_\mu(x)$ which describe state of the system give additional conditions to this equation

$$\begin{aligned}\int d^3 p f_p(x, \xi) p_l &= mn(x) u_l(x), \quad \int d^3 p f_p(x, \xi) = n(x), \\ \int d^3 p f_p(x, \xi) \varepsilon_p &= \frac{3}{2} n(x) T(x) + \frac{1}{2} mn(x) u(x)^2.\end{aligned}\quad (11)$$

The solution of the equation (10) taking into account conditions (11) we found in the form of a double series in the gradients of the parameters $\xi_\mu(x)$ (g is their small parameter) and small parameter ε defined by the estimations

$$\begin{aligned}u_l(x, t) &\sim \varepsilon, \quad T(x, t) - T_0 \sim \varepsilon, \quad E_l(x) \sim \varepsilon; \\ \frac{\partial v_n(x, t)}{\partial x_l} &\sim g \varepsilon, \quad \frac{\partial T(x, t)}{\partial x_l} \sim g \varepsilon, \quad \frac{\partial n}{\partial x_l} \sim g,\end{aligned}\quad (12)$$

which describes the proximity of the system to equilibrium ($A^{(m)}$ is contribution of the order g^m to A , $A^{(m, n)}$ is contribution of the order $g^m \varepsilon^n$ to A).

Zero order in gradients approximation $f_p^{(0)}$ has been investigated in our paper [1]. It was established that the main contributions to this distribution function have the form

$$\begin{aligned}f_p^{(0)} &= w_p^o + w_p^o \{A(p) p_n v_n + C(p) p_n E_n + \\ &+ B(p)(T - T_0)\} + O(g^0 \varepsilon^2); \\ w_p^o(\xi) &\equiv \frac{n}{(2\pi m T_0)^{3/2}} e^{-\frac{p^2}{2m T_0}}.\end{aligned}\quad (13)$$

Functions $A(p), B(p), C(p)$ satisfy the equations

$$\begin{aligned}\lambda_u A(p) p_n &= \int d^3 p' K(p, p') A(p') p'_n, \\ \langle \varepsilon_p A(p) \rangle &= 3/2;\end{aligned}\quad (14)$$

$$\begin{aligned}\lambda_T B(p) &= \int d^3 p' K(p, p') B(p'), \\ \langle B(p) \rangle &= 0, \quad \langle \varepsilon_p B(p) \rangle = 3/2;\end{aligned}\quad (15)$$

$$\begin{aligned}\{\mu \lambda_u A(p) - e/m T_0\} p_n &= \int d^3 p' K(p, p') C(p') p'_n, \\ \langle \varepsilon_p C(p) \rangle &= 0,\end{aligned}\quad (16)$$

where the notation

$$\langle g(p) \rangle \equiv \int d^3 p w_p^o g(p)$$

is introduced. In Eqs. (14)–(16) instead of the kernel $M(p, p')$ defined by the formula (3) the kernel $K(p, p')$ is used

$$M(p, p') w_{p'}^o \equiv -w_p^o K(p, p'). \quad (17)$$

In standard approach (see, for example, [3, 4]) the distribution function $f_p^{(0)}(x, \xi)$ is not calculated but assumed to be equal to the local Maxwell distribution $w_p(\xi(x))$ where

$$w_p(\xi) \equiv \frac{n}{(2\pi m T)^{3/2}} e^{-\frac{(p-mu)^2}{2m T}}. \quad (18)$$

The main in ε contributions to the distribution function of the first order in gradients $f_p^{(1)}$ have the structure

$$f_p^{(1)} = w_p^o \{ D(p)p_n + Q_{nl}(p)u_l + R(p)p_n(T - T_0) + S_{nl}(p)E_l \} \frac{\partial n}{\partial x_n} + w_p^o \left\{ F_{nl}(p) \frac{\partial u_n}{\partial x_l} + G(p)p_l \frac{\partial T}{\partial x_l} + H_{nl}(p) \frac{\partial E_l}{\partial x_n} \right\} + O(g\varepsilon^2). \quad (19)$$

This expression describes dissipative processes in the system. Function $D(p)$ gives the main contribution to $f_p^{(1)}$ and functions

$$G(p), \quad F_{nl}(p) \equiv F_1(p)\delta_{nl} + F_2(p)\Delta_{nl}(p) \quad (20)$$

($\Delta_{nl}(p) \equiv p_n p_l - \frac{1}{3}p^2\delta_{nl}$) allow to calculate viscosity and heat conductivity of the system. They satisfy the integral equations with additional conditions:

$$\{ a A(p) + \alpha(p) - \frac{1}{m} \} p_n = \int d^3 p' K(p, p') D(p') p'_n, \quad \langle \varepsilon_p D(p) \rangle = 0; \quad (21)$$

$$\{ \lambda_T G(p) - b_1 A(p) - \frac{1}{m} B(p) + B_1(p) \} p_n = \int d^3 p' K(p, p') G(p') p'_n, \quad \langle \varepsilon_p G(p) \rangle = 0; \quad (22)$$

$$A_1(p) - a_1 B(p) - \frac{1}{3} A(p) p^2 + 1 = \int d^3 p' K(p, p') F_1(p'), \quad \langle F_1(p) \rangle = 0, \quad \langle \varepsilon_p F_1(p) \rangle = 0; \quad (23)$$

$$-\frac{1}{m} A(p) \Delta_{nl}(p) = \int d^3 p' K(p, p') F_2(p') \Delta_{nl}(p'). \quad (24)$$

Here functions $\alpha(p), A_1(p), B_1(p)$ are defined by the formulas

$$\int d^3 p' M_n(p, p') w_{p'}^o \equiv w_p^o \alpha(p) p_n, \quad \int d^3 p' M_n(p, p') w_{p'}^o A(p') \equiv w_p^o A_1(p) p_n, \quad \int d^3 p' M_n(p, p') w_{p'}^o B(p') \equiv w_p^o B_1(p) p_n \quad (25)$$

and are absent in the standard theory because they take into account non-locality of our collision integral (see (1), (2)). Taking into account additional conditions for functions $A(p), B(p)$ (see (14), (15), scalar values μ (mobility of the polaron in a steady state), a, a_1, b_1 in equations (16), (21)–(23) are expressed through solutions of these equations. Therefore, these expressions are not needed for solution of integral equations (16), (21)–(23).

Neglecting of the dissipative processes, time equations for temperature and velocity can be written in the form

$$\begin{aligned} \frac{\partial T}{\partial t} &= -\lambda_T(T - T_0) + a_1 \frac{\partial u_l}{\partial x_l} + a_2 \frac{\partial E_l}{\partial x_l} + \\ &+ (a_3 u_l + a_4 E_l) \frac{\partial n}{\partial x_l} + O(g^0 \varepsilon^2, g^1 \varepsilon^2), \\ \frac{\partial u_l}{\partial t} &= -\lambda_u(u_l - \mu E_l) + b_1 \frac{\partial T}{\partial x_l} + b_2(T - T_0) \frac{\partial n}{\partial x_l} + \\ &+ O(g^0 \varepsilon^2, g^1 \varepsilon^2), \end{aligned} \quad (26)$$

where $a_1, \dots, a_4, b_1, b_2, \mu$ are scalar values some of which can be calculated from equations (16), (21)–(23). The kernel $K(p, p')$ of the integral equations (14)–(16), (21)–(24) has important properties which can be expressed in the terms of the bilinear form

$$\{ g(p), h(p) \} = \int d^3 p d^3 p' w_p^o w_{p'}^o g(p) K(p, p') h(p'), \quad (27)$$

that gives

$$\{ g(p), h(p) \} = \{ h(p), g(p) \}, \{ g(p), g(p) \} \geq 0. \quad (28)$$

These formulas show that eigenvalues λ_T, λ_u of operator with the kernel $K(p, p')$ are positive. According to (26) they describe relaxation phenomena in the system in the spatially homogeneous states (the above mentioned relaxation times $\tau_T = \lambda_T^{-1}$, $\tau_u = \lambda_u^{-1}$).

Expressions for energy and momentum fluxes taking into account dissipative contributions have the form

$$\begin{aligned} q_l &= n(c_1 u_l + c_2 E_l) + c_3 \frac{\partial n}{\partial x_l} - \kappa \frac{\partial T}{\partial x_l} + O(g^0 \varepsilon^2, g^1 \varepsilon^2), \\ t_{nl} &= nT \delta_{nl} - \eta \left(\frac{\partial u_n}{\partial x_l} \right)^s - \zeta \delta_{nl} \frac{\partial u_m}{\partial x_m} + \\ &+ d_1 \left(\frac{\partial E_n}{\partial x_l} \right)^s + d_2 \delta_{nl} \frac{\partial E_m}{\partial x_m} + \\ &+ d_3 \left(E_n \frac{\partial n}{\partial x_l} \right)^s + d_4 \delta_{nl} E_m \frac{\partial n}{\partial x_m} + O(g^0 \varepsilon^2, g^1 \varepsilon^2), \end{aligned} \quad (29)$$

where η, ζ are viscosities, κ is heat conductivity; $c_1, c_2, c_3, d_1, d_2, d_3, d_4$ are some coefficients;

$$(A_{nl})^s \equiv A_{nl} + A_{ln} - \frac{2}{3} \delta_{nl} A_{mm}.$$

3. CONCLUSIONS

On the basis of a generalization of the Chapman-Enskog method a new approach to derivation of hydrodynamic equations for weak density polaron gas has been elaborated taking into account the relaxation of temperature and velocity in the system. Non-locality of the collision integral of the used kinetic equation was taken into account also. Both circumstances lead to some modification of the standard theory [3, 4]. Solution of the obtained integral equation

with the help of expansion in the Sonine polynomial series will be discussed in another paper (in spatially uniform states this was done in [1]).

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К ГИДРОДИНАМИКЕ ПОЛЯРОННОГО ГАЗА В МЕТОДЕ СОКРАЩЕННОГО ОПИСАНИЯ БОГОЛЮБОВА

С.А. Соколовский

На основе обобщения метода Чепмена-Энскога разработан новый подход к выводу уравнений гидродинамики для поляронного газа малой плотности с учетом релаксации скорости и температуры системы. Учтена также нелокальность интеграла столкновений используемого кинетического уравнения. Оба обстоятельства ведут к модификации стандартной теории переноса.

ДО ГІДРОДИНАМІКИ ПОЛЯРОННОГО ГАЗУ В МЕТОДІ СКОРОЧЕНОГО ОПИСУ БОГОЛЮБОВА

С.О. Соколовський

На основі узагальнення метода Чепмена-Енскога розроблено новий підхід до виведення рівнянь гідродинаміки поляронного газу малої густини з урахуванням релаксації швидкості та температури. Також врахована нелокальність інтеграла зіткнень кінетичного рівняння, що використовується. Обидві обставини ведуть до модифікації стандартної теорії переносу.