GAUSSIAN FLUCTUATION FIELD OF ELECTRICAL POLARIZATION IN STRONGLY HEATED DIELECTRIC MEDIA

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(Received November 1, 2011)

The spatially nonuniform gaussian random field which describes fluctuations of electrical polarization in dielectric medium is built. It is done for the case when the space temperature distribution exists. This filed is the source of the thermal electromagnetic field by which the thermal radiation conduction is realized.

PACS: 05.40.+j

1. INTRODUCTION

Theoretical description of thermal radiation conduction in solid media is built traditionally on the basis of phenomenological reasons connected with geometrical optics (the radiation transfer theory). Microscopic theory of this phenomenon is absent up to now. Such a situation with the thermal radiation conduction in solid media is essentially differed from the qualitative point of view from the analogous phenomena in gases and generally speaking from that is occured in liquids. The availability of strong bonds between molecules (ions) in solid medium leads to such a mechanism of heat transfer which influences the temperature distribution in it. In frames of microscopic consideration, the heat transfer in solids is realized by transformations of thermal electromagnetic photons into thermal phonons and otherwise. In the same time the terms corresponding directly to such transformations are absent in the microscopic hamiltonian describing the interaction between molecules (ions) of the medium solid shell and the electromagnetic field. Transitions pointed out should appear in the “effective hamiltonian” of the system of solid shell excitations due to strong connection between molecules (ions). In the absence of microscopic theory, in the case when there are visible deviations in frameworks of the traditional approach on the basis of geometrical optics and we must built up a more detailed theory, it is necessary to introduce explicitly the electromagnetic field in the theory of thermal radiation conductance. It is generated by fluctuations of electromagnetic thermodynamic characteristics.

The foundations of theoretical study of thermal radiation conductance in solids on the basis of the thermodynamic fluctuation theory have been done in the work [1] (see also [2]). The representation of the fluctuation thermal electromagnetic radiation was introduced in it. In addition, the semi-phenomenological approach to the description of this radiation on the basis the fluctuation-dissipation theorem in frameworks of non-equilibrium thermodynamics was developed.

At once, this theory was formulated with the overlarge generality in the cited work. It was not possible to calculate explicitly the divergence of the energy flux density corresponding to thermal electromagnetic field for concrete types of solid media when it is averaged on thermal fluctuations and it was a functional on the temperature distribution. Just it is the value which is present in the evolution equation describing the heat transfer in medium.

It appeared that the consecutive application of representations proposed in works [1,2], with the necessity demands more detailed information about its physical nature. The idea of building of such a theory in that particular case when the medium is dielectric (or it is high-resistive semiconductor with covalent chemical bonds) have been done by authors in [3] (see also [4]). The essential element of this theory is the stochastic model of the field of spatially distributed thermal fluctuations of electrical polarization in medium. A strict mathematical construction of such a field is fulfilled in the present work.

2. STOCHASTIC MODEL OF THE HEAT RADIATION CONDUCTIVITY

The main idea of the building of the heat radiation conductance fluctuation theory consists in the introduction of thermal electromagnetic field, governed by Maxwell equations, connected with the medium under consideration. At this situation the field is generated by sources which have the thermal origin and, therefore, it should be connected with random fluctuations of electrodynamic characteristics of the

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medium. Due to this reason, equations of the field motion are stochastic. It leads to the necessity of introduction of an effective damping according to the fluctuation-dissipation theorem [2]. It compensates the antidamping of field values by fluctuations and, due to this, it is connected rigorously with the statistical dispersion of fluctuations.

We consider the fluctuation of electromagnetic field in media of the type pointed out in the introduction. Any visible fluctuations of electrical charge (and electrical current) are absent in not very small volumes of such media where thermodynamic description is applicable. Besides, the availability of covalent chemical bonds guarantee the absence of visible fluctuations of magnetization in volumes pointed out. Thus, thermal fluctuations of electrical polarization are main source of thermal electromagnetic radiation in medium of the type under consideration. Therefore, the thermal electromagnetic field $\mathbf{E}(\mathbf{r},t)$, $\mathbf{H}(\mathbf{r},t)$ should obey the Maxwell equation system:

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -[\nabla, \mathbf{E}]\,, \quad \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} = [\nabla, \mathbf{H}]\,, \qquad (1)$$

$$(\nabla, \mathbf{B}) = 0\,, \quad (\nabla, \mathbf{D}) = 0 \qquad (2)$$

in all space points with radius vectors $\mathbf{r}$. Here, $\mathbf{B} = \mu\mathbf{H}$ with the constant magnetic permeability μ and $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$. We suppose that the medium under consideration is isotropic and it is such that it is possible to disregard the time-dispersion of magnetic part in electromagnetic radiation for waves in red and infra-red spectrum regions which are connected with the heat transfer. The stochastic source in the equation system (1),(2) and the term connected with the damping, existing according to the fluctuation-dissipation theorem, are included in the electrical polarization $\mathbf{P}$. This vector consists of two parts:

$$\mathbf{P}(\mathbf{r},\omega) = \chi(\omega)\mathbf{E}(\mathbf{r},\omega) + \tilde{\mathbf{P}}(\mathbf{r},\omega), \qquad (3)$$

according to our model where $\chi(\omega)$ is the ordinary high-frequency electrical susceptibility of the medium (the spatial dispersion of thermal radiation is negligibly small),

$$\mathbf{P}(\mathbf{r},\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega t}\mathbf{P}(\mathbf{r},t)dt\,,$$

$\mathbf{E}(\mathbf{r},\omega)$ is the corresponding temporal Fourier image of electrical field and $\tilde{\mathbf{P}}(\mathbf{r},\omega)$ is the stochastic source describing fluctuations of the electrical polarization. At this time, the damping of thermal electromagnetic field is connected with the negativity of imaginary part of the susceptibility $\chi(\omega)$.

3. GAUSSIAN FIELD OF THERMAL FLUCTUATIONS

We take the following model of random field $\tilde{P}_k(\mathbf{r},\omega) = U(\hbar\omega/\kappa T)\varphi_k(\mathbf{r},\omega)$ where $U(\cdot)$ is the phenomenological field amplitude having the information about quantum origination of thermal electromagnetic radiation in it. It is submitted to only one condition, namely, it decreases to zero by sufficiently fast way in the region of large frequencies. The second factor presents a "standard" short-acting complex-valued vector of gaussian random field with zero average. Its gaussian property is connected with small values of fluctuations that it describes. The correlation radius is equal by the order to the average distance between medium molecules. In our theory, this gaussian field is characterized by the correlation function

$$\langle \varphi_k(\mathbf{r},\omega)\varphi^*_{k'}(\mathbf{r}',\omega')\rangle = (2\pi)^{-1}\delta(\omega-\omega')g_{kk'}(\mathbf{r}-\mathbf{r}') \qquad (4)$$

where the δ-function points out that the field temporal dependence represents "white noise", the dependence of the correlation function $g_{kk'}(\mathbf{r}-\mathbf{r}')$ on the $\mathbf{r}$ difference is the reflection of the stochastic spatial uniformity of the field $\varphi_k(\mathbf{r},\omega)$ and this function should be taken in such a form that it ensures the stochastic isotropy of the field $\varphi_k(\mathbf{r},\omega)$. The field $P_k(\mathbf{r},\omega)$ is not spatially uniform since the amplitude U depends on the temperature value $T(\mathbf{r},t)$ at the point $\mathbf{r}$ at time moment t. Thus, this field additionally depends on the "slow" time t as on the parameter. Due to the equation $(\nabla,\mathbf{D}) = 0$ or, equivalently to

$$\varepsilon(\omega)(\nabla,\mathbf{E}(\mathbf{r},\omega)) + 4\pi\nabla_k U(\hbar\omega/\kappa T(\mathbf{r},t))\varphi_k(\mathbf{r},\omega) = 0\,,$$

the field $\varphi_k(\mathbf{r},\omega)$ should satisfy the condition $\nabla_k\varphi_k(\mathbf{r},\omega) = 0$.

We notice that the question about the choice of fluctuation field model with the divergence equal to zero was not set in the Rytov theory [2]. It is connected with the fact that though effective fluctuation fluxes were introduced in that theory, it did not require their continuity.

Let us set the question about the existence of the gaussian field $\varphi_k(\mathbf{r},\omega)$ which has zero divergence strictly (not in average) $\nabla_l\varphi_l(\mathbf{r}) = 0$. Let $g_{l,l'}(\mathbf{r}-\mathbf{r}') = \langle\varphi_l(\mathbf{r})\varphi^*_{l'}(\mathbf{r}')\rangle$ be the field correlation function and

$$h_{l,l'}(\mathbf{q}) = \frac{1}{(2\pi)^3}\int_{\mathbf{R}^3} g_{l,l'}(\mathbf{r})e^{-i(\mathbf{q},\mathbf{r})}d\mathbf{r}$$

its Fourier image. The matrix function $g_{l,l'}(\mathbf{r})$ should be positively defined. It means that the value of corresponding matrix function $h_{l,l'}(\mathbf{q})$ is the positive matrix in each point $\mathbf{q}$, i.e. it takes place $\xi_l\xi_{l'}h_{l,l'}(\mathbf{q}) \geq 0$ for each vector $\xi_m; m = 1,2,3$. In order the gaussian field $\varphi_l(\mathbf{r})$ has zero divergence, it is necessary and sufficient that $q_lq_{l'}h_{l,l'}(\mathbf{q}) = 0$. It points out that the matrix $h_{l,l'}(\mathbf{q})$ is degenerate in each point $\mathbf{q}$. Therefore, in particular, it is not proportional to the Kronecker symbol and, consequently, the correlation function $g_{l,l'}(\mathbf{r})$ has no correlations along the direction. Therefore, it is not proportional to $\delta_{l,l'}\delta(\mathbf{r})$. Let us describe the asymptotic structure of correlation function assuming the existence of a small parameter r_* having the length dimensionality.

The condition of stochastic isotropy of random field $\varphi_l(\mathbf{r})$ leads to the fact that the function $h_{l,l'}(\mathbf{q})$ has the following general form $h_{l,l'}(\mathbf{q}) = \mathbf{q}^2 a((r_*\mathbf{q})^2)\delta_{l,l'} - q_l q_{l'} b((r_*\mathbf{q})^2)$ when some external vectors are absent. Here, the functions $a(\cdot)$ and $b(\cdot)$ are defined for a scalar physically dimensionless positive variable. The condition $\nabla_l \varphi_l(\mathbf{r}) = 0$ leads to $a \equiv b$. Then the negativity of the matrix $h_{l,l'}(\mathbf{q})$ demands $a(\mathbf{q}) \geq 0$ since $\xi_l \xi_{l'} h_{l,l'}(\mathbf{q}) = a \cdot (q^2\xi^2 - (\xi_l q_l)^2) \geq 0$ only in this case.

We express now the correlation function $g_{l,l'}(\mathbf{r})$ in terms of the function $a(\cdot)$,

$$g_{l,l'}(\mathbf{r}) = \int_{\mathbf{R}^3} e^{i(\mathbf{q},\mathbf{r})} a((r_*\mathbf{q})^2)\left(\mathbf{q}^2\delta_{l,l'} - q_l q_{l'}\right) d\mathbf{q} = $$

$$= A\delta_{l,l'} + B\frac{r_l r_{l'}}{\mathbf{r}^2},$$

where A and B are functions of scalar positive variable which is the ratio $r^2/r_*^2 \equiv \lambda^2$. Let us find integral representations for them. Calculating the trace in both sides of this formula, we find

$$3A + B = \frac{4\pi}{r_*^5}\int_0^\infty q^4 a(q^2)\left(\int_{-1}^{1} e^{iq\lambda\theta} d\theta\right) dq\,.$$

By the way, we find, by summing both sides of the formula with the multiplier $r_l r_{l'}$,

$$A + B = \frac{2\pi}{r_*^5}\int_0^\infty q^4 a(q^2)\left(\int_{-1}^{1} e^{iq\lambda\theta}(1-\theta^2) d\theta\right) dq\,.$$

From the obtained equation system for the functions A and B, calculating internal integrals and supposing infinite differentiability of the function $a(\cdot)$, we obtain $A = -B(1 + O(\lambda^{-1}))$ far from the point $\mathbf{r} = 0$ and, besides, we set that both functions are fast vanishing on r/r_*. At $\mathbf{r} = 0$, their values are equal

$$A = -B(1 + O(\lambda^{-1})) = \frac{8\pi}{3r_*^5}C\,, \quad C = \int_0^\infty q^4 a(q^2) dq\,.$$

Thus, due to smallness of the parameter r_*, we have

$$g_{l,l'}(\mathbf{r}) = \mathrm{const}\,\delta(\mathbf{r})\left(\delta_{l,l'} - \frac{r_l r_{l'}}{r^2}\right)(1 + O(\lambda^{-1}))\,. \quad (5)$$

4. CONCLUSION

We have shown in this work that the form of correlation function of thermal source should be essentially changed due to the account of the divergence absence condition for the stochastic electrical induction in the thermal electromagnetic radiation field. It points out that the approach proposed in [1, 2] when the thermal radiation conductivity theory is built should be modified.

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ГАУССОВСКОЕ ПОЛЕ ФЛУКТУАЦИЙ ЭЛЕКТРИЧЕСКОЙ ПОЛЯРИЗАЦИИ В СИЛЬНО РАЗОГРЕТЫХ ДИЭЛЕКТРИЧЕСКИХ СРЕДАХ

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Построено пространственно неоднородное гауссовское случайное поле, которое описывает флуктуации электрической поляризации среды при наличии пространственно неоднородного распределения температуры. Это поле служит источником теплового электромагнитного поля, посредством которого осуществляется радиационно-кондуктивный теплообмен.

ГАУССІВСЬКЕ ПОЛЕ ФЛУКТУАЦІЙ ЕЛЕКТРИЧНОЇ ПОЛЯРИЗАЦІЇ У СИЛЬНО РОЗІГРІТИХ ДІЕЛЕКТРИЧНИХ СЕРЕДОВИЩАХ

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Побудовано просторово неоднорідне гауссівське випадкове поле, яке описує флуктуації електричної поляризації середовища у випадку, коли існує просторово неоднорідний розподіл температури. Це поле є джерелом теплового електромагнітного поля, завдяки якому діє фізичний механізм радіаційно-кондуктивного теплообміну.