POLARIZED PHOTON EMISSION FROM RELATIVISTIC PLANAR CHANNELED POSITRONS IN CRYSTAL

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Radiation emitted from high-energy planar channeled positrons in crystal is studied in the framework of quantum electrodynamics to calculate the polarization characteristics of this radiation. Under our approach the general expression is derived for the amplitude of polarized photon emission from arbitrarily polarized relativistic planar channeled positrons in oriented crystal. The analytical expressions for the Stokes parameters of emitted photons are derived, computer codes are developed and polarization characteristics are calculated for the frequencies that are most interesting for the sources of polarized high-energy photons.

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1. INTRODUCTION

For nuclear physics research one should possess photon beams of maximum intensity and maximum polarization in the range of the giant resonance at energies 10... 20 MeV. A restricted number of methods of polarized photon generation, which exist today, do not satisfy these requirements completely [1-4]. At the same time it is known that in this range of energy the intensity of radiation from 1... 2 GeV channeled electrons is at maximum. Therefore, it is interesting to study the polarization of radiation under channeling and to estimate the possibility of the creation of photon beams with high polarization degree using off-axis collimation of photons under relativistic charged particles channeling in crystals.

The aim of this work is to derive the formulae for the Stokes parameters characterizing this radiation. It is known that in the channeling regime the particles momentum forms some small angle θ with respect to the crystal plane or to the crystal axes, which must be less than the critical Lindhard angle θ_c. For angles $\theta > 5\theta_c$ the agreement between the Coherent Bremsstrahlung theory and the experiments is good. We consider a relativistic charged particle incident onto a crystal approximately parallel to one of the crystal planes. In the planar channeling case for positively charged positrons, the channel is between the crystal planes, while for negatively charged electrons, the channel is provided by the crystal planes, because channeled positrons are pushed away from the atomic planes whereas channeled electrons are focused around the planes. This channel is the source of a potential well in the direction transverse to the particle‘s motion, which gives rise to transversely bound states for the particle. Transitions to lower-energy states lead to the phenomenon known as Channeling Radiation (CR). We obtain the formulae for the corresponding Stokes parameters, which characterize the linear and circular polarization of the CR from arbitrary polarized particles as the function of the set of variables $(\vec{p}_1, \vec{\varsigma}, \vec{k}, \vec{e})$. This set gives the angular dependence of the polarization of the emitted radiation. The calculation of the CR process is carried out by using the rules of quantum electrodynamics. The following analysis utilizes the methods used in [5-7] and based on the approach which was developed in [8,9].

2. WAVE FUNCTIONS, TRANSVERSE POTENTIAL AND RADIATION AMPLITUDE

A relativistic particle moving in a potential $U(x)$ periodic in the x direction, which is normal to the channeling planes, is described by the time-independent Dirac equation. Partitioning the wave function $\Psi(\vec{r})$, which is the solution of this equation, into large and small components

$$\Psi(\vec{r}) = \begin{pmatrix} \varphi(\vec{r}) \\ \chi(\vec{r}) \end{pmatrix} \qquad (1)$$

leads to a Pauli-type equation for the large components $\varphi(\vec{r})$. Since the potential is independent of y and z, the solution of the last equation can be written in the form:

$$\varphi(\vec{r}) = \sqrt{\frac{E+m}{2E}} \exp(i\vec{p}_{||}\vec{r}_{||}) f(x) w, \qquad (2)$$

where $w^* w = 1$.

This formulation allows us to transform a Pauli-type equation into a one-dimensional, relativistic Schrodinger equation for $f(x)$ with a relativistic particle mass:

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$$\left(-\frac{1}{2\gamma m}\frac{d^2}{dx^2}+U(x)\right)f_n(x)=\varepsilon_n f_n(x), \qquad (3)$$

where $\gamma = E/m$ is the relativistic factor and ε_n is the transverse energy level of the particle. The matrix element for CR is defined by

$$\begin{aligned}&M_{21}=\int \Psi_2^*\vec{\alpha}\vec{e}^*\exp(-i\vec{k}\vec{r})\Psi_1 d\vec{r}=\\&=\int(\varphi_2^*\exp(-i\vec{k}\vec{r})\vec{\sigma}\vec{e}^*\chi_1+\chi_2^*\exp(-i\vec{k}\vec{r})\vec{\sigma}\vec{e}^*\varphi_1)d\vec{r},\end{aligned} \qquad (4)$$

where $\vec{\alpha}$ and $\vec{\sigma}$ are the Dirac and Pauli matrices respectively, $\vec{k}$ is the photon momentum, $\vec{e}$ is the photon polarization vector. Then, substituting the above found wave function $\Psi(\vec{r})$ in the expression for the matrix element, one finds for the absolute square of the CR amplitude:

$$\begin{aligned}|M_{21}|^2=Cw_1^*\vec{e_1}\left(\vec{A}^*-i\left[\vec{B}^*\vec{\sigma}\right]\right)w_2\\ \times w_2^*\vec{e_2}^*\left(\vec{A}^*+i\left[\vec{B}^*\vec{\sigma}\right]\right)w_1,\end{aligned} \qquad (5)$$

where $C=(2\pi)^4\frac{(E+m)(E'+m)}{4EE'}\delta^2\left(\vec{p_\parallel}-\vec{p}'-\vec{k_\parallel}\right)$, and $\vec{A},\ \vec{B},\ I_1,\ I_2$ are given by the following expressions:

$$\begin{aligned}&A_x=2I_2(1+\frac{\omega}{2E'}),\ \ A_y=0,\ \ A_z=2I_1(1+\frac{\omega}{2E'}),\\&B_x=\frac{\omega}{E}(\theta\cdot I_1\cos\varphi-I_2),\\&B_y=\frac{\omega}{E'}\theta\cdot I_1(1+\frac{\omega}{E})\sin\varphi,\ \ B_z=\frac{\omega}{E'}\cdot\frac{m}{E}\cdot I_1,\\&I_1=\int\exp(-ik_x x)\cdot f_2^*(x)\cdot f_1(x)dx,\\&I_2=-\frac{i}{E}\int\exp(-ik_x x)\cdot f_2^*(x)\frac{df_1(x)}{dx}dx.\end{aligned} \qquad (6)$$

3. POLARIZATION CHARACTERISTICS

In our analysis we use the condition $\omega \ll E$, which is correct for this CR case. We also do not take into account here the interaction between the particle‘s spin and the potential of planes. For description of polarization, we use here a set of vectors $\{\vec{e_1},\ \vec{e_2},\ \vec{n}\}$, which are used in [7] and can be expressed via vectors $\vec{p_1}=\vec{p}-\vec{n}(\vec{n}\cdot\vec{p})$, $\vec{p_1}'=\vec{p}'-\vec{n}(\vec{n}\cdot\vec{p}')$ ($\vec{p}$, $\vec{p}'$ are the momentum of the particle before and after radiation respectively, and $\vec{n}$ is the direction in which photon is emitted) in the next form:

$$\vec{e_1}=\frac{\vec{p_1}}{|\vec{p_1}|},\ \vec{e_2}=\frac{\vec{p_1}^2\vec{p_1}'-\vec{p_1}\left(\vec{p_1}\cdot\vec{p_1}'\right)}{|\vec{p_1}|\sqrt{\vec{p_1}^2\vec{p_1}'^2-\left(\vec{p_1}\cdot\vec{p_1}'\right)^2}}. \qquad (7)$$

The set of vectors $\{\vec{e_1},\ \vec{e_2},\ \vec{n}\}$ forms an orthogonal basis and vector $\vec{e_1}$ lies in the radiation plane $(\vec{k},\vec{p})$. These vectors are related with $\theta,\ \varphi$ (spherical coordinates of the system in which the spectrum and angular characteristics are calculated; here relativistic particle moves along z direction and azimuthal angle φ is counted out of x direction, which is normal to the channeling planes, $\theta \ll 1$) by the next relations:

$$\begin{aligned}&\vec{e}_1=(-\cos\varphi,\ -\sin\varphi,\ \theta\ ),\\&\vec{e}_2=(\ \sin\varphi,\ -\cos\varphi,\ 0\ ),\\&\bar{n}=(\ \theta\cos\varphi,\ \theta\sin\varphi,\ \cos\theta\ ).\end{aligned} \qquad (8)$$

An arbitrary vector $\vec{R}$=(R_x,R_y,R_z) in the coordinate system $\{\vec{e}_1,\vec{e}_2,\vec{n}\}$ is written in the form:

$$\begin{aligned}&R_1\approx -R_x\cos\varphi-R_y\sin\varphi+R_z\theta,\\&R_2\approx R_x\sin\varphi-R_y\cos\varphi,\\&R_z\approx R_x\theta\cos\varphi+R_y\theta\sin\varphi+R_z.\end{aligned} \qquad (9)$$

Introducing the density matrices for relativistic particle and photon and after corresponding calculations we find the next general expressions of the Stokes parameters for the outgoing photon:

$$\begin{aligned}&\xi_1=(8/\Sigma L^e)(1+\omega/E')(\theta\cdot ReI_1\cdot I_2^*-\\&\quad-|I_2|^2\cos\varphi)\cdot\sin\varphi,\\&\xi_2=\varsigma_3\,(8/\Sigma L^e)\,(2\omega/E')\,(1+2\omega/E')\,2\theta\times\\&\quad\times\mathrm{Re}(I_1^*I_2)\cos\varphi\\&\quad-\varsigma_3\,(8/\Sigma L^e)\,(2\omega/E')\,(1+2\omega/E')\,|I_1|^2\,\theta^2-\\&\quad-\varsigma_3\,(8/\Sigma L^e)\,(2\omega/E')\,(1+2\omega/E')\,|I_2|^2-\\&\quad-\varsigma_3\,(8/\Sigma L^e)\left(\omega^2m^2/E'^2E^2\right)|I_1|^2-\\&\quad-\varsigma_2\,(8/\Sigma L^e)\,(2\omega m/E'E)\,\mathrm{Re}(I_1^*I_2)\sin\varphi-\\&\quad-\varsigma_1\,(8/\Sigma L^e)\,(2\omega m/E'E)\,|I_1|^2\,\theta+\\&\quad+\varsigma_1\,(8/\Sigma L^e)\,(2\omega m/E'E)\,\mathrm{Re}(I_1^*I_2)\cos\varphi,\\&\xi_3=(4/\Sigma L^e)\,(1+2\omega/E')\,|I_1|^2\,\theta^2+\\&\quad+(4/\Sigma L^e)\,(1+2\omega/E')\,|I_1|^2\cos 2\varphi-\\&\quad-(4/\Sigma L^e)\,(1+2\omega/E')\,2\theta\mathrm{Re}(I_1^*I_2)\cos\varphi,\end{aligned} \qquad (10)$$

where E and $E'=E-\omega$ are the energy of relativistic particle before and after radiation, respectively, m is the electron mass, ω is the energy of emitted photon, ζ_1,ζ_2,ζ_3 are the components of the unit spin vector $\vec{\varsigma}$ of the initial particle given in the coordinate system $\{\vec{e}_1,\vec{e}_2,\vec{n}\}$. A normalization factor ΣL^e is determined by the formula:

$$\begin{aligned}&\Sigma L^e=4\left(1+\omega/E'+\omega^2/2E'^2\right)\theta^2\,|I_1|^2+\\&+4\left(1+\omega/E'+\omega^2/2E'^2\right)|I_2|^2-\\&-8\left(1+\omega/E'+\omega^2/2E'^2\right)\theta\mathrm{Re}(I_1\times I_2^*)\cos\varphi+\\&+\left(\omega^2m^2/2E'^2E^2\right)|I_1|^2\,.\end{aligned} \qquad (11)$$

We can obtain from the conservation laws of energy and momentum next relation between the direction of $\vec{k}$, the difference of transverse energies before and after radiation, and radiation frequency:

$$\theta^2=\frac{2E(E-\omega)(\varepsilon_n-\varepsilon_{n'})-m^2\omega}{E\omega(E-\omega\cos^2\varphi)}. \qquad (12)$$

It may be shown, that in the cases of photons with energy $\omega \ll E$ there is the following relation for integrals I_1, I_2:

$$I_2=I_1\left(\frac{\varepsilon_n-\varepsilon_{n'}}{k_x}+\frac{k_x}{2E}\right). \qquad (13)$$

Then, using (12), the last expression may be written in the form:

$$I_2 = I_1 \frac{\gamma^{-2} + \theta^2 (1 - \frac{\omega^2}{E^2} \cos^2 \varphi)}{2\theta (1 - \omega/E) \cos \varphi}. \qquad (14)$$

This allows us to eliminate I_1, I_2 from (11). Thus in this case photon polarization is independent of the planar-continuum potential. As in bremsstrahlung, it is easy to see from (11) that circularly polarized CR can arise only from a polarized particle and the circular polarization of the photon beam is proportional to ω/E. In general, the circular polarization from longitudinally polarized particle is considerably greater than from transversely polarized particle, exactly in the same way as occur in the case of bremsstrahlung. The linear polarization of CR is not dependent upon the particle‘s spin and its degree is given by

$$P = \sqrt{\xi_1^2 + \xi_3^2}. \qquad (15)$$

Now we can obtain the final analytical expressions for the Stokes parameters and for the degree of linear polarization. These formulae are rather complicated and are not given here.

4. CONCLUSIONS

The results of the numerical calculations under these formulae for E =10 GeV and ω =10 MeV are presented in Figs. 1 and 2. They show that the degree of linear polarization of CR close to the maximum $P \approx 1$ in the greater part of observation angles and the planar CR will be almost completely linearly polarized in the direction normal to the channeling plane. Thus using developed method one can find the regions of angles where maximum linear polarization is observed and under off-axis collimation one can practically get photon beams with these characteristics. The circular polarization under these conditions is always to be a small of order ~1%. These results are in good agreement with the analysis of work [6].

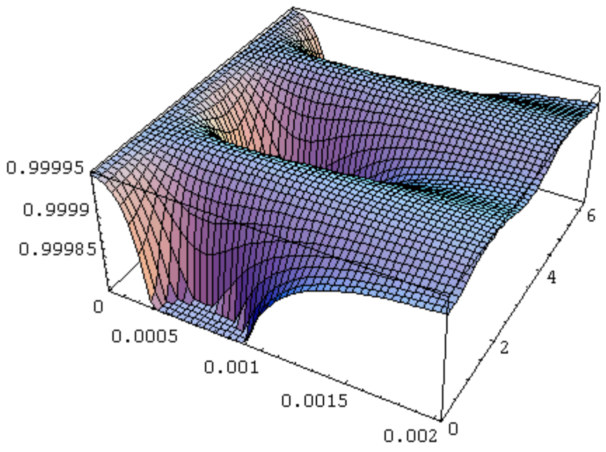

Fig. 1. *Surface plot of the function* $P(\theta, \varphi) = \sqrt{\xi_1^2 + \xi_3^2}$ *in the case of* $E = 10\,GeV$, $\omega = 10\,MeV$ *for Si crystal* $< 110 >$, $T = 293\,K$

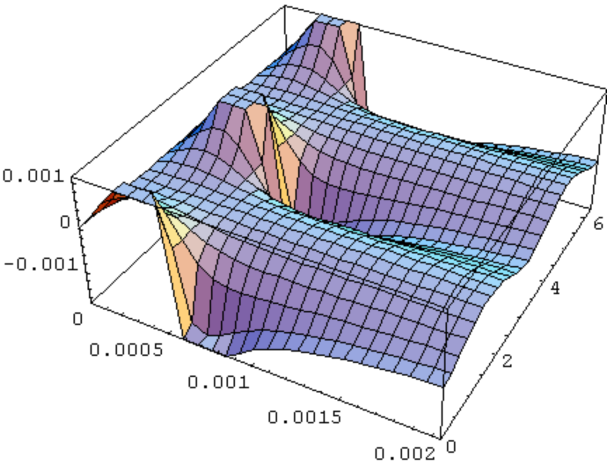

Fig. 2. *Surface plot of the function* $\xi_2(\theta, \varphi)$, $(\zeta_1 = \zeta_2 = 0, \zeta_3 = 1)$ *in the case of* $E = 10\,GeV$, $\omega = 10\,MeV$ *for Si crystal* $< 110 >$, $T = 293\,K$

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ПОЛЯРИЗАЦИЯ ИЗЛУЧЕНИЯ РЕЛЯТИВИСТСКИМИ ПОЗИТРОНАМИ ПРИ ПЛОСКОСТНОМ КАНАЛИРОВАНИИ В КРИСТАЛЛЕ

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В рамках квантовой электродинамики исследуется излучение релятивистских позитронов при плоскостном каналировании в кристалле. Получены аналитические выражения для параметров Стокса испущенных фотонов. Разработаны программы и проведен численный расчет поляризационных характеристик для частот излучения, где интенсивность излучения максимальна.

ПОЛЯРИЗАЦІЯ ВИПРОМІНЮВАННЯ ВІД РЕЛЯТИВІСТСЬКИХ ПОЗИТРОНІВ ПРИ ПЛОЩИННОМУ КАНАЛІРУВАННІ В КРИСТАЛАХ

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В рамках квантової електродинаміки розглянуте електромагнітне випромінювання площинно-каналіруючих релятивістських позитронів із кристалічної мішені. Отримані аналітичні вирази для параметрів Стокса випроміненого фотона. Розроблені програми і проведений чисельний розрахунок поляризаційних характеристик для частот, де інтенсивність випромінювання має максимум.