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Wave Diffraction by Periodic Multilayered Structures

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We offer a review of present day state of an approach to constructing of the theory of wave diffraction by multi-layered periodic partially transparent screens. A key point of the approach is definition and using of an operator of reflection of corresponded semi-infinite screen sets.

Introduction

We offer a review of present day state of a novel approach introduced in [1] to constructing the theory of electromagnetic wave interaction with periodic sets of screens. A periodic set is understood either as an infinite structure of equidistant planar screens, or as a semi-infinite or finite part of such a structure. The spectral operator of reflection $\hat{R}$ by a semi-infinite periodic structure is most essential in the developed theory. Actually the properties of a structure consisted of a finite number of equidistant screens can be found, if an effective description for field diffraction by boundaries of semi-infinite structure exists.

In finding the reflection operator $\hat{R}$ of a semi-infinite periodic screen set, the specific shift symmetry of such structure is employed. Actually the diffraction properties of the structure will be not changed if one or any finite number of boundary screens has been cut off. The said property of symmetry allows to derive an equation for the operator $\hat{R}$ on the assumption of the known spectral scattering operators of a single screen, which forms the building element in a semi-infinite set.

A field incident on a semi-infinite structure excites there an eigenwaves of the corresponding infinite periodic structure. The transmission operator, which permits finding the vector of spectral amplitudes of the excited eigenfield, can be expressed by $\hat{R}$. The eigenwave field can be studied and the equation for finding the propagation constants for these waves can be obtained using the known operator $\hat{R}$. Furthermore, if the eigenwave in a semi-infinite structure is propagating towards the free-space boundary, the reflection and transmission operators for such a field can also be expressed by $\hat{R}$. The operator method has been used to obtain the reflection and transmission operators for a periodic structure with finite number of screens. These operators can also be expressed by $\hat{R}$. Thus the knowledge of operator $\hat{R}$ allows obtaining a completely characterization of an electromagnetic properties of an infinite periodic screen set, as well as of its semi-infinite or finite-layered parts.

The developed theory has been used for the detailed study of the diffraction properties of periodic structures. Those structures can be composed of dielectric layers or of semi-transparent anisotropic screens which are dense strip gratings (including those of finite thickness), as well as of screens which are strip gratings operating in a multiwave mode. This theory has been applied also to a study of electromagnetic field transformation at the junction between regular and diaphragm waveguides, as well as of the field in a waveguide with finite number of diaphragms. This approach has also allowed studying the diffraction properties of multilayered sets of double periodic plane screens. Such structures are an effective model of artificial media.

1. Operator Method for Problems of Wave Diffraction by Planar Screens

The boundary value problem of electromagnetic wave diffraction is usually brought into a functional equation

$$(\hat{A}x)(\vec{r},\tau) = b(\vec{r},\tau), \qquad \vec{r} \in \Gamma, \qquad (1.1)$$

where $x(\vec{r},\tau)$ is the unknown coordinate and time function defining the diffracted field, $b(\vec{r},\tau)$ is the known function connected with the incident field, Γ is the obstacle boundary, and $\hat{A}$ means linear operator. To obtain the solution of the problem

$$x(\vec{r},\tau) = (\hat{A}^{-1}b)(\vec{r},\tau) \qquad (1.2)$$

the operator $\hat{A}$ must be inverted. The operator $\hat{A}^{-1}$ involves all information contained in the formulation of the problem which is necessary for its solution.

1.1. Generalized Scattering Matrix for a One Periodic Structure

As an example, consider the plane electromagnetic wave diffraction by a planar one periodic screen (see Fig. 1). Let the incident wave is linearly polarized (either $\vec{E}$ or $\vec{H}$ vector is parallel to the direction of the screen homogeneity). We shall assume harmonic time dependence $e^{-i\omega\tau}$ of field, which is omitted in the following. When a plane wave

$$u^{inc}(y,z) = e^{ik(y\sin\alpha + z\cos\alpha)} \qquad (1.3)$$

is incident on a periodic structure, the transmitted electromagnetic field is a superposition of space harmonics with discrete spatial spectrum

$$u^t(y,z) = \sum_{n=-\infty}^{\infty} b_n^t(h_0) e^{ih_n y + i\gamma_n z}, \qquad z > a, \qquad (1.4)$$

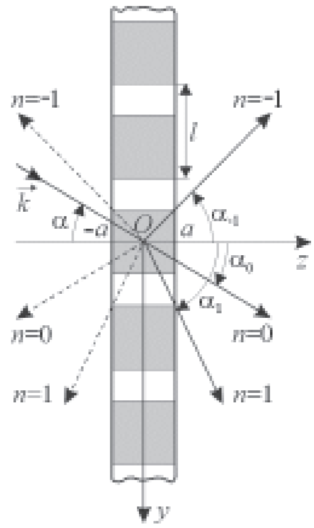

Fig. 1. *Plane wave incidence on a periodic structure*

where $h_0 = k\sin\alpha$, $h_n = h_0 + 2\pi n/l$, $\gamma_n = \sqrt{k^2 - h_n^2}$, and $\mathrm{Im}\gamma_n \ge 0$ if $\mathrm{Im}\gamma_n = 0$, then $\mathrm{Re}\gamma_n > 0$, an index t marks transmitted field variables. If γ_n is real, the n space harmonic propagates in the half-space $z > a$ at an angle α_n with respect to the positive Oz–axis, which is defined by $\tan\alpha_n = h_n/\gamma_n$. If $|h_n| > k$, the n space harmonic is a slow wave propagated along the Oy–axis. Its field decreases exponentially with distance from the surface of periodic structure. Therefore these space harmonics, which do not propagate in any non-zero angle direction from the screen, are inhomogeneous plane waves, whose field is localized near the surface of periodic structure.

Assume that the incident field is a homogeneous or inhomogeneous plane wave

$$u_m^{inc}(y,z) = e^{ih_m y + i\gamma_m z}, \quad (m = 0, \pm 1, \pm 2, ...). \qquad (1.5)$$

Substituting h_0 in (1.4) by h_m and proceeding to another summation index $s = m + n$, we obtain

$$u_m^t(y,z) = \sum_{s=-\infty}^{\infty} b_{s-m}(h_m) e^{ih_s y + i\gamma_s z}. \qquad (1.6)$$

It follows from (1.6), that when the plane wave (1.3) is incident on the periodic screen at an angle α, and another plane wave (1.5) is incident on the same screen from the direction which coincides with the propagation direction for any space harmonic, the sets of propagation directions for the waves of the transmitted field spectra appear to be the same.

All properties of the transmitted field explained above do also hold for the field reflected by the periodic structure. The propagation directions of the space harmonics of the same index of the reflected and of the transmitted fields are symmetric to the plane $z=0$.

The amplitude column-vector of the space harmonics of the transmitted field corresponds to each of the plane waves of unit amplitude $u_m^i(y,z)$ (in the case of an inhomogeneous wave, the field shows an unit amplitude at $z=0$). We construct now the infinite matrix $t=\|b_n(h_m)\|_{-\infty}^{\infty}=\|b_{nm}\|_{-\infty}^{\infty}$ and call it as the generalized periodic structure transmission matrix. With the t-matrix it is easy to obtain the transmitted field, if a superposition of waves in the form of (1.5) is incident on a periodic structure. Assume, that the wave amplitudes in this superposition define a column-vector $\{b_m^i\}_{m=-\infty}^{\infty}$. Then the vector of the transmitted field amplitudes can be found from the formula

$$b^t=tb^i=\left\{\sum_{m=-\infty}^{\infty}b_{nm}b_m^i\right\}_{n=-\infty}^{\infty}.$$

The infinite t-matrix determines a linear operator $\hat{t}$, which relates to each amplitude vector b^i of the incident field the b^t–vector of the transmitted field amplitudes. A generalized reflection matrix is similarly built up.

From now on, we will consider that operator $\hat{t}$ and reflection operator $\hat{r}$, solving the corresponding boundary value problem (1.1), are known.

1.2. Operator Method for Solving the Problem of Wave Diffraction by Two Periodic Screens

Coincidence in space harmonic propagation directions for the diffracted field, if plane waves with propagation constants h_m $(m=0,\pm1;\pm2\ ...)$ along the Oy–axis are incident on a periodic structure, is very important. For any system of parallel plane periodic structures, possessing different geometry and electric properties in the general but identical periodicity, this feature results in an electromagnetic field which shows a spectrum of plane waves, whose propagation directions are the same as those of the partial plane waves which are diffracted by each of the individual screens of the system.

Now let us consider the problem of plane wave diffraction by two parallel periodic screens (see Fig. 2). A plane wave with amplitude b_0^i is incident on this system from the half-space $z<-a_1$ at an angle α. Let the periodic structures show identical period l, let they be symmetric with respect to planes $z=0$ and Δz, respectively, and assume that the transmission and reflection operators are known for each structure (they are marked with indexes 1 and 2).

Denote the amplitude vector of the reflected field space harmonics by A, that of the field transmitted through the structure by B^t, and denote the amplitude vectors of the space harmonics in between the periodic screens which propagate (or are exponentially decaying) in positive or negative direction of the Oz–axis by C and D, respectively. The transmission and reflection operators which have been men-

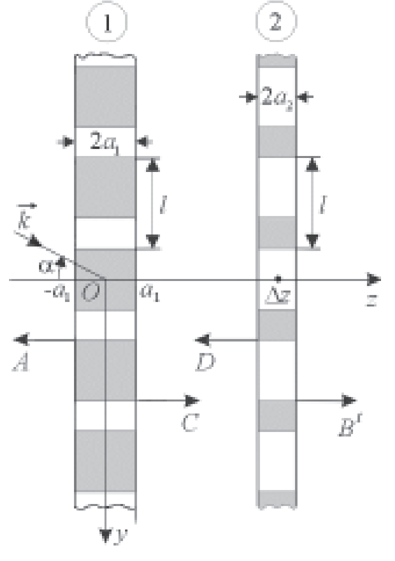

Fig. 2. *Plane wave incidence on a two-layer periodic structure*

tioned above, are written with respect to the screen located at $z = 0$. Hence the reflected and transmitted fields at the first screen (A and C) should be written related to the xyz–system, and fields D and B^t (generated by the second screen) should accordingly be written related to the $xy\tilde{z}$–system, where $\tilde{z} = z - \Delta z$. In order to study the C–field interaction with the second screen and the D–field interaction with the first one, these fields should be rearranged in terms of the obstacle-related coordinate systems. Obviously, to transform the $\sum_{s=-\infty}^{\infty} f_s^+ e^{ih_s y + i\gamma_s z}$–field from the xyz–system to the $xy\tilde{z}$–system and the $\sum_{s=-\infty}^{\infty} f_s^- e^{ih_s y - i\gamma_s \tilde{z}}$–field, accordingly, to the xyz coordinate system, vectors $\left\{f_s^+\right\}_{-\infty}^{\infty}$ and $\left\{f_s^-\right\}_{-\infty}^{\infty}$ should be multiplied by the same diagonal matrix $\varphi = \left\| \delta_{mn} e^{i\gamma_n \Delta z} \right\|_{n,m=-\infty}^{\infty}$, where δ_{mn} means Kronecker delta.

Let the vector describing the incident wave be called B^i. Its only non-zero component is b_0^i. The complex amplitude vectors of the fields are connected to the incident field vector B^i by

$$\begin{cases} A = \hat{r}_1 B^i + \hat{t}_1 \hat{\varphi} D, \\ B^t = \hat{t}_2 \hat{\varphi} C, \\ C = \hat{t}_1 B^i + \hat{r}_1 \hat{\varphi} D, \\ D = \hat{r}_2 \hat{\varphi} C, \end{cases} \qquad (1.7)$$

where indexes 1 and 2 mark the operators corresponded to the first and the second screen, $\hat{\varphi}$ is operator corresponded to the matrix φ. These equations yield the amplitude vectors of the transmitted and reflected fields

$$\begin{aligned} B^t &= \hat{t}_2 \hat{\varphi} (I - \hat{r}_1 \hat{\varphi} \hat{r}_2 \hat{\varphi})^{-1} \hat{t}_1 B^i, \\ A &= \left[\hat{r}_1 + \hat{t}_1 \hat{\varphi} \hat{r}_2 \hat{\varphi} (I - \hat{r}_1 \hat{\varphi} \hat{r}_2 \hat{\varphi})^{-1} \hat{t}_1 \right] B^i. \end{aligned} \qquad (1.8)$$

The solution of the diffraction problem in the form of (1.8) actually determines the transmission and reflection operators for the whole system. In principle, by applying the same algorithm with new operators, we may also solve the diffraction problem for a system, whose element consists of a two-layer periodic structure.

1.3. Operator Method as One of the Realizations of the Half-Inversion Method

In diffraction theory, the operator method is one of the effective realizations of a fairly general approach to the solution of the related boundary value problems which is called the method of operator partial inversion (sometimes called as “the half-inversion method”). The idea of the half-inversion method lies in that the diffraction problem, formulated as operator Eq. (1.1), is solved in two steps. As first step, the $\hat{A}$–operator is represented as sum of two operators $\hat{A} = \hat{A}_1 + \hat{A}_2$ in a way that the inverse to the $\hat{A}_1$–operator can be found, and that the $\hat{A}_2$–operator shows a small norm under the physical conditions of the diffraction problem. The second step then is obtaining the solution of equation

$$x = -\hat{A}_1^{-1} \hat{A}_2 x + \hat{A}_1^{-1} b. \qquad (1.9)$$

For example, in solving the problem of plane wave diffraction by a strip grating with the so-called “method of Riemann-Hilbert” [2], operator $\hat{A}_1$ is a static part of operator $\hat{A}$. Hence $\hat{A}_2$ is small in its norm, provided the wavelength exceeds the structure period. If in this problem the static part of the $\hat{A}$–operator which is related to a single strip [3] is taken for the $\hat{A}_1$–operator, the $\hat{A}_2$–operator will be the smaller in its norm, the smaller the strip width-to-wavelength ratio is and the bigger the grating period compared to the wavelength is.

The application of the operator method assumes that part of the operator, which corresponds to diffraction by one of the obstacles included in the studied structure, is subject for inversion. Represent third Eq. (1.7) as

$$C = \hat{t}_1 B^i + \hat{P}_{12} C, \qquad (1.10)$$

where $\hat{P}_{12} = \hat{r}_1 \hat{\varphi} \hat{r}_2 \hat{\varphi}$. By comparing (1.9) and (1.10), it is easily found that the operator part, which corresponds to the problem of diffraction by a periodic structure (shown in Fig. 1), is inverted. Operator $\hat{P}_{12}$ describes the interaction between the first and the second screen.

The problem becomes much more complicated, if the components of the complex structure have different periods. If the periods are multiples of a common base, then the biggest value should be taken for the period of the whole structure. If the period–to–period ratio is a rational number, the period is their least common multiple. In this case, the fields and operators should then be reduced to a new basis corresponding to the period of the whole structure. When the period-to-period ratio is not a rational number, the whole structure is not periodic.

It is not accidentally that we took the matrix spectral operators of scattering as an example. Firstly, such operators appear in the problems of wave diffraction by periodic structures, which are widely used and actively investigated. Secondly, since the properties of such operators have well been studied, and the physical interpretation of the results of their application is fairly simple, their basic introduction to methods of research for multilayer structures will henceforth be carried through with the example of objects generating scattered fields with discrete space spectrum. Thirdly, operators with infinite matrices take a special intermediate position between operators with matrices of finite order (or even in the one-dimensional case, between the complex transmission and reflection coefficients) and integral operators, which arise in problems of diffraction either of a field with continuous spectrum or by obstacles generating scattered fields with continuous space spectrum.

2. The Method of Study of Wave Diffraction and Propagation in Periodic Layered Structures

In the section title, the word "periodic" should be put into quotation marks: strictly speaking, the considered finite or semi-infinite set of screens is a structure of equidistant screens. However, for short, the term "periodic" will be applied thereafter also to obstacles which consisted of a finite number of elements, with the assumption that such a structure is a finite or semi-infinite part of a periodic infinite set.

2.1. Reflection Operator of Semi-Infinite Periodic Structure

Assume that half-space $z > 0$ is filled by an arrangement of equidistant and identical planar screens, each being a periodic strip grating (see Fig. 3). The strips in these gratings are considered to be infinitely thin and perfect conducting. The medium has $\varepsilon = \mu = 1$ in the gaps and in the half-space $z < 0$. From the left, a plane linearly polarized electromagnetic wave

$$u^i = b_0^i e^{ikz} \tag{2.1}$$

is incident on the boundary $z = 0$. To avoid complexity, we consider here the case of normal incidence and for definiteness consider vector $\vec{E}$ of the incident wave being in parallel to the grating strips, i. e. u^i in (2.1) means a unique nonzero $\vec{E}$–component of the electric field.

Let us solve the diffraction problem by the operator method. Consider the transmission $\hat{t}$ and reflection $\hat{r}$ operators for a single screen to be known. Introduce the reflection operator $\hat{R}$ of a semi-infinite periodic structure. When solving the diffraction problem, the reflection operator $\hat{R}$ will be found, so that for

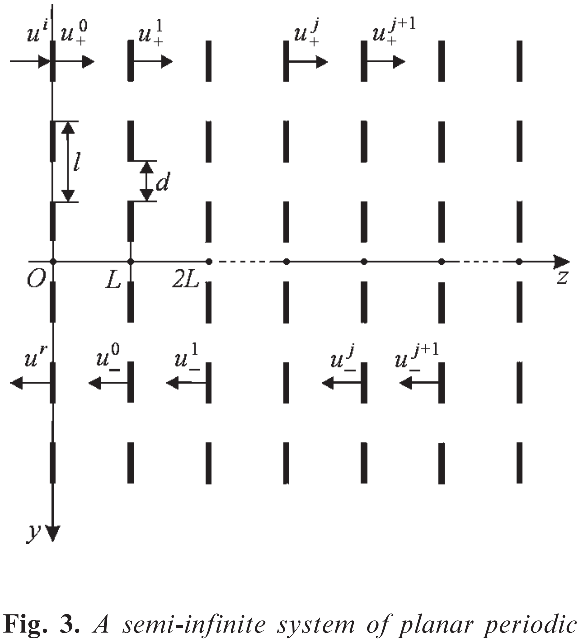

Fig. 3. *A semi-infinite system of planar periodic gratings*

any incident field with spectral amplitude vector $B^i = \left\{b_n^i\right\}_{-\infty}^{\infty}$, the sought vector $A^r = \left\{a_n^r\right\}_{-\infty}^{\infty}$ of spectral amplitudes of the reflected field can be defined by the formula

$$A^r = \hat{R}B^i. \qquad (2.2)$$

Enumerate the gaps between screens by assigning the number "0" to the near-boundary semi-infinite structure (see Fig. 3). Represent the field in each gap as a superposition of fields of plane waves

$$u^j = u_+^j + u_-^j, \qquad Lj < z < L(j+1),$$

where

$$u_+^j = \sum_{n=-\infty}^{\infty} b_n^j e^{i\gamma_n(z-Lj)} e^{ih_n y}, \qquad (2.3)$$

$$u_-^j = \sum_{n=-\infty}^{\infty} a_n^j e^{-i\gamma_n(z-Lj-L)} e^{ih_n y}, \qquad (2.4)$$

L means layered structure period, $j = 0, 1, 2, ...$ is gap number, and $h_n = 2\pi n/l$ for $\alpha = 0$. Consider first the fields in the region $z < 0$ and in the gap $j = 0$ between the near-boundary structure screens $0 < z < L$. It can easily be seen that the amplitude vectors $A^j = \left\{a_n^j\right\}_{-\infty}^{\infty}$, $B^j = \left\{b_n^j\right\}_{-\infty}^{\infty}$, and B^i of the fields in these space regions fulfill the equations

$$B^0 = \hat{t}B^i + \hat{r}\hat{\varphi}A^0, \qquad (2.5)$$

$$\hat{R}B^i = \hat{r}B^i + \hat{t}\hat{\varphi}A^0, \qquad (2.6)$$

$$A^0 = \hat{R}\hat{\varphi}B^0, \qquad (2.7)$$

where $\hat{\varphi}$ is the operator determined above with $\Delta z = L$. Equation (2.7) follows from the aforementioned shift symmetry of a semi-infinite structure.

Eliminate vectors B^0 and A^0 from the system (2.5)–(2.7) and obtain an equation with respect to the reflection operator $\hat{R}$

$$\hat{R} = \hat{r} + \hat{t}\hat{\varphi}(I - \hat{R}\hat{\varphi}\hat{r}\hat{\varphi})^{-1}\hat{R}\hat{\varphi}\hat{t}. \qquad (2.8)$$

Notice some features of Eq. (2.8). First along with Eq. (2.8), another equivalent form of notation is valid:

$$\hat{R} = \hat{r} + \hat{t}\hat{\varphi}\hat{R}\hat{\varphi}(I - \hat{r}\hat{\varphi}\hat{R}\hat{\varphi})^{-1}\hat{t}. \qquad (2.9)$$

Secondly, if operators

$$\tilde{R} = \hat{R}\hat{\varphi}, \qquad \tilde{r} = \hat{r}\hat{\varphi}, \qquad \tilde{t} = \hat{t}\hat{\varphi} \qquad (2.10)$$

are introduced, then, instead of Eq. (2.8) we obtain a more compact equation

$$\tilde{R} = \tilde{r} + \tilde{t}(I - \tilde{R}\tilde{r})^{-1}\tilde{R}\tilde{t}. \qquad (2.11)$$

Thirdly, if operator $\tilde{R}_0$ is the solution of Eq. (2.11), then the inverse operator $\tilde{R}_0^{-1}$ fulfills to this equation also.

In the general case, Eq. (2.8) can be solved numerically, for instance by the Newton method, or in some cases, by the method of successive approximations.

2.2. Eigenwaves of Periodic Structure

Let us now study the field in the periodic part of the structure in more detail. In the neighbored j and $j+1$ gaps the field amplitude vectors fulfill the equations

$$\begin{aligned} B^{j+1} &= \hat{t}\hat{\varphi}B^j + \hat{r}\hat{\varphi}A^{j+1}, \\ A^{j+1} &= \hat{R}\hat{\varphi}B^{j+1}, \qquad j = 0, 1, 2, ... \end{aligned} \qquad (2.12)$$

After elimination of A^{j+1} we obtain the recurrent formula

$$B^{j+1} = (I - \hat{r}\hat{\varphi}\hat{R}\hat{\varphi})^{-1}\hat{t}\hat{\varphi}B^j, \quad j = 0, 1, 2, ... \qquad (2.13)$$

Equation (2.13) allows to express the vectors of field amplitudes for any screen gap through the corresponding vectors of fields in the gap with $j=0$. It is easily seen that the vectors B^0 and A^0 are connected to the vector of the incident field amplitudes by

$$B^0 = (I - \hat{r}\hat{\varphi}\hat{R}\hat{\varphi})^{-1}\hat{t}B^i, \qquad A^0 = \hat{R}\hat{\varphi}B^0. \tag{2.14}$$

Provided that transmission operator $\hat{T}$ of the semi-infinite structure is introduced:

$$B^0 = \hat{T}B^i, \tag{2.15}$$

then from Eq. (2.14) follows, that

$$\hat{T} = (I - \hat{r}\hat{\varphi}\hat{R}\hat{\varphi})^{-1}\hat{t}, \tag{2.16}$$

and the recurrent formula (2.13) can be written as

$$B^{j+1} = \hat{T}\hat{\varphi}B^j. \tag{2.17}$$

An eigenfield of the infinite periodic structure is a superposition of eigenwaves. The eigenwave has two components in each screen gap propagating in opposite directions. Provided the vectors of spectral amplitudes for the k–th eigenwave are denoted as B_k^j and A_k^j, then for any j on the one hand, we have the relation

$$B_k^{j+1} = \hat{T}\hat{\varphi}B_k^j, \tag{2.18}$$

and on the other hand the eigenwave field in the neighboring screen gaps may only differ by a phase factor

$$B_k^{j+1} = e^{i\beta_k L}B_k^j. \tag{2.19}$$

It is easy to obtain the dispersion equation now for finding the k–th eigenvalue $e^{i\beta_k L}$ of operator $\hat{T}\hat{\varphi}$:

$$\det\left[I - \hat{t}\hat{\varphi}\mathrm{e}^{-i\beta_k L} - \hat{r}\hat{\varphi}\left(I - \hat{t}\hat{\varphi}\mathrm{e}^{i\beta_k L}\right)^{-1}\hat{r}\hat{\varphi}\right] = 0. \tag{2.20}$$

Thus operator $\hat{T}\hat{\varphi}$ contains information on the phase velocities of all eigenwaves, which are excited in the semi-infinite structure by an incident field.

2.3. Eigenfield Reflection and Transmission Operators at the Boundary of a Semi-Infinite Structure

Let now all half-space $z<0$ be filled with a periodic structure of screens possessing the same parameters as above. Consider the propagation of an eigenfield in a layered structure which is incident from the half-space $z<0$ through the free-space boundary in the plane $z=0$. Assume that in the screen gap which is closest to the boundary, the incident eigenfield components are described by

$$u_+ = \sum_{n=-\infty}^{\infty} b_n e^{i(z+L)\gamma_n} e^{ih_n y},$$

$$u_- = \sum_{n=-\infty}^{\infty} a_n e^{-iz\gamma_n} e^{ih_n y},$$

and those for the reflected eigenfield, accordingly, are

$$\upsilon_+ = \sum_{n=-\infty}^{\infty} b_n^- e^{i(z+L)\gamma_n} e^{ih_n y},$$

$$\upsilon_- = \sum_{n=-\infty}^{\infty} a_n^- e^{-iz\gamma_n} e^{ih_n y}.$$

Denote the vectors of spectral amplitudes of these fields by

$$B = \{b_n\}_{-\infty}^{\infty}, \qquad A = \{a_n\}_{-\infty}^{\infty},$$
$$B_- = \{b_n^-\}_{-\infty}^{\infty}, \qquad A_- = \{a_n^-\}_{-\infty}^{\infty}.$$

Represent the free space transmitted field as

$$u^t = \sum_{n=-\infty}^{\infty} b_n^t e^{iz\gamma_n} e^{ih_n y}, \qquad B^t = \left\{ b_n^t \right\}_{-\infty}^{\infty}.$$

Introduce operator of reflection $\hat{\rho}$ of the eigenfield and operator of its transformation into the free-space transmitted field (operator of transmission $\hat{\tau}$) by the formulas

$$\hat{\rho} B = A_-, \qquad \hat{\tau} B = B^t.$$

The expressions for $\hat{\rho}$ and $\hat{\tau}$ can be obtained as

$$\hat{\rho} = -\hat{T}\hat{\varphi}\hat{R}\hat{\varphi}\hat{T}\hat{\varphi}, \qquad \hat{\tau} = \hat{t}\hat{\varphi}(I + \hat{R}\hat{\varphi}\hat{\rho}).$$

Thus knowing the semi-infinite structure reflection operator $\hat{R}$, it is possible to determine operators of reflection $\hat{\rho}$ and transmission $\hat{\tau}$.

2.4. Operators of Transmission and Reflection for a Periodic Structure with Finite Number of Layers

Consider the incidence of a field with amplitude vector B^i on a periodic structure with finite number of screens *N*, Fig. 4. Introduce operators of reflection $\hat{r}_N$ and transmission $\hat{t}_N$ of the *N*–layer structure and find their expressions by using the operators of reflection and transmission for a semi-infinite periodic structure:

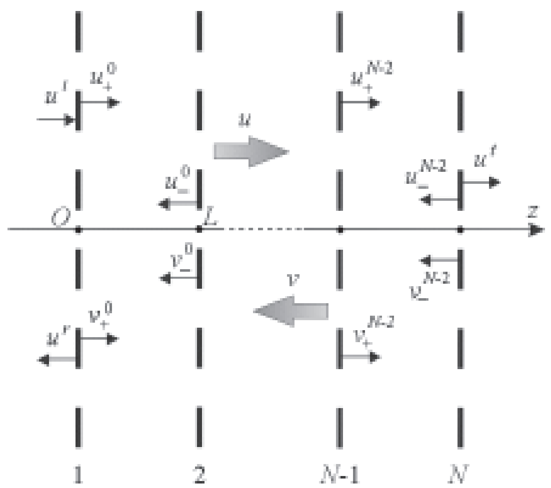

Fig. 4. *An N–screen periodic structure*

$$\hat{r}_N = \hat{R} + \hat{\tau}\tilde{T}^{N-2}\hat{\rho}\tilde{T}^{N-2}\left[I - \left(\hat{\rho}\tilde{T}^{N-2}\right)^2 \right]^{-1} \hat{T},$$

$$\hat{t}_N = \hat{\tau}\tilde{T}^{N-2}\left[I - \left(\hat{\rho}\tilde{T}^{N-2}\right)^2 \right]^{-1} \hat{T}.$$

A few important problems may be presented to illustrate the efficiency of the offered method in the theory of electromagnetic wave diffraction. First consider the problem of periodic structure eigenwave propagation through the junction of two semi-infinite systems of screens. From the left half-space the eigenwave of the corresponding periodic structure is incident on the junction of these two sets. The operator of reflection of this wave from the half-space boundary and the operator of excitation of the second layered half-space eigenfield (i. e. the operator of transmission) were found. We analyzed also the eigenfield transmission through and the reflection from an arbitrary gap between two different layered half-spaces. As a next example, operators of transmission and reflection for two layered slabs of identical strip screens were obtained. Each slab differs in its number of screens. The screens have been disposed in each slab with different periods also. The gap is arbitrary between slabs.

Thus in this section we outlines the formal procedure of the operator method for solving the problems of electromagnetic wave diffraction by multilayered periodic structures. The reflection operator of semi-infinite periodic set has fundamental significance for this method. The next sections will be dedicated to a physical analysis of solutions for a number of practical problems, as well as to an exposition of peculiarities which occur in using the operator method in concrete situations.

3. Wave Diffraction by Periodic Sequence of Dielectric Layers

Wave reflection by semi-infinite system of loss-less and lossy dielectric slabs was studied in [4, 5]. The transmission and reflection of finite number layers were analyzed in comparison with properties of semi-infinite structure.

3.1. Reflection Coefficient for Semi-Infinite Periodic System of Dielectric Layers

Let the half-space $z > 0$ be filled with a layered structure of equidistant dielectric slabs of the same thickness h (see Fig. 5). An incident electromagnetic wave is

$$u^i = E_x^i = e^{ikz}. \qquad (3.1)$$

If the wave (3.1) is incident on a single slab $0 \le z \le h$ in the free space, the reflected wave shows the form $u^r = re^{-ikz}$ and the transmitted wave accordingly reads $u^t = te^{ik(z-h)}$, where

$$r = \frac{(\varepsilon-1)\left(e^{ik\sqrt{\varepsilon}h} - e^{-ik\sqrt{\varepsilon}h}\right)}{\left(\sqrt{\varepsilon}+1\right)^2 e^{-ik\sqrt{\varepsilon}h} - \left(\sqrt{\varepsilon}-1\right)^2 e^{ik\sqrt{\varepsilon}h}},$$

$$(3.2)$$

$$t = \frac{4\sqrt{\varepsilon}}{\left(\sqrt{\varepsilon}+1\right)^2 e^{-ik\sqrt{\varepsilon}h} - \left(\sqrt{\varepsilon}-1\right)^2 e^{ik\sqrt{\varepsilon}h}}.$$

If introduce $\tilde{R} = R\varphi$, $\tilde{r} = r\varphi$, and $\tilde{t} = t\varphi$, where $\varphi = e^{ikd}$, $d = L - h$, L is structure period, the Eq. (2.8) becomes a quadratic algebraic equation for obtaining the reflection coefficient of semi-infinite set of dielectric layers:

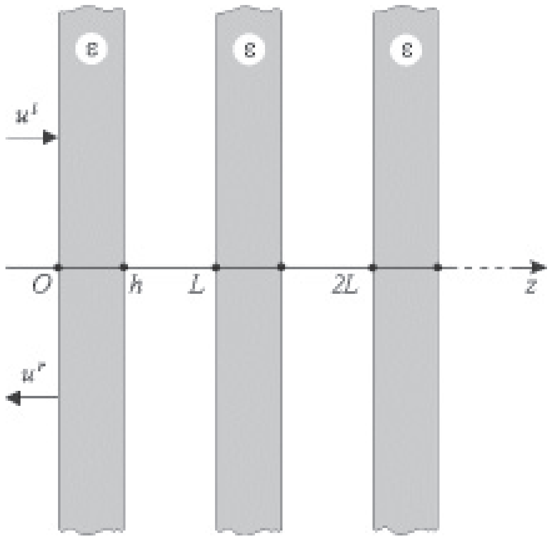

Fig. 5. *A semi-infinite set of dielectric slabs*

$$\tilde{R}^2 + \tilde{R}\frac{\tilde{t}^2 - \tilde{r}^2 - 1}{\tilde{r}} + 1 = 0. \qquad (3.3)$$

Two roots of Eq. (3.3) fulfill the equality $\tilde{R}_1\tilde{R}_2 = 1$, so that $|R| \le 1$ for one of the roots is always met, what is physically meaningful.

The reflective properties of such a structure were studied versus the relative thickness of the slab h/L, the frequency parameter L/λ, and the value of ε [4, 5]. For small L/λ –ratios all layered half-space can be considered as homogeneous with an effective permittivity

$$\varepsilon_{ef} = \frac{h\varepsilon + (L-h)}{L}. \qquad (3.4)$$

If the wavelength λ is commensurable with the characteristic sizes (L and h) of the structure, the reflection coefficient behavior is governed by two main factors. First, if the thickness of a single dielectric slab is close to integer number of half wavelength in the dielectric, then the transmission coefficient $|t|$ is close to one, being equal to one in the resonance case. Such resonant character of transparency of a single slab determines the electrodynamics behavior of their semi-infinite set as well: the reflection coefficient $|R|$ vanishes in the same resonance points, irrespective of the slab spacing. The studied structure is matched completely with free space in these points.

The second factor, which essentially affects the reflective properties of the structure, is connected to its periodicity. It is well known, that an eigenmode in the periodic structure can exist only for certain structure–to–wavelength parameter ratios. In particular, forbidden zones appear, where the phase mismatch between the waves, which undergo multiple reflections by the slabs, prevents propagated wave formation. In fact, if the parameters correspond to a forbidden zone of the infinite periodic structure, the reflection coefficient $|R|$ of a semi-infinite set should be one. The dispersion equation was analyzed for a field in a periodic set of dielectric slabs.

4. Wave Diffraction by Periodic Sequence of Dense Strip Gratings

4.1. Semi-Infinite Periodic Structure of Dense Strip Gratings

The sets of dense strip gratings were studied in [4]. Assume the grating strips are perfect conducting and – at first – infinitely thin. Fig. 3 shows the strip grating. If a wave

$$u^i = E_x^i = e^{ikz}, \qquad E_y^i = E_z^i = 0, \tag{4.1}$$

is incident on a single screen lying in the plane $z = 0$ and $\kappa = l/\lambda < 1$, only the zero spectral harmonic both the reflected and transmitted field will be a non evanescent plane wave. Sufficiently far from the plane $z = 0$, the total field is well described by a main wave

$$E_x = \begin{cases} e^{ikz} + a_0 e^{-ikz}, & z < 0, \\ b_0 e^{ikz}, & z > 0. \end{cases} \tag{4.2}$$

The distance Δz from the plane $z = 0$, at which the diffracted fields defined by full partial wave superposition included evanescent waves and (4.2) practically coincide, essentially depends on the ratio l/λ; we may consider $\Delta z \sim l$ if $l \ll \lambda$. Hence it is clear that for an analysis of the reflection by a semi-infinite set of gratings which fulfill the conditions $\kappa < 1$ and $L > l$ the values a_0 and b_0 from (4.2) should be used for *r*– and *t*–coefficients, respectively. These values can be obtained from the rigorous solution of the problem of plane wave diffraction by a strip grating (see, e. g. [2]) or by using Lamb's approximate formula [6]

$$b_0 = i\kappa 2 \ln\cos\frac{\pi d}{2l}\left(1 + i\kappa 2\ln\cos\frac{\pi d}{2l}\right)^{-1}. \tag{4.3}$$

From the boundary conditions there also follows $1 + a_0 = b_0$.

Reflection of an incident *H*–polarized wave $(u^i = H_x, \quad H_y = H_z = 0)$ is similar to the considered one. For *H*–polarization the Lamb's approximation of the transmission coefficient is

$$B_0 = \left(1 + i\kappa 2\ln\sin\frac{\pi d}{2l}\right)^{-1}. \tag{4.4}$$

The relation $1 - A_0 = B_0$ follows from the boundary conditions.

The $|R|$–coefficient of semi-infinite structure is notably small in a very broad range of d/l and L/λ values for *H*–polarized incident field. For *E*–polarization, the structure shows a considerably worse transparency for small κ.

The zones of total reflection are connected to the band-gap zones of the periodic screen set. To study the eigenmodes of an infinite structure, the corresponding dispersion equation can easily be obtained. It can clearly be seen, that for a single-wave mode, the fields diffracted by a single strip grating, fulfill the averaged equivalent boundary conditions. For small κ, the analytic Lamb's approximation formulas (4.3) and (4.4) can be used. For an infinite structure, matching the fields at the boundary between two neighbored regions, one obtains equations for the eigenwave propagation constant β for *E*–polarization

$$\cos\beta L = \cos\frac{2\pi L}{\lambda} + i\frac{b_0 - 1}{b_0}\sin\frac{2\pi L}{\lambda}, \tag{4.5}$$

and for *H*–polarization

$$\cos\beta L = \cos\frac{2\pi L}{\lambda} - i\frac{A_0}{1 - A_0}\sin\frac{2\pi L}{\lambda}. \tag{4.6}$$

Occurrence of the regions of total reflection which are repeating in steps of 0.5 on the scale of relative values of the screen disposition period (and are connected to the forbidden bands of the infinite set of screens) in a semi-infinite sequence of screens is the common property for such class of structures. However, in the strip-grating case, a specific standing-wave mode with nodes coincident with those points, where the planes of semi-transparent screens are located, exists in the points $L/\lambda = n \cdot 0.5$ $(n = 1, 2, ...)$, which are exactly located on the boundary of each of the regions of total reflection. It can be proven, that for $\varphi = 1$ a unique solution is the $R = -1$ value (or for H–polarization the $R = 1$ value). From (4.5) and (4.6), the range of parameter values can easily be shown, for which the considered structure totally reflects an incident plane wave.

We showed that for $\kappa < 1$ in the partial reflection mode, the phase of the R–coefficient (for normal plane-wave incidence) is independent from the d/l–ratio.

4.2. Anisotropic Artificial Dielectric

It seems to be of interest to find performances of the equivalent dielectric of a multilayer set of gratings by using the reflection coefficient value for the layered half-space and its relation to the effective refraction index. The electrodynamics properties of such anisotropic structure are described by two tensors $\hat{\varepsilon}$ and $\hat{\mu}$, and in order to recover all components of these tensors, we should generalize the solution to plane linearly polarized waves, which are incident at any arbitrary angle. However, for most cases, no necessity to recover all $\hat{\varepsilon}$ and $\hat{\mu}$ components emerges. It is sufficient to determine the effective value of the refraction index n for normal incidence but with E– and H– polarizations of the waves. For this purpose with the known $\tilde{R}$–values, we may use the Fresnel formulas or define the β from the dispersion equation. In this case an unambiguous solution of Eqs. (4.5) or (4.6) is ensured by the condition $|\beta L - kL| < 2\pi$.

4.3. Sequence of Gratings of Finitely Thick Strips

Note that – within the developed approach – it is fairly easy, at least for the H–polarization case, to investigate limits for using the idealization "infinitely thin strips". With this objection in mind, employ, for example, the results of work [2], where analytical expressions for the transmission and reflection coefficients of an H–polarized wave, which is incident on the periodic grating of perfect conducting, finitely thick bars, are derived for the long-wave approximation. For normal incidence, these formulas in our notation take the form

$$A_0 = -i\kappa\left(\frac{\pi h}{d} - 2\ln\sin\frac{\pi d}{2l}\right)\times \times\left[1 - i\kappa\left(\frac{\pi h}{d} - 2\ln\sin\frac{\pi d}{2l}\right)\right]^{-1}, \tag{4.7}$$

$$B_0 = 1 - A_0, \tag{4.8}$$

and they are valid under the condition that $h/\lambda \ll 1$, where h means bar thickness.

The reflection coefficients were found calculations for the H–polarized wave normally incident on a semi-infinite set of gratings of finitely thick perfect conducting bars and for the structure with finite number of layers for some parameter values. A comparison of the graphs for corresponding dependencies shows, that for H–polarization the electrodynamic properties of multilayer structures, which are periodic gratings of finitely thick strips, practically do not differ for $\kappa < 0.5$ and $h/l < 0.05$ from the properties of structures with infinitely thin strips in a fairly wide range of parameter values.

4.4. Periodic Structures with Period "Failure"

As a matter of fact, a practical multilayer structure may differ from the considered ideal model, for example, in a failure in the strict periodicity of screen disposition within a set. It is important to study the influence of possible structure period failures on the character of reflection and transmission coefficient dependencies versus the key problem parameters.

Let us consider first the reflective properties of a structure, which consists of periodically disposed M screens with period L and of one more the same screen at a distance of Δ from the M–layer sequence. A change in Δ between the group of periodically disposed screens and the non-regular one practically has no effect on the zones of total reflection, while the reflection coefficient behavior inside the zones of partial reflection changes: instead of the points of perfect resonant transparency (which occur at $\Delta = L$), resonances are observed which are not so regular and different in depth and where the reflection coefficient does not vanish. With increasing L/λ, the transparency in the zone of partial reflection decreases from zone to zone.

The second model of period "failure" is concerned with two layers of M and N periodically (with identical period of L) disposed identical screens in a distance of Δ.

The third possible study of a broken periodicity in element disposition is concerned with two layers of periodically (with period of L) disposed screens, which are distant from each other by $\Delta = 2L$ or $\Delta = 3L$. This models the situation of 1 or 2 screens missing inside the finite-layered periodic sequence.

And finally, the fourth example concentrates on two multilayer periodic structures of identical screens (with M and N being the number of screens and L the screen disposition period), one located at distance $\Delta = 2L$ from the other. However, in this gap at a distance of Δ_1 from the left-hand layer (with M screens), there is one more screen disposed, so that distance from this screen to the right-hand layer (with N screens) is equal to $\Delta_2 = 2L - \Delta_1$.

The analysis allows the conclusion that the properties of layered structures within the zones of total reflection are sufficiently stable against any failure in periodicity, while a small failure in the zones of partial reflection can already exert an essential influence.

5. Multimode Wave Diffraction by Periodic Set of Strip-Gratings

Assume, as above, the screen strips perfect conducting and infinitely thin, suppose, however, the strip disposition period l to wavelength λ ratio being no longer small [7]. In this case, the diffraction properties of gratings and of their sets essentially differ from the corresponding properties in the above-considered long-wave mode. The electromagnetic field interaction with such screens should be described by the corresponding scattering operators, because now not a single plane wave, but some space harmonics are propagating in the diffracted field for $l/\lambda > 1$. Reflection operator $\hat{R}$ is found numerically by solving the nonlinear operator Eq. (2.8).

A fairly close problem is the problem of wave transformation by the junction of two rectangular waveguides, a regular one and another one loaded by a semi-infinite or a finite set of diaphragms. The problem of wave diffraction by a periodic system of slots of the plane waveguide walls was studied also.

5.1. On Numerical Solution of Operator Equation

For obtaining the solution in the multimode case the numerical method of successive approximations can be used by realizing the iterative process

$$\tilde{R}_n = \tilde{r} + \tilde{t}\left(I - \tilde{R}_{n-1}\tilde{r}\right)^{-1}\tilde{R}_{n-1}\tilde{t}, \qquad n = 1, 2, \dots, \tag{5.1}$$

where $\tilde{R}_0$ is the initial approximation (e. g. $\tilde{R}_0 = \tilde{r}$), and $\tilde{R}_n$ is the approximate solution of Eq. (2.11), which is obtained from the n–th iteration.

5.2. A Strip-Grating Periodic Structure (Multi-Mode Conditions)

The electrodynamics properties of a strip grating depend essentially on the κ–parameter. In the analysis of diffraction properties of a periodic set of gratings operated in a single-wave mode $(\kappa < 1)$ one is then allowed to treat the screens as homogeneous along the Oy–axis and semitransparent films, on which the equivalent boundary conditions are fulfilled. The described approach is justified only in the case that the screen gap L is sufficiently large compared to wavelength. Within the framework of the discussed approximate solution it is, however, impossible to define its limits of applicability and to evaluate the error of the results. The exact solution opens such possibility only, when the grating properties are described for $\kappa < 1$ with operators $\hat{r}$ and $\hat{t}$ as well, and their periodic set properties are calculated with operators $\hat{R}$, $\hat{T}$.

We proceed now to the analysis of the electrodynamic properties of a periodic set of gratings in the multi-mode case $1 < \kappa < 2$. This means that in the space spectrum of the diffracted fields, three plane waves propagating in different directions will occur for a normal incident plane electromagnetic wave. The screen interaction with the fields of these waves, as well as with the local fields of higher space harmonics of the spectrum, results in the very complicated dependencies of amplitudes and phases of the reflected and transmitted fields of the system parameters. In analyzing the diffraction properties of such periodic screen sets, it was most convenient to begin with a study of the eigenfield structure (the eigenwave family) and the dispersion dependencies of the eigenwave propagation constants.

The most important conclusion, which follows from a consideration of the eigenwave structure, consists in dividing the waves into two groups: into those with even and with odd field distributions. Indeed, for the waves with odd indices the space harmonics with positive and negative numbers of equal magnitude show identical complex amplitudes, while the waves with even indices show amplitudes which are opposite in sign. Eigenvectors with even distribution are orthogonal to vectors with odd distribution; hence the corresponding (even or odd) fields can be excited independently of each other. In particular, if a plane wave is normal incident on a semi-infinite structure, the odd eigenwaves cannot be excited.

Since for normal incidence, the odd eigenwave is not excited, then even for the multiwave region, in the even eigenwave forbidden zones, the structure is completely reflected. However, in the band-gap zone the reflection coefficient of the zero space harmonic $|a_0|$ is not equal to 1 (except for, possibly, at some isolated points). This is quite natural, because for $1<\kappa<2$ the reflected energy is transported by the zero and the first (numbered by ± 1) space-spectrum harmonics. The total energy flux of the reflected field in the cutoff zone is equal to the flux in the incident plane wave.

5.3. Junction of Regular and Diaphragm Waveguides

The junction of two rectangular waveguides consisting of a regular one and another one of the same cross-section, but periodically loaded by diaphragms was considered. A rectangular waveguide periodically loaded with diaphragms is a transmission line with a specific dispersion characteristics being strongly dependent on structure parameters. Strong dispersion and presence of the frequency “cutoff” regions allow using sections of such waveguides as frequency filters. In this case, because a waveguide section with finite number of diaphragms shows frequency characteristics which are considerably different for the fundamental and the higher propagating waveguide modes (in the multi-mode case), such a waveguide section is also effectively used as mode filter or mode structure converter.

5.4. Junction of Regular Plane and Periodically Slotted Waveguides

The propagation of electromagnetic waves in a plane waveguide with slots periodically disposed in its walls was considered. The interest in such a structure arises in the design of waveguide components or microwave antennas, because of a frequent necessity to consider the transformation of waveguide waves by a waveguide section with transversal slots. Despite the diffracted fields possessing in this case a continuous space spectrum, the approximate problem can be solved with a matrix instead of integral operators. For simplicity, any interaction between the waveguide slots by means of the free-space slot-radiation fields is supposed to be negligible. In such approximation, the field radiated from any slot to outside the waveguide does not imply any change in the complex amplitudes of the waveguide waves at the other slotted waveguide sections. The error of such solution is the smaller, the narrower the slots are and the bigger the period of their displacement is in comparison to wavelength.

If the final goal of the research is finding the reflected and transmitted fields in a waveguide with finite number of periodically disposed identical slots, then with operators $\hat{t}$ and $\hat{r}$ known, the above-described method allows to easily obtain the reflection operator $\hat{R}$ at the junction of a regular waveguide and a semi-infinite waveguide with periodically disposed slots, and thus all necessary characteristics of the fields. If the interest is concentrating on obtaining the parameters of the field radiated in a free space then the found solution also allow to obtain all necessary information. Let, for example, the waveguide possess a finite number of periodically disposed slots. Then with operator $\hat{R}$ known it is easily to find the waveguide fields in front of the first slot, between the slots, and behind the last slot.

An essential point of this approach lies in the possibility of studying systems with large number of slots, including also systems of several groups of slots which are periodically disposed in each group, but with different period for each group, and also the waveguide multi-mode case.

6. Wave Scattering by Structures of Planar Double Periodic Screens

Electromagnetic wave scattering by layered sets of double periodic planar structures are considered in this section.

6.1. Operators of Scattering by Planar Double Periodic Arrays

It is considered an infinite planar array, whose elements are periodically disposed in the mesh nodes plotted in nonorthogonal coordinates s_1 and s_2 as shown in Fig. 6. The element position in the plane $z = 0$ is determined by radius vector with two indices ν_1 and ν_2 according to $\vec{\rho}_{\nu_1\nu_2} = \nu_1 l_1 \vec{e}_1 + \nu_2 l_2 \vec{e}_2$ where $\vec{e}_1$ and $\vec{e}_2$ are unit vectors directed towards s_1 and s_2, and l_1 and l_2 are the corresponding mesh periods. A plane electromagnetic wave is incident on the grating from the upper half-space. A polarization of the incident field is defined by polarization vector. An electromagnetic field for each of the space harmonics is represented as a superposition of *TE* and *TM* waves. The constructive methods of obtaining the operators of reflection and transmission for double periodic arrays of complex-shaped strip particles and rectangular perforated metal screens of finite thickness have been developed.

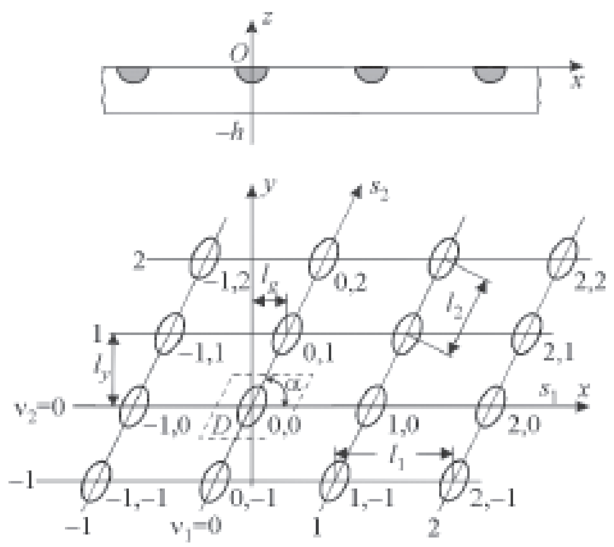

Fig. 6. *A double periodic planar array*

As array elements of strip periodic structures some planar metal strips of any shape are chosen in particular C– and S–shaped elements. The strip length is assumed be larger than its width. The width may vary along the strip. The longitudinal component of the strip surface current is assumed to essentially exceed the transversal component, so that the latter may be neglected. The problem of obtaining the spectral transmission and reflection operators is reduced to the numerical solution of the integral equation for an unknown surface current density by the method of moments [8, 9]. The developed method allows obtaining the reflection and transmission spectral operators for strip periodic structures in free space as well as placed on a magnetic-dielectric substrate.

Diffraction properties of double periodic finite thickness metal screens with rectangular perforation were described by generalized scattering matrices. These planar screens have waveguide channels of rectangular cross section which are periodically disposed in two directions. For simplicity, the perforation shows dimensions that only the fundamental mode can propagate. The operator method was used to determine the generalized reflection and transmission matrices for such screen. In the considered problem a key point of the operator method was obtaining the operators of transmission and reflection for a periodic array of semi-infinite rectangular waveguides which is a simplest part of structure with respect to its electrodynamic description [10].

6.2. Reflection and Transmission Operators for a Semi-Infinite Periodic Structure of Double Periodic Screens

Let the half-space be filled with a system of equidistant identical plane-parallel double periodic structures. For simplicity, we consider the normal incidence of plane electromagnetic wave and the frequency range, in which only the principal space harmonic of the field must be taken into account.

We distinguished the two classes of periodic structures: symmetric and chiral ones. The chiral structures, unlike the symmetric, are those whose mirror image and structure itself cannot coincide by shifting and turning in the structure plane. The chiral feature of the periodic structure can result either from chirality of its elements or from the chiral arrangement of structure. For example, if non-chiral elements are disposed asymmetrically with respect to a rectangular mesh, the whole structure can show a chiral feature.

The dispersion equation for electromagnetic waves in a layered periodic system is derived. The relationship between the eigenwawe propagation constant and the reflection operator of the semi-infinite structure is established. Some simple examples were considered [11].

Assume first, that the off-diagonal elements of the matrices $\tilde{r}$ and $\tilde{t}$ are zero. Examples of such structures are arrays of cross-shaped strip elements with strips which are oriented along the orthogonal directions of periodicity, further arrays of strip rings with a split in one of the directions of periodicity, planar arrays consist of disks or a perforated metal screen with square holes in the orthogonal mesh. The system of equations for the field in the periodic layered structure splits into two independent systems with respect to the amplitudes of two eigenwaves linearly polarized along the *Ox*– and *Oy*–axes, respectively. Two eigenwaves which are linearly polarized along these axes, with different propagation constants can propagate in such a structure.

Assume now that the main-diagonal elements of the matrices of the reflection and transmission operators are equal $\tilde{r}_{xx} = \tilde{r}_{yy}$, $\tilde{t}_{xx} = \tilde{t}_{yy}$. The non-diagonal elements shall be non-zero. Such structures may be formed by planar arrays with equal periods along the *Ox*– and *Oy*–axes, further with strip elements being rectilinear sections which are oriented at an angle of 45° against the directions of array periodicity, or by a strip ring with a split again oriented at 45° towards the periodicity direction. The matrices $\tilde{r}$ and $\tilde{t}$ of such infinitely thin structures show identical non-diagonal elements: $\tilde{r}_{xy} = \tilde{t}_{xy}$. The dispersion equations for the propagation constant of eigenwave in periodic systems of such type (e. g. for thin strip structures or for strip structures in a dielectric layer) are derived and its solution analysis has been fulfilled analytically.

The properties of a structure consisting of S–shaped strip elements were considered. The S–shaped strips are plane-chiral elements. Electromagnetic waves with left- and right-hand circular polarizations are reflected from and transmitted through the S–shaped element grating, thereby changing both amplitude and polarization in a different way. The grating obviously possesses resonance properties. The resonance is observed for that frequency, for which the strip length is about multiply half the free-space wavelength. A numerical solution of the dispersion equation of a periodic system of S–shaped element gratings was found, i. e. the eigenwave propagation constants versus the interlayer spacing length–to–wavelength ratio. The wave reflection by a semi-infinite system of gratings of S–shaped strip elements with same parameters as in the above analysis of the infinite structure is considered.

The results related to study of wave reflection by layered set of metal screens with double periodic system of rectangular holes are presented in [12].

Conclusion

In the conclusion we would like to comment that the development of the present theory is certainly not comprehensive. In particular, the solution methods for the main nonlinear operator equation in respect of reflection operator of semi-infinite set require a further development. The problems related to optimization of the algorithms of the numerical solution for such equations, as well as those related to the argumentation of existence and uniqueness of the solution, need to be investigated also.

Much attention was paid to study by using the developed method the diffraction properties of periodic structures exciting diffracted fields with continuous space spectrum. In particular, first discussed are the solutions of problems connected with the junction of a regular waveguide and a waveguide with periodically disposed transversal slots [13]. The approach to analysis of wave scattering by a semi-infinite periodic set of cascade arrangement strips as well as a semi-infinite planar strip grating has been considered [14]. However the numerical solution of the last problem and the physical analysis of scattering are not fulfill yet.

References

1. L. M. Lytvynenko, I. I. Reznik, D. L. Lytvynenko, "Diffraction of waves by semi-infinite periodic structures", Dokladi AN USSR, 1991, no. 6, pp. 62-66 (in Russian).
2. V. P. Shestopalov, L. M. Lytvynenko, S. A. Masalov and V. G. Sologub, *Diffraction of waves by gratings*. Kharkov, USSR: Kharkov State University, 1973, 288 p. (in Russian).
3. L. M. Lytvynenko, S. L. Prosvirnin, *Spectral operators of scattering in problems of wave diffraction by plane screens.* Kiev: Naukova Dumka, 1984, 240 p. (in Russian).
4. D. L. Lytvynenko, L. M. Lytvynenko, I. I. Reznik, "Diffraction of waves by semi-infinite set of semitransparent screens", Preprint of Institute of Radio Astronomy, 1993, no. 65, 39 p.
5. Litvinenko D. L., Lytvynenko L. M., Prosvirnin S. L., "Analysis technique for wave diffraction by multilayered periodic structures", *Radio Physics and Radio Astronomy*, vol. 2, no. 4, pp. 485-491, 1997 (in Russian).
6. H. Lamb, "On the reflection and transmission of electric waves by a metallic grating", *Proc. London Math. Society* (1), vol. 29, pp. 523-545, 1898.
7. Lytvynenko L. M., Prosvirnin S. L., "Analysis of wave diffraction by a sequence of identical strip gratings. Multi-wave conditions", *Radio Physics and Radio Astronomy*, vol. 4, no. 3, pp. 276-286, 1999 (in Russian).
8. Prosvirnin S. L., "Transformation of polarization when waves are reflected by a microstrip array made of complex-shaped elements", *Journal of Communications Technology and Electronics*, vol. 44, no. 6, pp. 635-639, 1999.
9. T. D. Vasilyeva, S. L. Prosvirnin, "Electromagnetic wave diffraction by the plane array of chiral strip elements of complex shape", *Physics of wave processes and radio systems*, vol. 1, no. 4, pp. 5-9, 1998 (in Russian).
10. Gribovsky A. V., Prosvirnin S. L., Reznik I. I., "Reflected phase array formed by finite-depth rectangular waveguides", *Radio Physics and Radio Astronomy*, vol. 2, no. 1, pp. 52-60, 1997 (in Russian).
11. Prosvirnin S. L., Vasilyeva T. D., "Eigenwaves of periodic layered structure of complex arrays", in *Proc. 8-th Int Conf on Electromagnetics of Complex Media, Lisbon, Portugal,* 27-29 Sept 2000, pp. 241-244.
12. Gribovsky A. V., Lytvynenko L. M., Prosvirnin S. L., "Electromagnetic wave diffraction by a multilayer structure of infinite metal screens with rectangular perforation", *Radio Physics and Radio Astronomy*, vol. 5, no. 2, pp. 166-170, 2000 (in Russian).
13. Pogarsky S. A., Chumachenko V. A., "The E_{0p}-wave diffraction by a periodic finite number set of slits in a circular waveguide", *Radio Physics and Radio Astronomy*, vol. 9, no. 1, pp. 66-72, 2004 (in Russian).
14. Vorobyov S. N., Lytvynenko L. M., Prosvirnin S. L., "Operator method in the problem of electromagnetic waves diffraction by semi-infinite strip grating", *Radio Physics and Radio Astronomy*, vol. 10, no. 3, pp. 273-283, 2005 (in Russian).

Дифракция волн на периодических многослойных структурах

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Представлен обзор работ по развитию подхода к построению теории дифракции волн на многослойных периодических частично прозрачных экранах, основу которого составляют определение и использование операторов отражения соответствующих полубесконечных систем экранов.

Дифракція хвиль на періодичних багатошарових структурах

Л. М. Литвиненко, С. Л. Просвірнін, К. Шунеман

Надається огляд робіт з розвитку підходу до побудови теорії дифракції хвиль на багатошарових періодичних частково прозорих екранах, який грунтується на визначенні та використанні операторів відбиття відповідних напівнескінченних систем екранів.