A. Ptak*, M. Machura, U. Gornik*****
* Institute of Ewromethis and Computer Science
** Institute of Mathematics and Computer Science,
*** Czestochowa University of Technology
(E-mail:olaptak@zim.pcz.pl)

Fuzzy Strategy for Flow Problems

Real situations allow to establish that flow capacities as well as flows themselves have a fuzzy character. An analysis of formed risk can concern underestimation and re-evaluation of capacity or real flow importance. The strategy of flow paths sequence determination is worked out under fuzzy parameter conditions.

В реальных ситуациях емкости потоков, как и сами потоки, имеют нечеткий характер. Дан анализ рисков недооценки или переоценки емкости или реального значения потока. Выработана стратегия определения последовательности пути потока в условиях нечетких параметров.

K e y w o r d s: maximal flow, fuzzy sets.

Structures and characteristics of flow paths. The flow network has a set number of nodes and edges (*nv*, *ne*). It is possible to create a set of paths with usage of an algorithm, an example is presented in Fig. 1 [1—3].

The presented algorithm is based on information concerning the flow network shown, for example, in form of the flow capacity matrix $a[i, j]$ [3, 4]. The source *s* and the sink *t* are also defined. The first two nested loops allow the selection of nodes neighboring at currently analyzed reference point. If that point (node?) is the sink *t*, then one goes towards the successive flow path (label cd). In the node where the path forks, each new direction is continuation of a new path. This is expressed by the counter *k*. Real connection is verified when the weight (flow capacity) of a given edge is checked: whether it is different from «∞» (lack of connection), «0» (full use of flow capacity through the realization of flows), «–» (the same node; edge length is zero). When above mentioned conditions are fulfilled, then a successive edge is added to currently created flow path (with the number $i + k - 1$). The matrix of weights presenting the flow capacities of successive edges of every path (*w*) is also created. If the added node is the sink, then the path is finished (record of path). Each fork increases the number of the path by $k - 1$, where, in this case, *k* denotes the number of edges going out of the current node. The number of successive edges (*p*) is the same for all paths. It is realized in the external loop at the change of path number (*i*). The algorithm performance can be traced on the example shown in Fig. 2.

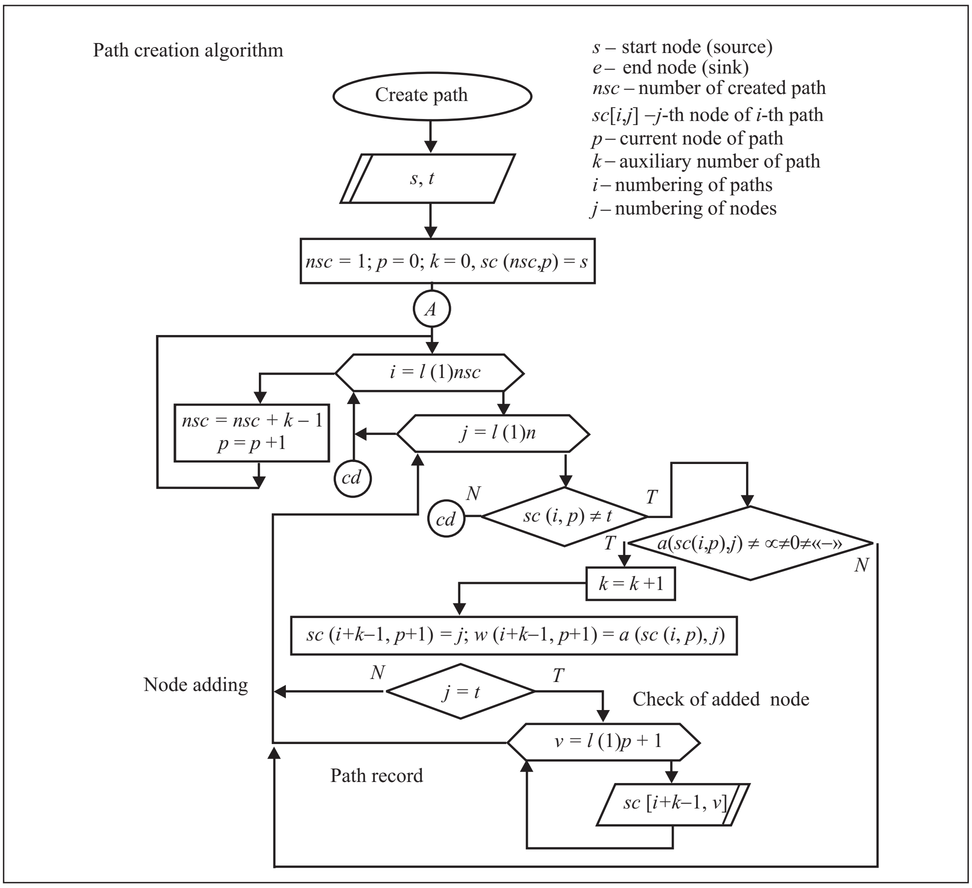
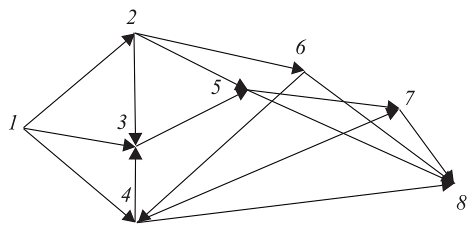


Fig.1. Algorithm of creation of flow paths

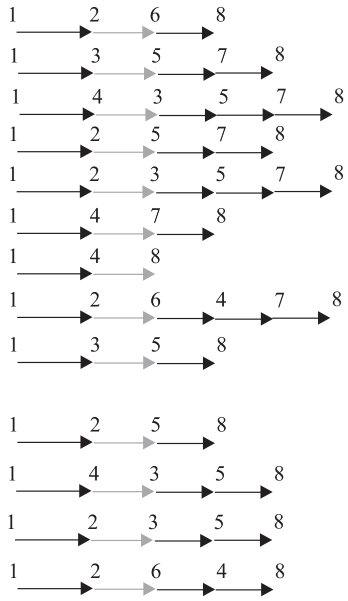
Table 1. **Weight matrix of network connections**

w (i, j)	1	2	3	4	5	6	7	8
1		5	2	8				
2			6		9	4		
3					6			
4			8				11	12
5							10	9
6				7				8
7								7
8								

N o t e. Number (where) *i* — row vector number (where from); *j* — column



a



b

Fig. 2. Example of the algorithm performance: *a* — network graph; *b* — creating paths

The method of selection of successive paths for flow realization decides on the approach to the optimal solution. It is possible to start from the longest paths or from paths with the largest (or smallest) flow capacity [5, 6].

Path selection heuristics for realization of flows. Selection of paths with the minimal flow capacity does not always allow the determination of total network saturation [2, 7]. The sum of small flows can stop larger flows. For the example in Fig. 2, data concerning characteristics can be presented in the following way (Tables 1—3).One can classify the following characteristics:

$l\ (i)$ is a path length (number of edges),where i is a path number;

$p\ (i)$ is a path flow capacity;

dispersion of flow capacity values (variance or standard deviation) for network is rs, for each path is $r\ (i)$ and each layer is $wr\ (j)$, where j is a layer number;

$d\ (i)$ is a edge location (layer number) with the smallest flow capacity in a given path;

$k\ (j)$ is a number of nodes in every layer (location distribution of edges in network).

Having a set of data, one can construct criteria set, which permits testing network saturation. Examples of such sets include paths selection criteria for successive flows realization until the moment of losing connection between the source and the sink [8]. Possible path selection mechanisms are:

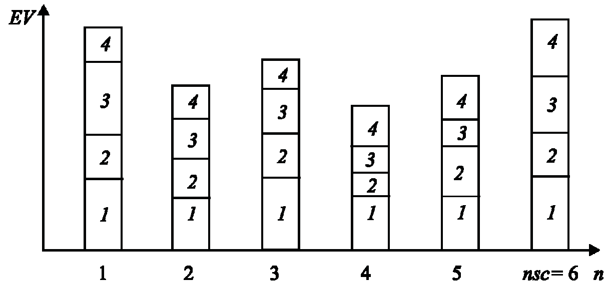


Fig. 3. Influence of criteria (1, 2, 3 and 4) on rank characteristic of paths (1, ..., 6)

Table 2. **Paths characteristic**

Path namber	Node number					l (i)	p (i)	r (i)	d (i)
	1	2	3	4	5				
1	5	4	8			3	4	2,86	2
2	2	6	10	7		4	2	8,19	1
3	8	8	6	10	7	5	6	1,76	3
4	5	9	10	7		4	5	3,69	1
5	5	6	6	10	7	5	5	2,96	1
6	8	11	7			3	7	2,89	3
7	8	12				2	8	4,00	1
8	5	4	7	11	7	5	4	5,76	2
9	2	6	9			3	2	8,22	1
10	5	9	9			3	5	3,56	1
11	8	8	6	9		4	6	1,19	3
12	5	6	6	9		4	5	2,25	1

Table 3. **Layers characteristic**

Layer number	1	2	3
$wr(j)$	4,96	6,8	3,5

a) maximal path length; minimal flow capacity; maximal flow capacity dispersion; minimal edge weight in neighborhood of layer with the smallest flow capacity dispersion;

b) maximal flow capacity; minimal flow capacity dispersion; minimal path length; maximal distance with minimal weight from the source and the sink;

c) minimal flow capacity; minimal weight in maximal fork; maximal dispersion in layer with minimal weight; minimal weight out of the source and the sink;

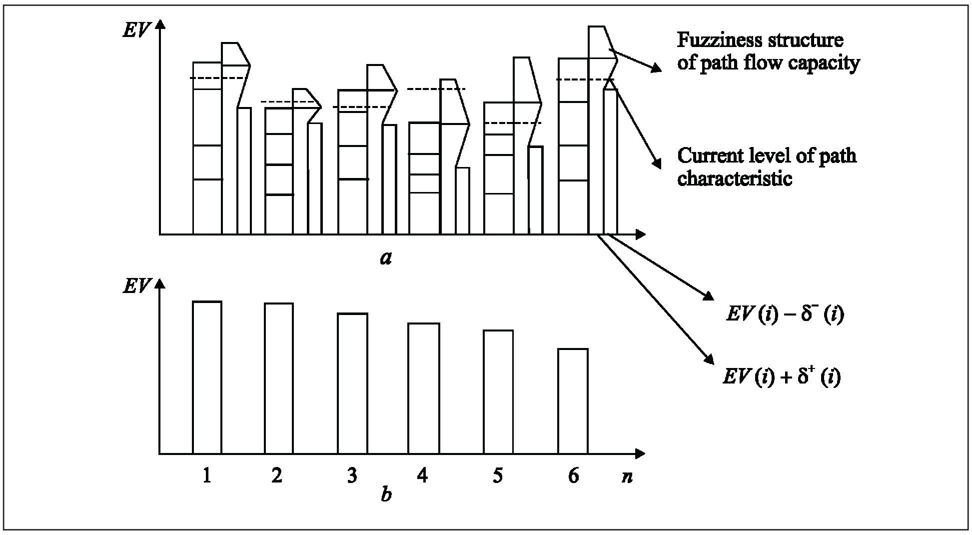


Fig. 4. Influence of flow capacity fuzziness on path characteristic according to selected criteria set (*a*) and on selection sequence of paths (*b*) (with reference to situation shown in Fig. 3)

d) minimal path length; maximal flow capacity; maximal flow capacity dispersion; minimal weight in layer with maximal flow capacity dispersion.

The terms «maximal» and «minimal» in criteria sets were used for simplification, pointing only at the tendency of paths ordering. Furthermore, each criterion is connected with a specific weight $wk(z, i)$, where z is a criteria set code; i is a criterion successive number. The following expression defines the final characteristic of the analyzed path:

$$EV(z,p) = \sum_{i=1}^{np} x(i)\, wk(z,i),$$

where i is a path number; np is a criteria number in set z; $x(i)$ is a path location number in order, according to selected (i-th) criterion, set of paths. The criteria weights in selected successive sets are decreasing: $wk(z, 1) > wk(z, 2) > ... > wk(z, np)$.

The influence of individual criterion on the final estimation of path usefulness for the flow realization is diversified (Fig. 3). The paths in 6-1-3-5-2-4 order were selected for the flow realization.

Thus, the selection of concrete heuristic for network saturation test [8] comes down to the selection of the criteria set, which depends on network characteristics (paths and layers).

Influence of fuzziness of flow capacity parameters on results of paths scheduling. Fuzziness of flow capacity influences paths selection sequence for

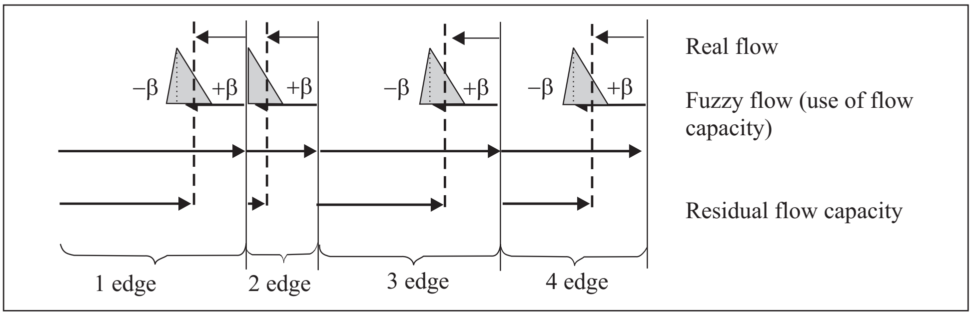


Fig. 5. Correction of network segments flow capacity at fuzzy flow realization

the realization of flows [1]. Fig. 4 presents an example of influence of fuzziness on paths selection sequence.

After selecting a path, one can realize the flow operation subtracting the path flow capacity from the current flow capacity value of all the edges:

$$p^{(j)}(i) = p^{(j)}(i) - p(i)_{-\beta}^{+\beta},$$

where j — edge number; i — path number; $(+\beta, -\beta)$ — flow fuzziness parameters [5]. Flow realization along the path is shown in Fig. 5.

Conclusions. To estimate the scale of fuzziness influence on the network saturation level, it is necessary to analyze all permutations of lower and upper flows limitations for every edge of all paths making additions with lower and upper flow capacity limitations.

The number of path permutations is in the pessimistic variant (for every set of paths one realizes as many flows — from the source to the sink — as paths) equals *nsc*!

The number of variations of edges, taking into consideration lower and upper flow capacity of each of them, equals $2^{nkr(s)}$; where $nkr\ (s)$ — number of edges of s-th path.

Fuzziness of flow conditions through the selected path dictates allowance for lower and upper flow limit.

Optimization of network saturation requires analyzing $2nsc!\prod_{s=1}^{nsc} 2^{nkr(s)}$ data sets.

У реальних ситуаціях ємність потоків, як і самі потоки, мають нечіткий характер. Дано аналіз ризиків недооцінки чи переоцінки ємності чи реального значення потоку. Вироблено стратегію визначення послідовності шляху потоку в умовах нечітких параметрів.

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