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Analysis and Optimization of M/G/1 Vacation Queueing Systems with Server Timeout

We consider a single-server vacation queueing system that operates in the following manner. When the server returns from a vacation it observes the following rule. If there is at least one customer in the system, the server commences service and serves exhaustively before taking another vacation. If the server finds the system empty, it waits a fixed time c. At the expiration of this time the server commences another vacation if no customer has arrived; otherwise, it serves exhaustively before commencing another vacation. Analytical results are derived for the mean waiting time in the system. The timeout scheme is shown to be a generalized scheme of which both the single vacation and multiple vacations schemes are special cases, with $c=\infty$ and $c=0$ respectively. The model is extended to the N-policy vacation queueing system. In both schemes we use a linear cost model to obtain an optimal operating value of c.

Рассмотрена односерверная система массового обслуживания (СМО) с перерывами, работающая в таком режиме: при включении сервера после перерыва, если, по крайней мере, один клиент находится в системе, сервер начинает обслуживание и продолжает его до наступления очередного перерыва. Если обнаруживается, что система пуста, сервер находится в режиме ожидания фиксированное время c. По истечении этого времени наступает следующий перерыв в работе сервера, если новый клиент не появился. В противном случае, клиент обслуживается до наступления очередного перерыва. Получены аналитические оценки для среднего времени ожидания в системе. Показано, что схема прерываний является обобщенной схемой, в которой единичная и множественная схемы прерываний — частные случаи соответственно при $c=\infty$ и $c=0$. Модель распространяется на СМО с N-стратегиями перерывов. В обеих схемах использована линейная модель затрат для получения оптимального параметра срабатывания c.

K e y w o r d s: vacation queueing systems, timeout policies, performance analysis, N-policy with timeout.

Introduction. Vacation queueing systems have been extensively analyzed by several authors. A survey of vacation queues is given in [1], and an excellent text on the subject is [2]. There are several models of queueing systems, including the single vacation system and the multiple vacation system. In the single vacation queue, a server's vacation begins whenever the system becomes empty. At the end of the vacation, the server returns to begin serving the customers that arrived during its vacation, if such customers exist; otherwise, it waits until a customer arrives when a busy period commences. The time to serve customers and the du-

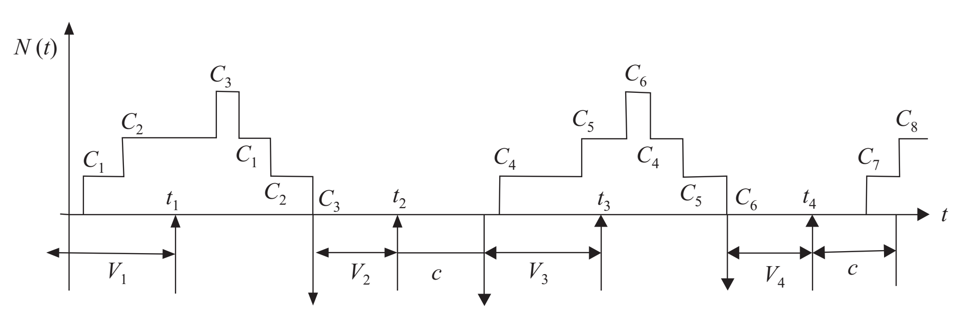

Fig. 1. Vacation scheme with timeout

ration of a vacation are assumed to be mutually independent. The multiple vacation queue operates in a manner similar to the single vacation queue with the exception that if no customers are found at the end of a vacation, the server immediately commences another vacation.

A variety of problems can be modeled by the vacation queueing system. These include machine breakdowns, maintenance in communication and computer systems, and token passing local area networks. In these systems the server becomes absent from a particular service center either because it is busy elsewhere serving other customers or is unavailable due to system breakdown.

In the vacation models that have been analyzed in the literature, server timeouts have not been considered. In this paper we consider vacation queueing systems with server timeouts. Specifically, we consider a system that operates in the following manner. When the server has finished serving a customer and finds the system empty, it takes a vacation whose duration is independent of both the service time and the inter-arrival time. At the end of the vacation the server returns to serve those customers, if any, who arrived during its vacation. It will commence another vacation when the system becomes empty. If no customer arrived during the vacation, the server waits for a fixed time *c*. If no customer arrives by the end of this period, the server commences another vacation. If a customer arrives before the period expires, the server commences service and serves exhaustively before commencing another vacation.

Fig. 1 illustrates this scheme. Assume that the system has been operational for some time and the server returns from a vacation of duration V_1 at a time labeled t_1 and found two customers C_1 and C_2 waiting. Before completing the service of these customers, another customer C_3 arrives. After serving the three customers the server takes another vacation of duration V_2. Upon returning from that vacation at a time t_2 it found no customer waiting. It waits for a time of duration *c* without any customer arriving. It thus leaves for another vacation of duration V_3 without serving a customer. It came back from the vacation at the time labeled t_3 and found two customers C_4 and C_5 waiting. Before completing the service of these customers, another customer C_6 arrives. After serving these three customers, the server takes another vacation of duration V_4. Upon returning from the vacation at the time labeled t_4 it found no waiting customers. However, before the expiration of the timeout, customer C_7 arrives to initiate another busy period, and the process continues.

This vacation queueing is essentially a hybrid multiple and single vacation scheme that was introduced in [3]. The main objective of the model in [3] was to demonstrate how vacation queueing systems with exponentially distributed service times and finite population could be modeled by the stochatic Petri net. In this paper we assume that the population is infinite and service times have arbitrary distribution.

One important application of the vacation scheme with server timeout is to enhance resource utilization in the single vacation model. Specifically, if there is a problem in the arrival process that prevents customers from arriving for service, the server may be idle indefinitely after returning from a vacation in the single vacation. But when the idle time is bounded as described above, the server can be used to perform other functions at the expiration of the timeout rather than wait indefinitely. Thus, when the source is subject to breakdown, such as the disruption of the communication links along which messages arrive in a communication system, the server timeout scheme prevents the server from waiting indefinitely for customers to arrive after returning from a vacation.

Two other hybrid vacation schemes have been proposed. In [4], if the server returns from the $(i-1)$-th vacation and finds the system empty, it takes another vacation with probability p_i and waits for the first customer to arrive with probability $1-p_i$. In the later case the server takes a vacation after serving exhaustively. In [2], if the server returns from a vacation and finds the system empty, it takes at most J vacations repeatedly until it finds at least one customer waiting in the system when it returns from a vacation. If no customer arrives by the J-th vacation, the server waits until a customer arrives.

Analysis of the model. We assume that customers arrive at the system according to a Poisson process with rate λ. The time, X, to serve a customer has a general distribution with cumulative distribution function (CDF) $F_X(x)$, mean $E[X]$ and second moment $E[X^2]$. The Laplace—Stieltjes transform of X is $M_X(s)$, which is defined by (see, for example, [5]):

$$M_X(s)=E[e^{-sX}]=\int_0^\infty e^{-sx}dF_X(x).$$

The duration, V, of a vacation is also assumed to have a general distribution with CDF $F_V(x)$ and Laplace—Stieltjes transform $M_V(s)$. The mean of V is $E[V]$ and its second moment is $E[V^2]$. X and V are assumed to be mutually independent. Let the random variable A denote the number of customers in the system at the beginning of a busy period. The probability mass function (PMF) of A is $p_A(a)$ whose z-transform is given by $G_A(z)$. That is,

$$G_A(z)=E[z^A]=\sum_{a=0}^{\infty} z^a p_A(a).$$

The mean of A is $E[A]$ and its second moment is $E[A^2]$. Let the random variable B denote the number of customers left behind by an arbitrary departing customer, and let L denote the number of customers in the system at an arbitrary point in time. The PMF of B is $p_B(b)$ whose z-transform is given by $G_B(z)$. The mean of B is $E[B]$ and its second moment is $E[B^2]$. Similarly, The PMF of L is $p_L(l)$ whose z-transform is given by $G_L(z)$. The mean of L is $E[L]$ and its second moment is $E[L^2]$. Let W_q denote the waiting time in the system, and let the utilization factor ρ be defined as $\rho=\lambda E[X]$. The main result of the analysis is the following.

Theorem. The mean waiting time in the system is given by

$$E[W_q]=\frac{\lambda E[V^2]}{2\{[1-e^{-\lambda c}]M_V(\lambda)+\lambda E[V]\}}+\frac{\lambda E[X^2]}{2(1-\rho)}.$$

P r o o f. As shown in [6], a vacation queueing system can be analyzed by the following decomposition:

$$G_L(z)=G_B(z)\,G_{L(\mathrm{M/G/1})}(z),$$

where $G_{L(\mathrm{M/G/1})}(z)$ is the z-transform of the number of customers in the system in a standard M/G/1 queue (i.e., one in which the server never takes a vacation). In [7] it is shown that

$$G_B(z)=\frac{1-G_A(z)}{(1-z)\,E[A]},$$

$$G_{L(\mathrm{M/G/1})}(z)=\frac{(1-\rho)(1-z)\,M_X(\lambda-\lambda z)}{M_X(\lambda-\lambda z)-z}.$$

Thus, applying Little's law [8] we obtain the mean waiting time as

$$E[W_q]=\frac{1}{\lambda}\frac{d}{dz}G_L(z)\Big|_{z=1}-E[X]=\frac{E[A^2]-E[A]}{2\lambda E[A]}+\frac{\lambda E[X^2]}{2(1-\rho)}.$$

This means that once $G_A(z)$ is known we can obtain $E[W_q]$. The remainder of the proof is devoted to deriving the expressions for $G_A(z)$ and $M_W(s)$, the Laplace—Stieltjes transform of the waiting time.

Consider the following three mutually exclusive events associated with the server's experience after returning from a vacation.

1.The server returns from vacation, waits and commences another vacation without serving a customer; the probability of this event is $M_V(\lambda)\, e^{-\lambda c}$.

2. The server returns from vacation, waits and serves at least one customer before taking another vacation; the probability of this event is $M_V(\lambda)\{1-e^{-\lambda c}\}$.

3. The server returns from vacation and finds at least one waiting customer; the probability of this event is $1-M_V(\lambda)$.

Under event 2, a busy period is initiated with exactly one customer in the system. Similarly, under event 3, a busy period is initiated with at least one customer in the system. Therefore, if we define p_k as the probability of event k, given that a busy period was initiated before the server commences another vacation, where $k=2,3,$ then we obtain the following result:

$$G_A(z)=zp_2+\frac{M_V(\lambda-\lambda z)-M_V(\lambda)}{1-M_V(\lambda)}\,p_3,$$

where

$$p_2=\frac{M_V(\lambda)\{1-e^{-\lambda c}\}}{1-e^{-\lambda c}M_V(\lambda)},\;\; p_3=\frac{1-M_V(\lambda)}{1-e^{-\lambda c}M_V(\lambda)}.$$

From this we obtain the result:

$$E[A]=p_2+\frac{\lambda E[V]}{1-M_V(\lambda)}\,p_3,\;\; E[A^2]=E[A]+\frac{\lambda^2 E[V^2]}{1-M_V(\lambda)}\,p_3,$$

$$E[W_q]=\frac{\lambda E[V^2]}{2E[A][1-e^{-\lambda c}M_V(\lambda)]}+\frac{\lambda E[X^2]}{2(1-\rho)}=$$

$$=\frac{\lambda E[V^2]}{2\{[1-e^{-\lambda c}]\,M_V(\lambda)+\lambda E[V]\}}+\frac{\lambda E[X^2]}{2(1-\rho)},$$

which completes the proof. We consider limiting cases for c:

$$\lim_{c\to 0}E[W_q]=\frac{E[V^2]}{2E[V]}+\frac{\lambda E[X^2]}{2(1-\rho)},$$

$$\lim_{c\to \infty}E[W_q]=\frac{\lambda E[V^2]}{2\{M_V(\lambda)+\lambda E[V]\}}+\frac{\lambda E[X^2]}{2(1-\rho)}.$$

These are the results obtained in [9] for the multiple vacation system and single vacation system, respectively. Note that $E[W_q]$ monotonically decreases as c increases since

$$\frac{dE[W_q]}{dc} = \frac{-\lambda^2 E[V^2]\, M_V(\lambda)\, e^{-\lambda c}}{2\{[1-e^{-\lambda c}]\, M_V(\lambda)+\lambda E[V]\}^2} < 0$$

which is consistent with the fact that the single vacation scheme ($c=\infty$) has a smaller mean waiting time than the multiple vacation scheme ($c=0$).

Also, in [2] it is shown that the Laplace—Stieltjes transform of the waiting time is given by

$$M_W(s) = \frac{\lambda(1-\rho)\{1-G_A(1-s/\lambda)\}}{E[A]\{s-\lambda+\lambda M_X(s)\}}.$$

Applying this result to our model yields

$$M_W(s) = \frac{(1-\rho)\{\lambda[1-M_V(s)]+s(1-e^{-\lambda c})\, M_V(\lambda)\}}{\{s(1-e^{-\lambda c})\, M_V(\lambda)+\lambda E[V]\}\{s-\lambda+\lambda M_X(s)\}}$$

which, on differentiation and evaluating at $s=0$, gives the same value of $E[W_q]$ obtained earlier.

Although the model has been analyzed with fixed timeout c, the analysis can be extended to the case where the timeout is a random variable T with mean $E[T]$ and Laplace—Stieltjes transform $M_T(s)$. In this case only a slight modification is required in the results. Specifically, we replace the factor $e^{-\lambda c}$ with $M_T(\lambda)$.

Optimal timeout design. As stated earlier,

$$\frac{dE[W_q]}{dc} = \frac{-\lambda^2 E[V^2]\, M_V(\lambda)\, e^{-\lambda c}}{2\{[1-e^{-\lambda c}]\, M_V(\lambda)+\lambda E[V]\}^2} < 0$$

which means that $E[W_q]$ decreases as c increases. Thus, $c=\infty$ provides the smallest mean waiting time. However, one of the benefits of a vacation queueing system is to engage the server in other activities when the queue is empty. This means that any idle time incurs some cost to the system operator. For ease of analysis, we assume that a linear cost is associated with idle times. Thus, the cost incurred in the time interval c is kc, where $k>0$. To further simplify the analysis we assume that $k=1$. We also assume that there is a unit cost per unit mean waiting time. Therefore, we formulate the following optimization problem:

$$\text{Minimize } S(c) = E[W_q]+c$$
$$\text{subject to } c \ge 0.$$

The solution to the problem satisfies the condition $\frac{d}{dc} S(c) = 0.$ That is,

$$\frac{-\lambda^2 E[V^2] M_V(\lambda) e^{-\lambda c}}{2\{[1-e^{-\lambda c}] M_V(\lambda) + \lambda E[V]\}^2} + 1 = 0.$$

Let $x = e^{-\lambda c}$. Then we obtain

$$2M_V^2(\lambda) x^2 - \{\lambda^2 M_V(\lambda) E[V^2] + 4M_V^2(\lambda) + 4\lambda M_V(\lambda) E[V]\} x +$$
$$+ \{2\lambda^2 (E[V])^2 + 2M_V^2(\lambda) + 4\lambda M_V(\lambda) E[V]\} = 0.$$

The solution to the equation is

$$x = \frac{\{\lambda^2 E[V^2] + 4M_V(\lambda) + 4\lambda E[V]\} \pm \lambda \sqrt{E[V^2]\{\lambda^2 E[V^2] + 8M_V(\lambda) + 8\lambda E[V]\}}}{4M_V(\lambda)}.$$

It can be shown that

$$\lambda \sqrt{E[V^2]\{\lambda^2 E[V^2] + 8M_V(\lambda) + 8\lambda E[V]\}} < \lambda^2 E[V^2] + 4M_V(\lambda) + 4\lambda E[V].$$

Thus, we use the smaller of the two solutions as the feasible solution and obtain

$$x^* = \frac{\{\lambda^2 E[V^2] + 4M_V(\lambda) + 4\lambda E[V]\} - \lambda \sqrt{E[V^2]\{\lambda^2 E[V^2] + 8M_V(\lambda) + 8\lambda E[V]\}}}{4M_V(\lambda)}.$$

From this we obtain $c^* = -1/\lambda \ln\{x^*\}$.

The *N*-policy with timeout. The *N*-policy was introduced in [10] and operates as follows. The server goes on vacation at the end of a busy period. The vacation ends with the arrival of the *N*-th customer since the end of the last busy period.

The *N*-policy scheme with timeout operates as follows. Consider cycles of busy periods and let T_{il} denote the arrival time of the *i*-th customer in the *l*-th cycle, where $i = 1, 2, \ldots, N$, and $l = 1, 2, \ldots$. Starting from the arrival of the first customer in a given cycle, if less than $N - 1$ other customers arrive within the time interval *c*, then the server's vacation ends and service is performed exhaustively at the end which another vacation begins. Assume that the system is empty and, therefore, the server is on the *l*-th vacation. The vacation ends at time T_l given by $T_l = \min(T_{1l} + c, T_{Nl})$ $l = 1, 2, \ldots$

That is, the vacation ends at time T_{Nl} (when the *N*-th customer arrives) or $T_{1l} + c$ (if less than $N-1$ customers arrive within the timeout period *c* measured from when the first customer arrived), whichever comes first. This means that each busy period begins with either *N* customers, if $T_{NL} < T_{1l} + c$, or *n* customers, $1 \le n \le N-1$, otherwise. The server serves exhaustively and takes another vaca-

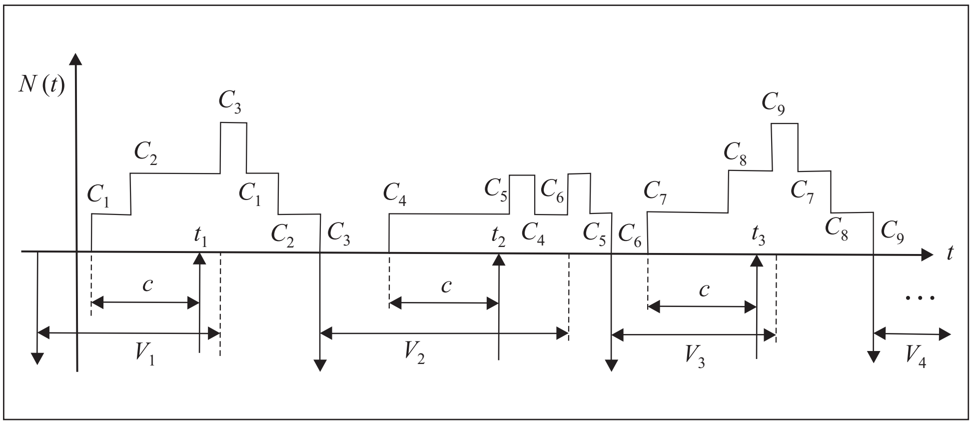

Fig. 2. N-policy scheme with timeout

tion when the system becomes empty, whatever number of customers the busy period begins with.

The scheme is illustrated in Fig. 2, where it is assumed that $N = 3$. In the figure, during the server's first server vacation customer C_1 arrives and thus activates the timer. By the time the timeout expired only one other customer, C_2, arrived. Since the timeout has expired with less than 3 customers in the system, the server initiates a busy period with two customers at time t_1. Another customer, C_3, arrived during this busy period. After serving all 3 customers, the server takes a vacation. While the server is on vacation, customer C_4 arrives to activate the timer for the next cycle. The time expires at time t_2 and the server initiates a busy period at that time with only one customer. During this busy period, customers C_5 and C_6 arrive and are served before the server commences another vacation. While the server is on vacation, customer C_7 arrives and activates the time. Before the timer expires customer C_8 arrives. The timer expires at time t_3 and the server initiates a busy period at that time with two customers. During that busy period customer C_9 arrives and is served before the server commences another vacation. The process continues, as shown in the figure. The intervals labeled V_i, $i = 1, 2, ...$, indicate the vacation intervals under the normal N-policy.

The analysis of the model is similar to that for the single vacation with server timeout. In the current scheme, a busy period will commence with exactly N customers in the system with probability q_N which is the probability of $N-1$ or more Poisson arrivals in an interval of length c and thus is given by

$$q_N = 1 - \sum_{n=0}^{N-2} \frac{(\lambda c)^n}{n!} e^{-\lambda c}.$$

Similarly, a busy period will commence with n customers, $1 \le n \le N-1$, with probability q_n, which is the probability of exactly $n-1$. Poisson arrivals in an interval of length c and is given by

$$q_n = \frac{(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c}, \quad 1 \le n \le N-1.$$

Therefore,

$$G_A(z) = z^N q_N + \sum_{n=1}^{N-1} z^n q_n = z^N \left[1 - \sum_{n=0}^{N-2} \frac{(\lambda c)^n}{n!} e^{-\lambda c}\right] + \sum_{n=1}^{N-1} z^n \frac{(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c},$$

$$E[A] = N\left[1 - \sum_{n=0}^{N-2} \frac{(\lambda c)^n}{n!} e^{-\lambda c}\right] + \sum_{n=1}^{N-1} \frac{n(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c} =$$

$$= N - \sum_{n=1}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c}.$$

If we define D as the event that there are N customers at the beginning of a busy period and E as the event that there are less than N customers at the beginning of a busy period, then $p_D = q_N$, $p_{E|n} = q_n$.

Mimicking the method used in [2] we have the following conditional Laplace—Stieltjes transform of the waiting time:

$$M_W(s|D) = \frac{(1-\rho)\{[\lambda/(s+\lambda)]^N - [M_X(s)]^N\}}{N\{[\lambda/(s+\lambda)] - [M_X(s)]\}} + \frac{\lambda(1-\rho)\{1-[M_X(s)]^N\}}{N\{s-\lambda+\lambda M_X(s)\}},$$

$$M_W(s|E,n) = \frac{(1-\rho)\{[\lambda/(s+\lambda)]^n - [M_X(s)]^n\}}{n\{[\lambda/(s+\lambda)] - M_X(s)\}} + \frac{\lambda(1-\rho)\{1-[M_X(s)]^n\}}{n\{s-\lambda+\lambda M_X(s)\}}.$$

Thus, the unconditional Laplace—Stieltjes transform of the waiting time is given by

$$M_W(s) = M_W(s|D)\, p_D + \sum_{n=1}^{N-1} M_W(s|E,n)\, p_{E|n} =$$

$$= M_W(s|D)\, p_D + \sum_{n=1}^{N-1} M_W(s|E,n) \frac{(\lambda c)^{\nu-1}}{(n-1)!} e^{-\lambda c}.$$

From this we obtain the mean waiting time as

$$E[W_q] = \frac{N-1}{2\lambda} p_D + \frac{\lambda E[X^2]}{2(1-\rho)} p_D + \sum_{n=1}^{N-1} \left\{\frac{\lambda E[X^2]}{2(1-\rho)} + \frac{n-1}{2\lambda}\right\} \frac{(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c} =$$

$$= \frac{N-1}{2\lambda} + \frac{\lambda E[X^2]}{2(1-\rho)} - \sum_{n=1}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c} = \frac{E[A]-1}{2\lambda} + \frac{\lambda E[X^2]}{2(1-\rho)}.$$

Before considering the limiting values of c, we recall that the expected value of A is given by

$$E[A] = N - \sum_{n=1}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c} =$$

$$= N - (N-1)e^{-\lambda c} - \sum_{n=2}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c} = e^{-\lambda c} - \sum_{n=2}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c}.$$

Thus, $\lim_{c\to 0} E[A] = 1$ and $\lim_{c\to\infty} E[A] = N$. This implies that

$$\lim_{c\to 0} E[W_q] = \frac{\lambda E[X^2]}{2(1-\rho)},$$

$$\lim_{c\to\infty} E[W_q] = \frac{N-1}{2\lambda} + \frac{\lambda E[X^2]}{2(1-\rho)},$$

which are the results for a standard M/G/1 queue with no server vacation and a standard N-policy scheme without timeout, as shown in [10] respectively. We observe that

$$\frac{dE[W_q]}{dc} = \lambda e^{-\lambda c} \left\{ \sum_{n=1}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} - \sum_{n=2}^{N-1} \frac{(N-n)(\lambda c)^{n-2}}{(n-2)!} \right\} > 0$$

which means that the expected waiting time monotonically increases with c. Therefore, the optimal value of c is zero.

Summary. We have derived expressions for the mean waiting time of a vacation queueing system in which the server does not immediately take another vacation upon returning from a vacation and finding the system empty, as in the multiple vacation scheme, or wait indefinitely for a customer to arrive, as in the single vacation scheme. In the proposed model, if the server returns from a vacation and finds the system empty, it waits for a fixed time c. If at the expiration of this time no customer arrives, the server will take a vacation; otherwise it serves arriving customers exhaustively before taking another vacation. The results of the analysis are consistent with those of the multiple vacations scheme (where $c = 0$, and the single vacation scheme where $c = \infty$). A linear cost model was assumed to obtain the optimal value of c for the assumed model.

The model is also extended to the N-policy scheme where the timeout is measured from the arrival of the first customer since the end of the last busy period. The results have also been shown to be consistent with earlier results for the case when $c=0$, which is the standard M/G/1 queue, and the case when $c=\infty$, which is the *N*-policy scheme without timeout. It is shown that the expected waiting time increases monotonically with *c*, which means that $c=0$ gives the minimum expected waiting time.

Розглянуто односерверну систему масового обслуговування (СМО) з перервами, що працює у такому режимі: при включенні сервера після перерви, якщо хоча б один клієнт перебуває у системі, сервер починає обслуговування і продовжує його до наступної перервиикщо виявляється, що система є пустою, сервер перебуває у режимі очікування фіксований час *c*. По закінченні цього часу починається наступна перерва у роботі сервера, якщо новий клієнт не з'явився. У протилежному випадку клієнт обслуговується до початку чергової перерви. Отримано аналітичні оцінки для середнього часу очікування в системі. Показано, що схема переривань є узагальненою схемою, в якій одинична та множинна схеми переривань — окремий випадок відповідно при $c=\infty$ і $c=0$. Модель розповсюджується на СМО з *N*-стратегіями переривань. У обох схемах використано лінійну модель витрат для отримання оптимального параметра спрацьовування *c*.

1. *Doshi B.T.* Queueing Systems with Vacations - A Survey // Queueing Systems.— 1986. — Vol. 1. — P. 29—66.
2. *Takagi H.* Queueing Analysis — A Foundation of Performance Analysis. Vol. 1: Vacation and Priority Systems. — Amsterdam: North-Holland, 1991.
3. *Ibe O.C., Trivedi K.S.* Stochastic Petri Net Analysis of Finite-Population Vacation Queueing Systems// Queueing Systems. — 1991. — Vol. 8.— P. 111—128.
4. *Kella O.* Optimal Control of the Vacation Scheme in an M/G/1 Queue// Operations Research. —1990. — Vol. 38. — P. 724—728.
5. *Ibe O.C.* Fundamentals of Applied Probability and Random Processes. — Burlington, Massachusetts: Elsevier Academic Press, 2005.
6. *Fuhrmann S.W., Cooper R.B.* Stochastic Decompositions in the M/G/1 Queue with Generalized Vacations// Operations Research. — 1985. — Vol. 33. — P. 1117—1129.
7. *Kleinrock L.* Queueing Systems Vol. 1: Theory.— N. Y.: John Wiley & Sons, 1975.
8. *Little J.D.C.* A Proof of the Formula: $L=\lambda W$//Operations Research. — 1961. — Vol. 9. — P. 383 — 387.
9. *Levy Y., Yechiali U.* Utilization of Idle Time in an M/G/1 Queueing System// Management Science. — 1975. — Vol. 22. — P. 202—211.
10. *Yadin M., Naor P.* Queueing Systems with a Removable Service Station//Operational Research Quarterly. — 1963. — Vol. 14. — P. 393—405.

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