A. Katkow, A. Ulfik
Faculty of Management
Czestochowa University of Technology
(E-mail: ulfik@zim.pcz.pl)

Network Implementation of Investment Management

This paper presents simulation of selection elements of portfolio applying classical Markowitz's portfolio analysis theory using parallel computational environment – cellular automata. Simulation and its results for parallel computational environment are presented.

Рассмотрено моделирование выбора элементов портфеля на основе классической теории анализа портфеля заказов Марковитца с использованием параллельной вычислительной среды — клеточного автомата. Приведены результаты моделирования в параллельной вычислительной среде.

K e y w o r d s: simulation, portfolio selection, cellular automata

The use of computers in portfolio analysis is necessary because of computational complexity. Imitating occurrences in nature lets us have very good results. The most popular usage of such processes are genetic algorithms and neural networks. Another one is a cellular automaton. Cellular automata were created in fortieth of the last century to emulate processes occurring in nature. Soon after this invention, cellular automata occurred very interesting and useful. First physicists with big success used them to simulation of complicated issues. Today cellular automata are being used also in mathematic, mechanic, economy, graphic (for example creating textures and fractals), sociology (for example spreading epidemics or crowd behavior), computer games and many other fields. In spite of this, there are still some researches done to find new application for cellular automata. One of them can be problem of selection of portfolio in stock market.

Portfolio analysis. H. Markowitz in 1952 had published his first paper about portfolio selection [1]. It has started a real revolution for capital markets and created completely new technique of making investment decisions today, called portfolio analysis. Although this theory was showing how to choose the best stocks to get the highest income with lowest level of risk, it was very hard to apply it in practice because of computational complexity. After few years another scientist W. Sharp has simplified this model and made portfolio analysis able to apply in practice [2]. He has added to this theory two coefficients giving investors a clear hint which stocks are giving better results than a general tendency on considered market. As factor reflecting market tendency he has suggested stock exchange index. On stock market in Poland investors most commonly use Warsaw Stock Exchange Index.

Stock market in Poland is currently strongly developed. Apart from stock, investors can invest money in bonds, futures contracts and also other financial instruments. The main aim for investors is to estimate correctly future value of securities and then choose those with the biggest profit to buy as well as to find the best moment to sell. Problem is securities with high level of expected income are characterized by high level of risk. Investors can reduce this risk by investing in more than one security.

According to theory of portfolio analysis we can predict future values of stock basing on historical quotation. From this data a rate of return is counted, interpreted as expected profit and standard deviation witch is measure of dispersion and is interpreted as risk connected with this expected profit. Investments are usually interested in shares with large profit and low risk level. Portfolio analysis studies how these values will change if we will invest in more than one share. This theory also shows how to choose assets during constructing of portfolio to diversifcate risk which means, that risk of portfolio would be lower than risk of shares that are this portfolio's components.

Rate of return and risk. In classical Markowitz's model of portfolio selection the most important profiles of assets are rate of return and standard deviation. These two values have to be calculated for all shares taken by investor into his consideration. Positive value of return rate is interpreted as expected profit and negative value — as expected loss. This quantity is count in period of time R_t with following formula:

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}},$$

where P_t — the price of securities in period t; P_{t-1} — the price of securities in period $t-1$; D_t — paid-out dividend in period t.

Return rate is appointed for every period of time t, there for it becomes function of time. The real value of income depends on many factors and investor can not be sure that he will get calculated profit. This is the reason that we use expression: expected rate of return. Expected rate of return from securities R is counted with following formula:

$$R = \frac{\sum_{t=1}^{N} R_t}{N},$$

where R_t — the rate of return in period t; N — number of all analyzed rates of return.

Defined in this way level of profit or loss always is accompanying the investment risk. Risk in portfolio analysis is calculated using statistics. Standard deviation S is interpreted as amount of risk. Below is shown formula to calculate variation of stock's rate of return

$$S^2 = \frac{1}{n-1}\sum_{t=1}^{n}(R_t - R)^2,$$

where S^2 — the variation of rate of return; n — number of all analyzed rate of return.

Both values: expected income and risk can be shown on a graph. It is called map of risk and income. This graph makes possible retrieval companies with possibly lowest risk and largest income. In the centre of this graph investors put «market tendency» often understood as those variables counted for stock index. In Poland investors use Warsaw Stock Exchange Index called WIG. In Table 1 shown return rates and standard deviation of WIG calculated on base of different history periods.

Fig. 1 represents map of risk and income for counted all companies' quotation from polish stock market in different time periods. Fig. 1, *a* represents results of computation with one year history, Fig. 1, *b* — with two years and the last with five years of history of quotation. On all figures axis of rate of return — R and standard deviation — S cross in coordinates of WIG market as grey dot.

Too long history can include information that don't have any influence on current situation [3, 4]. Because of that there is no point of calculating rate of return and standard deviation on very long quotation history. On Fig. 1, *a—e* shown above maps of risk and income based on long history are less legible then first one — based on one year history. As we consider longer quotation history, we will find stocks with high level of rate of return interpreted as level of expected income and high level of standard deviation interpreted as risk. Those stocks most probably will not repeat its history in future and there for its consideration by investors is pointless. Fig. 1, *a—e* are showing that the shorter quotations' history is, the better prognosis we get.

Expected rate of return and standard deviation are two basic characteristics of individual securities. When we consider more than one security there is one

Table 1. **Rate of return and standard deviation of WIG depending on quantity of historical quotation**

Time period, year	Number of quotations	*R*	*S*
1	246	0,001395	0,013586
2	498	0,001532	0,011759
3	751	0,001172	0,010756
4	1003	0,000515	0,032951
5	1254	–0,000057	0,029594

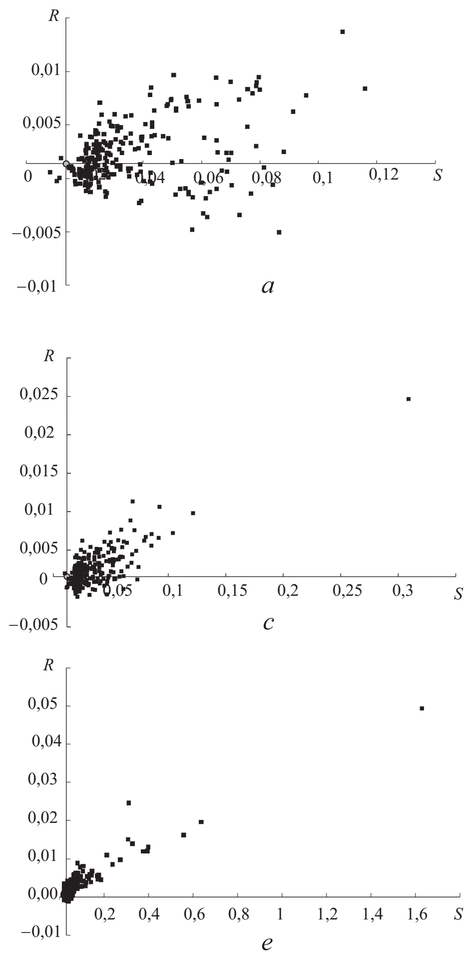

Fig. 1. Map of risk and income for companies' quotation of all companies from polish stock market from *a* — 01.04.06 to 01.04.07; *b* — 01.04.05 to 01.04.07; *c* — 01.04.04 to 01.04.07; *d* — 01.04.03 to 01.04.07; *e* — 01.04.02 to 01.04.07

more important value. It is the coefficient of correlation between two securities. It defines connection of rates of return of two stocks. The coefficient of correlation of rates of return of stock 1 and 2 ρ_{12} can be calculated by following formula:

$$\rho_{12} = \frac{\sum_{i=1}^{n} (R_{1i} - R_1)(R_{2i} - R_2)}{S_1 S_2},$$

where R_{1i} — rate of return in period I; R_1 — expected rate of return from first stock; S_1 — standard deviation of rate of return first stock; R_{2i}, R_2, S_2 — the same for second stock; n — number of all analyzed rate of return.

Effective portfolios. Portfolio is effective when rate of return is higher than for any other portfolio with the same risk; level of risk is lowest from portfolios with the same rate of return.

Problem of selection of effective portfolio is not simple in spite of existence of theoretical solution. With choosing portfolio investor has to first choose securities in which he wants to invest, and then establish how the invested capital will divided among securities.

Two element portfolios. Investors are seeking the possibility of investing capital in stocks with high rate of return. However these securities very often characterize with high level of risk. For investor will be interesting such investments, in which with growth of rate of return, the risk will go down. The portfolio analysis gives us such possibility.

Portfolio is set of stock, which we have or we want to buy. The rate of return from two component portfolio is the sum of rates of return individual values multiplied by their parts in investment

$$R_p = x_A R_A + x_B R_B$$

where x_A — the part of share A in portfolio; R_A — the rate of return of stock A; x_B, R_B — the same for stock B; $0 \le x_A \le 1$; $0 \le x_B \le 1$; $x_A + x_B = 1$.

Calculations of risk for two elements investment portfolio S_p^2 are more complicated. Variation of two component portfolio is define as:

$$S_p^2 = x_A^2 S_A^2 + x_B^2 S_B^2 + 2x_A x_B S_A S_B r_{AB}$$

where r_{AB} — the coefficient of correlation between the rate of return of stock A and B; S_p — standard deviation of two component portfolio.

Many elements portfolios. For more than two elements of portfolio, formulas for rate of return and standard variation are as follow:

$$R_p = \sum_{i=1}^{N} x_i R_i ,$$

$$S_p^2 = \sum_{i=1}^{N} x_i^2 S_i^2 + 2\sum_{i-1}^{N} \sum_{j=1}^{N} x_i x_j S_i S_j r_{ij} .$$

Markowitz's portfolio analysis gives investors the best portfolio, depending on their expectations. Disadvantage of this theory is its numerical complexity. Even thorough we can use very fast multi threads computers and other types of parallel machines, we still can not find the best solution using classical methods for finding the best solution. First of all we have to gather many data and with use of this data, calculate expected rate of return and standard deviation. We have to count this characteristics for all considered stock companies. We also

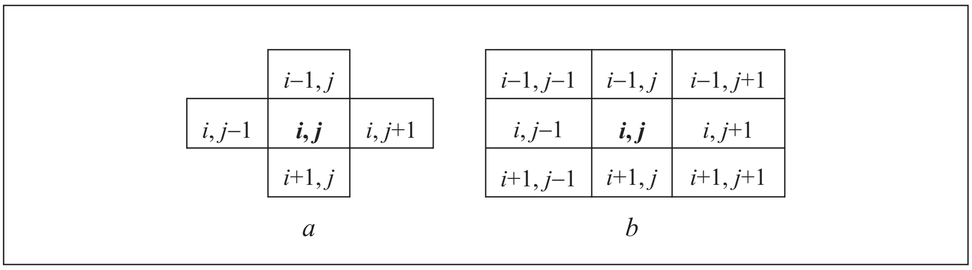

Fig. 2. Two dimensional cellular automata with neighborhood von Neumann (*a*) and Moore (*b*)

need to have coefficients of correlation between all considered companies. There are $\frac{n(n-1)}{2}$ coefficients of correlation, so for 50 stocks we have to count 1225 coefficients of correlation, for 100 stocks – 4950 coefficients and for 265 stocks (on 01.04.2007 Warsaw Stock Exchange had 265 companies) 34 980 coefficients of correlation.

When we will count all necessary characteristics we have to choose portfolio. Number of possible portfolios is $2^n - 1$, where n is number of stocks. For 5 stocks we can have 31 possible portfolios, for 10 — already 1023. For 50 stocks we will have 1 125 899 906 842 623 possible portfolio and with 265 stocks this number will reach $5{,}9 \cdot 10^{79}$. Additionally every portfolio can have different percentage of every stock that it includes. To solve this problem we not only need very fast computer but also special technique that will allow to find the most effective portfolio and solution proposed in this paper is cellular automata.

Cellular automata. Cellular automata (CA) are discrete models used mainly in physic, mathematics and computability theory. They were created in forties of last century by Stanisiaw Ulman and later developed by his colleague also working at Los Alamos National Laboratory — John von Neumann. CA are structures of the same cells putted into lattice. It is usually one, two or three-dimensional and involves large number cells (theoretically this number is infinite but it is impossible to implement such model). Every cell has defined type and starting value. It also has its algorithm called function of conversion. This function defines what will be future value depending on values of neighboring cells in current time.

There are two types of neighborhoods in CA: von Neumann’s (Fig. 2, *a*) and Moore’s (Fig. 2,*b*). Von Neumann’s neighborhood is made up of four cells adjacent vertically and horizontally. Eight cells surrounding given cell from right, left, top, bottom and on a slant make up Moore’s neighborhood. In every iteration current value of every cell is calculated on base of values of adjacent cells from previous iteration [5].

The best known example of CA is game «Life» created by John Conway. In this game every cell receives a starting state: it can be active or unactive. This game simulates real environment where animals can be born or day when they don't have enough food. In this game there are settled rules of behavior of every cell. If it is inactive and three other active cells surround it, it comes to life so becomes active. If cell is active and in its surrounding are two or three other active cells, it stays active also. In every different case cell is inactive; it stays inactive if such she was earlier or she days if earlier conditions are not fulfilled. So simple rules leads to astounding solutions [6]. The final effect of CA can be systematize as following:

CA achieves stable state, in which nothing does not it change (all cells stay in determinate state);

state of CA changes cyclically after some quantity of iteration;

CA achieves chaotic state in which it is hard to find any order;

in CA we can find stable local configurations with long time of life.

The CA are already applied in many fields of science. In every issue it is necessary to establish three basic parameters CA:

type of cells creating CA, which means to establish what kind of information they have to contain;

starting value of every cell;

function of passage which is algorithm deciding what will be the state of cells in current iteration on the base of values of neighboring cells in previous iteration.

It is purposeful to check how CA can simulate mechanisms occurring on the stock market [7, 8] and how this simulation can be useful for potential capital markets investors.

Simulation. To find the most effective portfolio we have to establish work of CA and their three basic parameters: type of cells, starting value and function of passage from one iteration to the other. In suggested solution all cells of cellular automata work collectively exchanging information between each other [9].

All cells of CA have one aim: choose the most effective portfolio so they have to include information about composition of this portfolio and during iterations improve it. To be able to choose portfolio, they also have to have access to all input data. Every cell can be interpreted as artificial investor choosing the most efficient portfolio. This artificial investor is working on his own choosing the best result but at the same time he exchanges all information with other artificial investors which are in CA arranged in grid.

At the beginning of simulation all cells of CA get randomly chosen portfolio. Then they calculate the basic characteristics of portfolio: rate of return and standard deviation. Then every cell communicates with its neighbors with Moore's or von Neumann's neighborhood (it is that moment when they work

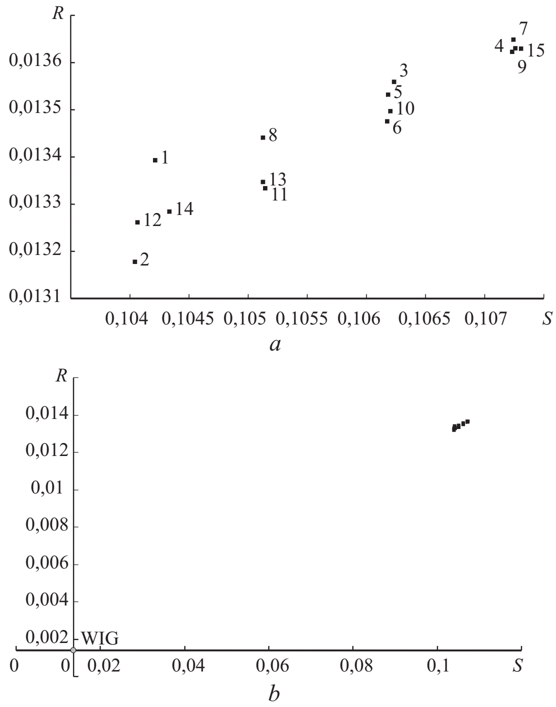

Fig. 3. Map of risk and income with all portfolios from simulation shown above (*a*) and their relation to WIG (*b*)

collectively). If in another cell there is portfolio with better characteristics then cell gets this portfolio from its neighbor. If portfolios in adjoining cells have worse characteristics, then cell stays with its original portfolio (in fallowing formula this process is called **Compare P[k]**). In next step all cells work individually by looking for new, better portfolio. They get this portfolio as completely new, randomly chosen one. If new portfolio is better it stays as current one if not, then it is passed over (in fallowing formula this process is called **Select P[k]**). This is repeated until all cells in CA get the same , most effective portfolio.

Implementation of simulation described above may be given by following code.

Input:

R[i] – array of rates of return of *n* stocks (for i = 1, ..., n)

S[i] – array of standard deviations of *n* stocks (for i = 1, ..., n)

ρ[i][j] – array of coefficients of correlation between socks i and j (for i and j=1, ..., n)

Output:

P[k] – array of portfolio for all cells

Start
Random(P[k])
Repeat
  Compare P[k]
  Select P[k]
until P[i] = P[j] **for** all i and j
Stop

On 01.04.07 on Warsaw Stock Exchange there were 265 stock companies that have enough long history, to be able to calculate their characteristics. For simulation were chosen all quotation with at least 50 days history. Some of companies had just started and had too less quotation. If we would have counted expected rate of return and standard deviation would not realize in future. All characteristics were counted on base of quotation since 01.04.06 till 01.04.07.

Simulation were done in application written in Borland C++ Builder. In those simulation were used two-dimensional CA with 10 000 cells (100 rows and 100 columns in grid). There were formed two component portfolios. The aim of this simulation was achieving stable state by CA, in which all cells have the same portfolio. This simulations was repeated several times with necessary

Table 2. **Efficient portfolios chosen by cellular automata and their characteristics with coefficient of correlation between stock**

Number of simulation	Composition of portfolio (%)	R	S	ρ_{ij}
1	EPL96 + APT4	0,01339289	0,10421372	0,0191
2	EPL96 + ZAP4	0,01317766	0,10404359	0,0505
3	EPL98 + EDR2	0,01355884	0,10623490	0,0176
4	EPL99 + EFK1	0,01363012	0,10725951	0,0216
5	EPL99 + CSS1	0,01353179	0,10618511	0,0762
6	EPL98 + GRL2	0,01347554	0,10617583	0,1121
7	EPL99 + INK1	0,01364862	0,10724388	0,1263
8	EPL97 + CAR3	0,01344098	0,10512677	–0,0683
9	EPL99 + GRJ1	0,01362272	0,10723304	–0,0124
10	EPL98 + TLX2	0,01349710	0,10620266	0,1048
11	EPL97 + AGO3	0,01333366	0,10514575	–0,0061
12	EPL95 + SKA5	0,01326135	0,10406501	0,0671
13	EPL97 + AMB3	0,01334683	0,10512602	0,0332
14	EPL96 + ASL4	0,01328401	0,10433455	0,0125
15	EPL99 + PCG1	0,01362945	0,10730811	0,0516

number of iteration. In cells portfolios changed when new randomly chosen portfolio was more effective then present one.

In simulation as main aim was assumption to achieve portfolio with maximum income and risk as less as it is possible. In Table 2 we can see results of 15 simulations. All results were unique and often did not appear in next execution of simulation even though input data were the same. Because aim was to have maximum income in all portfolios CA has chosen stock EPL which had very high level of expected rate of return. As other component were chosen different companies with different characteristics. In Table 2 are shown those characteristics for all companies chosen by CA in simulation results shown in Table 3 and WIG. Fig. 3, *a* presents map of risk and income with portfolios chosen by CA during simulation; numbers near dots are numbers of portfolios from Table 2. Fig. 3, *b* shows its relation to WIG; WIG is marked as grey dot.

As we can see on above figure all portfolios chosen by cellular automata had better characteristics than characteristics of WIG. In Table 4 we can see how prices of stock companies has changed in two months time. That prices of 4 com-

Table 3. **Stock companies chosen by cellular automata and their characteristics**

Name	*R*	*S*
WIG	0,001395	0,013586
AGO	0,000312	0,027090
AMB	0,000750	0,022069
APT	–0,003240	0,060476
ASL	0,002426	0,068524
CAR	0,003889	0,022262
CSS	0,003505	0,023912
EDR	0,004858	0,043015
EFK	0,003108	0,041203
EPL	0,013736	0,108290
GRJ	0,002367	0,020355
GRL	0,000693	0,020296
INK	0,004958	0,028927
PCG	0,003041	0,078690
SKA	0,001860	0,020722
TLX	0,001771	0,030702
ZAP	–0,000230	0,016641

Table 4. **Stock companies chosen by cellular automata**

Name	Price of stock		Rate of return in 2 months period (%)
	on 01.04.07	on 31.05.07	
WIG	57197,48	63064,92	10,26
AGO	46,82	45,75	–2,29
AMB	19,01	17,37	–8,63
APT	22,10	21,69	–1,86
ASL	5,15	5,15	0
CAR	62,25	120,50	93,57
CSS	45,00	43,00	–4,44
EDR	111,50	119,00	6,73
EFK	38,10	38,90	2,10
EPL	34,00	35,50	4,41
GRJ	67,25	71,35	6,10
GRL	39,10	41,50	6,14
INK	35,80	45,00	25,70
PCG	1,24	2,02	62,90
SKA	44,00	54,00	22,73
TLX	16,20	17,30	6,79
ZAP	65,70	103,60	57,69

panies has drop down to lower level than it was month ago. But at the same time we can see that all others has raided. Prices of 5 companies has raised much more than quotation of WIG (bold font) and this increase was insignificant as for two months time. Because of that cellular automata ca be useful tool for capital investors for choosing the best portfolio for their investments.

Розглянуто моделювання вибору елементів портфоліо на базі класичної теорії аналізу портфеля заказів Марковитця з використанням паралельного обчислювального середовища — клітинного автомата. Наведено результати моделювання у паралельному обчислювальному середовищі.

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