A. Katkow, D.Sc.
Czestochowa University of Technology
(E-mail: katkow@zim.pcz.pl , 42-200 Czestochowa, Poland)

Algorithms of Random Search of Optimum Portfolio of the Investment in the Structure of Coupled Map Lattices

The problems of the optimization of investment, with the use of analysis of a portfolio of investments, should be classified as complex in the computational sense of problem. To solve such problems it is expedient to use parallel computing systems. Realization of more effective algorithms, based on the use of iterative process of the random search, is examined in the paper. The problems of optimization of investment portfolio in the parallel computational environment namely in the coupled map lattices are investigated.

Проблемы оптимизации инвестиций с помощью анализа портфеля инвестиций следует классифицировать как сложные вычислительные задачи. Для решения таких проблем целесообразно использование параллельных вычислительных систем. Рассмотрена реализация эффективных алгоритмов, основанных на использовании итерационных процессов случайного поиска. Исследованы проблемы оптимизации портфеля инвестиций в параллельных вычислительных сетях, а именно в решетках связанных отображений.

K e y w o r d s: portfolio analysis, random search, coupled map lattices.

Optimization of the investment activity of enterprises or individual clients with the great quantity of components of the investment portfolio and a significant quantity of securities is connected with the performing of the great quantity of computational operations. The innovation process, which purpose in this area of investment activity is the acceleration and simplification of the computational process, consists of the use of computing systems with parallel realization of calculations processes. On the other hand this process consists of the development and use of new effective algorithms for solution of the problem of control of the investment process. The paper studies the use of contemporary computing systems, based on realization of computational operations with application of coupled map lattices and algorithmic proposals based on the development of effective algorithms with realization of the process of risk minimization within the desired level with the profitableness of investments on the basis of the random search algorithms.

The constructing of the investment portfolio consists of two processes connected together [1]. The first stage of the portfolio creation consists of searching for an answer to the question: what firms can be included on as the investment portfolio elements. The complexity of the first stages consists in selection of the necessary quantity of firms which will enter in the created investment portfolio. We will call a set of firms, which can form the portfolio, as a basic set of companies. In principle it is possible to compose a basic set of all firms, whose securities are located in the free to sale stock exchange quotation. It is also possible to select the most reliable and thriving firms of their number. In any case their quantity must compose a significant group, that can guarantee that the investment portfolio would have solid foundation. The quantity of firms, which should be selected to be included in the portfolio of the total number of firms from the basic set, is also an important stage of decision.

It is customary to assume that the investment portfolio, which consists of 10 parts, appears as sufficiently well formed, if it is created on the basis of a set of several dozens or may be hundreds of firms, which take part in the process of the stock exchange quotation. About 250 firms take part in the auction process at the Warsaw Stock Exchange of securities. If we attempt to form the investment portfolio of 10 parts, then the number of possible combinations of possible realization of the portfolio reaches the significant magnitude and it can be evaluated by the formula C_n^m, where n is the total quantity of the firms and m — quantity of component parts of the portfolio. The number of possible combinations in the portfolio composition reaches 2^{1017}. Thus the total number of combinations reaches an enormous value. If we consider that the task about the selection of the portion of each portfolio component contribution still must be solved for each portfolio, then solution of the problem about location of the investment portfolio optimum requires applying the most contemporary computers. It is interesting in this context to examine the application of parallel computing techniques to decide the problem of the investment portfolio optimum.

The selection of firms for including in the investment portfolio begins from analysis of financial condition of the firms. The financial state of the firm is determined on the basis of the analysis of the profitableness of firm's securities and of the investments risk. The dependence of the price of the firm stocks on a price change of securities of other participants in the financial processes on the stock exchange is also of great significance. This influence is expressed mathematically as a coefficient of correlation between the price of this firm share and securities of other firms. The second stage of creation of the investments portfolio consists in determination of the part of the portfolio composed by contribution of each of the firms, whose securities were included in its contents. This problem is connected with the determination of value of minimum risk and it is a task of mathematical programming.

The second stage of the problem of creation of the investments portfolio can be recorded for the population of securities, which consists of N elements, in a following manner for standard variation of portfolio as a goal function:

$$\min S_p^2 = \sum_{i=1}^{N} x_i^2 S_i^2 + 2\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j S_i S_j r_{ij}, \tag{1}$$

for given level rate of return

$$R_p = \sum_{i=1}^{N} x_i R_i, \tag{2}$$

and a sum of weight coefficient x_i

$$1.0 = \sum_{i=1}^{N} x_i \tag{3}$$

as boundary conditions [2]. In this equations S_i — the standard deviation of returns of the company shares; S_p — variance portfolio; R_p — expected rate of return of the portfolio; r_i — expected rate of return on shares of the company i; x_i — part of the company shares i in the portfolio; r_{ij} — the correlation coefficient yields shares of the company i and company j. The goal function is a nonlinear function in this case. To solve this problem (1) it is possible to use, for example, the method of Lagrange's coefficients.

As it was mentioned above, the process of creating the optimum portfolio of investments can be divided in two stages, one of which was connected with selection of the portfolio components from a set of securities, which is expedient to invest. The second part of solution of the problem of creating the optimum investments portfolio consists of the search for the coefficients of the participation of each selected security in the portfolio composition that leads to minimization of the goal function (1) within the limitations (2), (3). Both these tasks can be successfully solved by using the parallel computational environment of the coupled map lattice, two algorithms of random search being realized in each of them. The first algorithm of the random search realizes the selection of portfolio components from the base of securities, and the second algorithm of the random search achieves a search for the optimum coefficients of each security participation in the portfolio composition.

The idea of creation of the architecture of parallel computational environment, which by the principles of functioning is similar to the neuron network, belongs to John von Neumann. It is well known architecture Cellular Automata (CA). Coupled Map Lattices (CML) is a generalization of cellular automata [3]. CML is a cellular automaton with information representation in the form of real numbers [4]. In CML structure the computing components in the lattice sites operate with real numbers and their interaction is not limited to Neumann or Moore local neighborhood. In these structures, computing components can initiate processes of information exchange between other computing components that are not located in close proximity. This means that there is an opportunity to influence the local process of a random search of an optimal portfolio by means of the results of a random process of finding the optimal portfolio, implemented in any computational element of CML structures. CML like CA is placed in the category of architectures of the Single Instruction Multiply Data type, according to classification by M. Flynn. In this architecture a single flow of instructions and plural flow of data take place. According to characteristic feature of CML architecture they create computational environment of identical computational elements, connected in such a manner that they create the grid of the computational structure. It is known that high computing speed of CML is caused by wide parallelization of computational process. This circumstance permits optimization both on the component parts of the portfolio, and the values of the portfolio components contribution to one and the same computational environment. In this structure each cell of lattice is connected with four adjoining the same computational cells (P_{ij}), forming the computational network with large computing power.

A very important feature of CML structures, unlike CA is the ability to implement the interaction with computing components located anywhere in the CML structure and not only with neighboring elements as in the case of CA structures. Each of the processors in the computational cell has Random Access Memory, where it has information about the base set of shares, and carries out the same computational algorithm of random search. The essence of the first algorithm of the random search consists in the random searching of the investments portfolio of components of the base of securities. The essence of the second algorithm consists in the fulfillment of the operation of random searching of a value of weight coefficients of the compound parts of the portfolio. Thus the strategy of the search for such investments portfolio is realized in each cell which is nearer as to its parameters to the optimum than the investments portfolios created in the neighboring cells. For this purpose the first algorithm creates portfolio from the randomly selected composition of shares and the second algorithm generates the random contribution of each security or stock to the portfolio composition. In each cell on the basis of those random selected portfolio components the random search for the portfolio parameters is achieved which correspond to the goal function (1) and boundary conditions (2), (3). As a result of comparison the portfolio is determined, which parameters are better and these parameters are transferred to the local neighboring cells.

The portfolio parameters are memorized in each of the local neighboring cells as locally optimum portfolio. Then in each cell a new portfolio composition is created again with the new coefficients of stocks participation in their composition. The parameters of the newly created portfolio are compared with those of the local optimum portfolio. As a result of comparison two versions are possible:

1. A new portfolio has the parameters, which appear to be worse than in the local optimum portfolio. In this case a new portfolio does not remain and the process of creation of the new portfolio is initiated in the cell.

2. A new portfolio has the parameters, which appear to be better than in the local optimum portfolio. In this case the portfolio version is represented for the comparison by the portfolios parameters in the neighboring cells.

3. As a result the comparison of the investments portfolio parameters created in this cell and four neighboring cells such parameters are selected from them which minimize the goal function to the best degree (1).

The example of selection of the component parts of the portfolio from three components with the use of random search method let us examine below. We will simulate a task in the CML structure. Let us accept the following numerical values of the rates of return and risk as variation of stock's rate of return for characteristic of the securities, included in the portfolio. This is test example from [2]:

$$\begin{aligned} &S_1 = 0.25;\ R_1 = 0.05;\ r_{12} = 0.15;\\ &S_2 = 0.21;\ R_2 = 0.1;\quad r_{13} = 0.17;\\ &S_3 = 0.28;\ R_3 = 0.15;\ r_{23} = 0.09;\\ &\qquad R_p = 0.1. \end{aligned} \tag{4}$$

The algorithm of the calculation part of contribution of the components to the composition of the briefcase of investments is represented below.

Input:

X[i] - array of initial value of unknowns
Rate[i] - array of rate of return (i=1,…,n)
Var[i] - array of variation rate of return
Cov[ij] - array of covariation (i=1,…,n,j=1,…,m,i≠j)
Varpold - variation of the portfolio (old)
Varpnew - variation of the portfolio (new)
e - error of the solution (Varpnew-Varpold)
E - maximum of the solution error

Output:

X[i] - array of solution
Varpnew - variation of the portfolio
k - a number of random search iteration

```
k=0
while e <= E do
  k=k+1
  X[1] = random from 0 to 1.0
  X[2] = random from 1.0-X[1]
  X[3] = 1.0 – X[1] – X[2]
  calculate Varpnew
  calculate e
end while
```

We have $n = 3$, $m = 3$ for the test example (4). The algorithm work consists of three stages. In the first stage the random search for weight coefficients X[i], which satisfy the boundary conditions (3) is performed, then Varpnew is calculated, a variation in the rate of return of the portfolio with the assigned rates of return of the corresponding stocks, including in the portfolio (1), and at the end the value e is calculated, which is interpreted as an error and shows how much the value of variation in the rate of return of the portfolio approaches the optimum value of variation on this iteration. Iterative process is ended, if value e becomes less than E which means that the degree of variation approximation to its optimum value becomes less than E. It is possible to finish the fulfilment of the computational process after performing the specific number of iterations.

We simulated the process of solution of the problem about the selection of the optimum portfolio of investments in the CML environment. The results of the process of simulation for test problem (4) are presented in Figure.

Let us calculate the values

$$x_1 = 0.245,\ x_2 = 0.508,\ x_3 = 0.245,\ S_p^2 \min = 0.1668.$$

These results were obtained after 20 iterations in the CML environment. In this case on the diagram the values x_1, x_2, S_p^2 scaled with the coefficient that is equal to 100.

Let us examine possible types of interaction between CA cells with the search for the optimum portfolio of investments. The least rapid method of communication between the cells and the simplest of them is based on the idea of realization of synchronous operation similar to Jacobi iteration technique and the below exchange of information between the cells is named that of Jacobi's type. The essence of this method of exchange consists in the complete realization of the processes of exchange between all CML cells with the fulfillment of one iteration. As a result of the exchange of information between the CA cells one version of portfolio with the best parameters is selected. The new versions of the portfolio of investments in the cells are not created until it will end the process of exchange as a result of which the best portfolio version will be selected. This

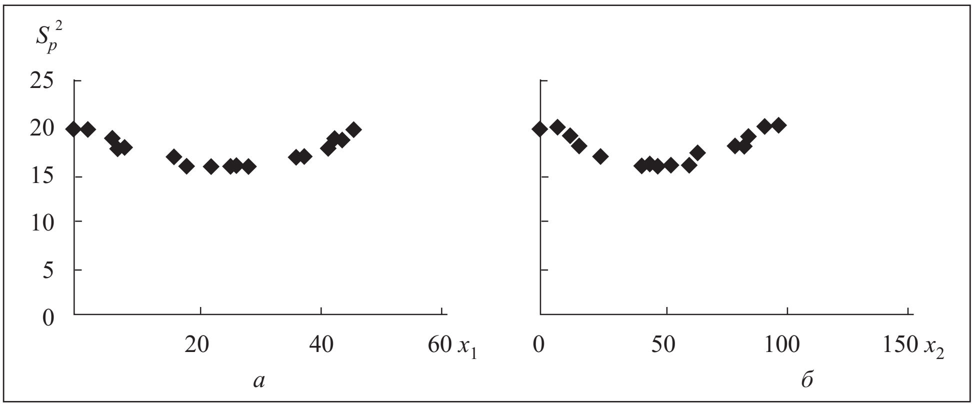

Diagram x_1 and x_2 values as results of random search of minimum goal function $S_p^2()$

version of the portfolio of investments is memorized in each CML cell. On the following iteration the new version of the portfolio of investments is created in each cell, however for the exchange of information between the neighboring cells it is represented only that version of the portfolio, which as to its parameters is equal or exceeds that, which was obtained on the previous iteration. After the new version of the portfolio of investments is created in each CML cell it begins the process of information exchange between cells as a result of which the portfolio of investments with the best parameters is determined.

The process described above is repeated to that moment, when as a result of search the version of portfolio with the optimum parameters is obtained. This situation occurs when as a result of iteration in each cell the portfolio is created, which parameters equal or differ to insignificant degree from the portfolio of investments, created on the previous iteration. The exchange of information between the cells is accomplished locally asynchronously on the basis of signals, which are established by each cell, when it is ready to information exchange. Each cell after the selection of the new portfolio of investments, approaches the interrogation of adjacent cells for the purpose of determination of their current state. When a cell is in the process of scanning the state of adjacent cells fixes their readiness for the realization of exchange, there occurs the transfer of the portfolio parameters from each peripheral cell to the central cell, where the portfolio of investments with the best parameters is selected as a result of comparison. This portfolio is written in the memory of central cell and transferred to the peripheral cells, where it is also written in the memory of cells.

As an example the implementation of the iteration process of random search for the type of Jacobi method with a random selection of component in CML is presented.

```
          Input:
X [i]      - array of initial values
E [i]      - array error of solution
E          - the maximum error of solution
e          - error of solution
F [i] - array of flags

          Output:
X [i] - array of calculated values

while E [i]≤ e
          for i from 1 to q do
                    F [i] = 0
          end for
s = 0
          while s <q do
          i = random from 1 to q
          calculate X [i]
          F [i] = 1
          for i from 1 to q do
                              s = s + F[i]
          end for
     end while
     for i from 1 to q do
               evaluate X [i]
     end for
end while.
```

In this algorithm $q = w * w$, where w is a number of row and column in the lattice structure, and in this case we investigate interaction of the computing element only with neighboring computing elements as in CA structures. The operation calculate X[i] is performed with respect to algorithm, shown above. Performing of the operation evaluate X[i] means, that the results of calculation in the lattice cells are transmitted to neighboring cell in each cell of the lattice .

Each element of CML architecture in the iteration does the following calculation:

$$M_K = M_S + M_R + M_P + M_B,$$

where M_K — the amount of arithmetic operation, performed in one step of iterative process; M_S — arithmetic operations performed in calculating the expected rate of return on the portfolio; M_R — arithmetic operations performed in calculating the risk of the portfolio investment; M_P — arithmetic operations, carried out a comparison of the results, selected by characterizing the individual elements cellular automat on computing the investment portfolios; M_B — arithmetic operation for choice of new version of component portfolio set.

The total amount of mathematical operations M_C, carried out by cells from CML during the search for the optimal portfolio on one step of iteration process can be described as

$$M_C = q * M_K + M_{WE} + M_{WY},$$

where q is quantity of CML cells and M_{WE}, M_{WY}, respectively, operations of the opening and launch of input streams for the introduction of information for the initialization memory of each calculation element and output stream for removal results of computing process in CML.

Let us assume that with the sequential realization of computational process the same quantity of operations with the use of one CML cell is carried out. It is natural that in this case the total time of the problem solution would be approximately q times more.

Some investigations of network realization of investment management are represented in [5].

Summary. The innovation process of using the algorithms of the random search for the solution of the problem of creation of the optimum portfolio of investments is investigated on the basis of the classical Markowitz's model. The algorithm of solution of test problem with the use of a process of random search is proposed and the results of its simulation are given. The estimation of the effectiveness of solution of the problem in CML environment is produced. It shows that with solution of the problem in CML environment the rate of obtaining the solution increases approximately q times, where q is quantity of CML cells.

Проблеми оптимізації інвестицій за допомогою аналізу портфеля інвестицій слід класифікувати як складні обчислювальні задачі. Для вирішення таких проблем є доцільним використання паралельних обчислювальних систем. Розглянуто реалізацію ефективних алгоритмів, базованих на використанні ітераційних процесів випадкового пошуку. Досліджено проблеми оптимізації портфеля інвестицій у паралельних обчислювальних мережах, а саме у решітках зв'язаних відображень.

1. *Markowitz H.* Portfolio Selection: Efficient Diversification of Investments. — London : John Wiley & Sons, 1959.
2. *Naugen R. A.* Teoria nowoczesnego inwestowania. — W-wa : WIGPRESS, 1996.
3. *Kaneko K.* Theory and Application of Coupled Map Lattices. — Chichester : J. Willey & Sons, 1993.
4. *Kulakowski K.* Automaty komorkowe. — Krakow : Wydawnictwo AGH, 2000.
5. *Katkow A., Ulfik A.* Network implementation of investment management // Electronic modeling. — 2007. — **29,** № 4.

Submitted on 04.11.09