Melt segregation and matrix compaction: closed governing equation set, numerical models, applications

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Partially molten systems are commonly modeled as an interpenetrating flow of two viscous liquids and are therefore described in terms of fluid mechanics [Drew, 1983; McKenzie, 1984; Nigmatulin, 1990]. In the gravitational field a liquid filling a viscous permeable porous matrix is in mechanical equilibrium only if its pressure gradient is equal to the hydrostatic one, and the pressures of the liquid and matrix are the same. If the liquid and matrix densities differ, with the liquid forming an interconnected network, the two conditions cannot be satisfied simultaneously, and the liquid segregates from the matrix while the latter compacts. The averaged momentum and mass conservation equations for a multi-phase medium are formulated separately for every phase. Considering the energy conservation equation, this results in $4N+1$ equations for a N-phase medium while the number of unknowns is $5N$ ($3N$ velocity components, N pressures, temperature, and N–1 independent phase fractions). Therefore, for the problem to be fully determined it becomes necessary to add N–1 coupling equations. Khazan (2010; on review at GJI) and Khazan and Aryasova (Rus. Earth Phys., 2010, in press) derived a general equation (the mush continuity equation, MCE) closing the governing equation set. Its simplified 1D form valid for a two-phase system in the case of low-melt-fraction mush and linear matrix rheology, together with equation describing the rate of the inelastic porosity change [Scott, Stevenson, 1986], constitute a closed subset of governing equation set:

$$\frac{\partial}{\partial z}\frac{k(\varphi)}{\mu}\left(\frac{\partial(p_l-p_m)}{\partial z}-\Delta\rho g\right)=\varphi\frac{p_l-p_m}{\eta},$$

$$\frac{\partial\varphi}{\partial t}=\varphi\frac{p_l-p_m}{\eta}, \qquad (1)$$

where p_l, and p_m are the melt and matrix pressure, respectively, φ is the melt fraction or porosity, $k(\varphi)\propto\varphi^n$ is the matrix permeability, $\Delta\rho=\rho_m-\rho_l$ is the difference between the matrix, ρ_m, and melt, ρ_l, densities, η and μ are matrix and melt viscosities, correspondingly, n=2 to 3, g is acceleration due to gravity; Z axis points upward. Let L be the thickness of the partially molten zone, and φ_0 be the maximum of the initial porosity distribution. In terms of dimensionless coordinate $\zeta=z/L$, time $\tau=t\Delta\rho gL/\eta$, melt overpressure $\Pi=(p_l-p_m)/\Delta\rho gL$, and porosity $\psi=\varphi/\varphi_0$ the equations may be written as

$$\frac{\partial}{\partial\zeta}\psi^n\left(\frac{\partial\Pi}{\partial\zeta}-1\right)-\gamma_c\psi\Pi=0, \quad \frac{\partial\psi}{\partial\tau}=\psi\Pi,$$

where

$$\gamma_c=\frac{L^2}{\delta_c^2}, \quad \delta_c=\sqrt{\frac{k(\varphi_0)\eta}{\varphi_0\mu}}, \qquad (2)$$

γ_c and δ_c are referred to as the compaction/segregation parameter and length, respectively. If coordinate is normalized by the compaction length, the first of Eqs. (2) does not contain γ_c [Grègoire et al., 2006] but it appears instead in the boundary conditions.

In what follows two characteristic situations referred to as segregation and compaction are considered. The former is a model of the evolution of a bounded partially molten zone. Its outer boundary coincides with solidus where the porosity and pressure difference vanish. The boundary and initial conditions are $\Pi(\tau,0)=\Pi(\tau,1)=0$, $\varphi(0,\zeta)=4\zeta(1-\zeta)$. For compaction (of bottom sediments, e. g.), it is assumed that the bottom is impermeable, and porosity at τ=0 is the same throughout the layer, so that: $\partial\Pi(\tau,0)/\partial\zeta=1$, $\Pi(\tau,1)=0$, $\psi(0,\zeta)=1$.

The solutions to Eqs. (2) are shown in Fig. 1 for segregation, and Fig. 2 for compaction. One may

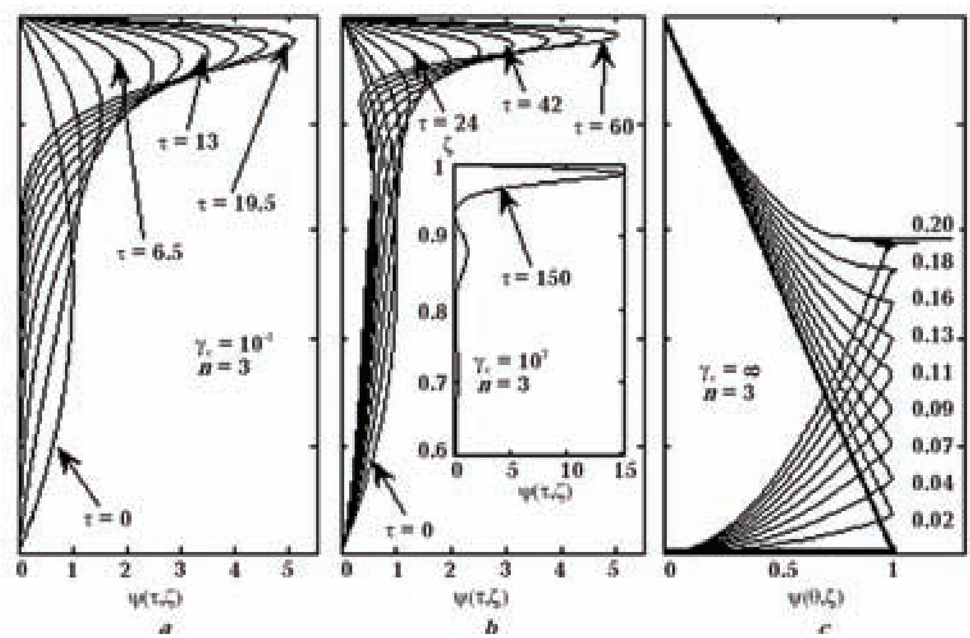

Fig. 1. Evolution of porosity $\psi(\tau, \zeta)$ at segregation: *a* — $\gamma_c = 10^{-2}$, *b* — $\gamma_c = 10^2$, *c* — at $\gamma_c \to \infty$ Eqs. (2) reduce to $\Pi \equiv 0$, $\partial\psi/\partial\zeta\theta = -\partial\psi^n/\partial\zeta$ with и being a formally introduced time variable $\theta = \gamma_c\tau$. Note that the first and the second waves at $\tau = 150$ (*b*, inset) contain 57 and 13 % of the melt, respectively, with the rest of the melt residing in the tail.

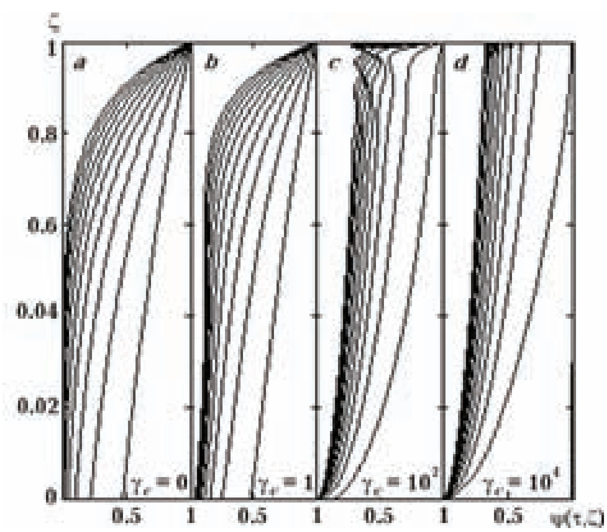

Fig. 2. Compaction of the bottom sediments at: *a* — $\gamma_c = 0$, *b* — $\gamma_c = 1$, *c* — $\gamma_c = 10^2$, *d* — $\gamma_c = 10^4$ ($n = 3$).

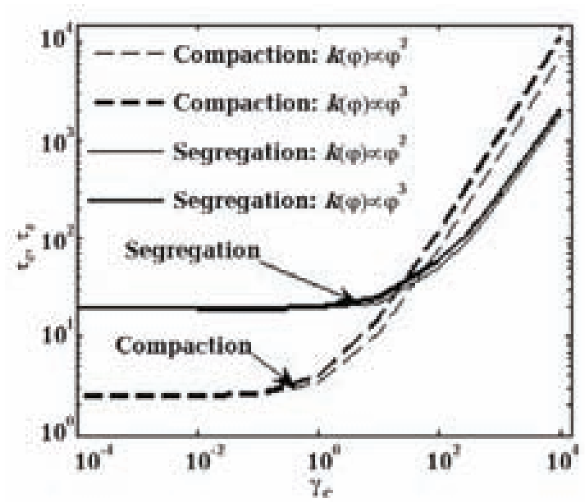

Fig. 3. Characteristic time of compaction τ_c and segregation τ_s vs. γ_c.

see from Fig. 1 that at low $\tilde{a}_c$ practically all the melt segregates to the roof with amplitude of the porosity growing with time (Fig. 1, *a*). The pressure gradient remains hydrostatical in the enriched layer and evolves to zero in the low-melt fraction tail. At high γ_c (Fig. 1, *b*) a wave structure develops with the dimensionless pressure being generally low and sigh reversible, and a significant part of the melt remaining trapped in the tail. The case $\gamma_c = \infty$ (Fig. 1, *c*) corresponds to $\Pi \equiv 0$ (or $p_l = p_m$). The porosity amplitude remains the same (i. e. no segregation occurs), and it takes $\partial\psi/\partial\zeta$ a finite time to reach the -8 implying a numerical instability, which is absent from the finite γ_c models. The variation of porosity while a liquid is expulsed from a compacting porous layer (Fig. 2) is similar to that at segregation indicating that a property to generate a wave-like structure becomes more pronounced with increasing γ_c, i. e. at high melt viscosity. A large pluton layering [Wager, Brown, 1968] may be due to this wave structure, which is supported by the observation [Brown, 1973] that layered intrusions are commonly intrusions of a tholeiitic basalt while those of the alkali basalt parentage are rare, which may be due simply to about an order higher viscosity of the tholeitic magmas.

The dependence of characteristic compaction, τ_c and segregation, τ_s, times on γ_c, is shown in Fig. 3. In dimensional variables an approximate fit to these results may be written as follows:

$$t_c \approx 2.49\frac{\eta}{\Delta\rho g L} + \frac{L}{V_D}, \quad t_s \approx 19{,}5\frac{\eta}{\Delta\rho g L} + 0.25\frac{L}{V_D},$$

$$V_D = \frac{\Delta\rho g k(\varphi_0)}{\mu\varphi_0}, \tag{3}$$

where V_D is the Darcy's velocity. The L^{-1} scaling of the compaction/segregation times at low γ_c effectively constrains the thickness of compacting porous sediments as well as the maximum possible thickness of the partially molten zone. Really, if the mushy layer thickness increases gradually, due to, for instance, sedimentation with a rate of r_d then the steady sediment thickness, L_d, may be estimated as $L_d/r_d = t_c$ wherefrom $L_d \approx 1.7\sqrt{\eta r_d/\Delta\rho g}$. Similarly, let a protokimberlite melt result from a decompression melting of a diapir floating at a velocity V_d with its temperature varying along an adiabate. The melting starts when the diapir top reaches the intersection level of the adiabate and solidus, and maximum possible thickness of the molten zone, L_s, may be estimated as $L_s \approx \approx 4.5\sqrt{\eta V_d/\Delta\rho g}$. The estimates of L_d and L_s are valid if $\gamma_c >> 2$ for the compaction problem or $\gamma_c >> 80$ for a segregation one. Also, Darcy's velocity is to be large, namely $V_D >> r_d$ at compaction and $V_D >> 0.25V_d$ at segregation. These estimates relate to an evolution of a porous layer filled with a low viscosity liquid, and may be used to estimate, for instance, a steady thickness of porous marine sediments, or a maximum possible thickness of a partially molten zone filled with a low viscosity magma (kimberlite, carbonatite) at the moment of segregation. To illustrate the latter case, adopt the following parameter values: $\eta = 10^{19}$ Pas, $\Delta\rho = 100$ kg·m^{-3}, $V_d = 3$ cm·y^{-1}, $k(\varphi) = a^2\varphi^3/270$ [Wark et al., 2003], grain size $a = 1$ mm, $\varphi_0 = 0.01$, $\mu = 0.1$ Pas. Then compaction/segregation length $\delta_c = 6$ km, $V_d = 30$ cm·y^{-1}, $t_s = 0.2$ My, $L_s = 8$ km, $\gamma_c = 1.7$. As soon as melt segregates, new partially molten zone grows, and the sequence of the events repeats until the whole diapir passes by the melting level. A diapir size, D, can be estimated based upon diameters $D = 20$ to 80 km of low-amplitude uplifts known to correlate with kimberlite fields [Kaminsky et al., 1995]. Therefore, the decreasing dependence of the segregation time on a mushy layer thickness implies formation of clusters of $D/L_s = 3$ to 10 low volume eruptions of almost the same age and composition. An attractive feature of the model is that it relates the kimberlite origin to a localized incipient melting.

References

Brown P. E. A layered plutonic complex of alkali basalt parentage; the Lilloise Intrusion, East Greenland // J. Geol. Soc. London. — 1973. — **129**. — P. 405—418.

Drew D. A. Averaged field equations for two-phase flow // Ann. Rev. Fluid Mech. — 1983. — **15**. — P. 261—291.

Grégoire M., Rabinowicz M., Janse A. J. A. Mantle mush compaction: A key to understand the mechanisms of concentration of kimberlite melts and initiation of swarms of kimberlite dykes // J. Petrol. — 2006. — **47**. — P. 631—646.

McKenzie D. The generation and compaction of partially molten rock // J. Petrol. — 1984. — **2**. — P. 713—765.

Nigmatulin R. I. Dynamics of multi-phase media. — New York: Hemisphere, 1990.— **1**. — 507 p.

Scott D. R., Stevenson D. J. Magma ascent by porous flow // J. Geophys. Res. — 1986. — **91**. — P. 9283—9296.

Wager L. R., Brown G. M. Layered Igneous Rocks. — Edinburgh, London: Oliver, Boyd, 1968. — 588 p.

Kaminsky F. V., Feldman A., Varlamov V., Boyko A., Olofinsky L., Shofman I., Vaganov V. Prognostication of primary diamond deposits // J. Geochem. Explor. — 1995. — **53**. — P. 167—182.

Wark D., Williams C., Watson E., Price J. Reassessment of pore shapes in microstructurally equilibrated rocks, with implications for permeability of upper mantle // J. Geophys. Res. — 2003. — **108(B1)**. — DOI:10.1029/2001JB001575.