

Autowave solutions of a nonlocal model for geophysical media taking into account the hysteretic character of their deformation

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Geophysical media as open thermodynamic systems actively display synergetic properties, ability to creation of localized dissipative structures, and an order. Experimental investigations show that dynamics of physical processes in nonequilibrium media is determined substantially by hierarchy and discreteness of a media structure, the set of internal relaxing processes, the nonlocality of interaction between structure elements, the directed exchange of energy between the degrees of freedom. In the papers [Danylenko, Skurativsky, 2007; Danylenko et al., 2008] it is proposed to take into account these features of internal media structure in the dynamical equation of state. This leads to the nonlocal nonlinear mathematical model for structured media:

$$\begin{aligned} \frac{d\rho}{dt} + \rho \frac{\partial u}{\partial x} = 0, \quad \rho \frac{du}{dt} + \frac{\partial p}{\partial x} = \gamma \rho^m, \\ \tau(\dot{p} - \chi \dot{\rho}) = \varkappa \rho^n - p + \Phi_1 + \sigma \left\{ p_{xx} + \rho^{-1} p_x \rho_x - \eta (\rho_{xx} - \rho^{-1} \rho_x^2) \right\} - \\ - h \left(\dot{p} + \eta \left[2\rho^{-1} \dot{\rho}^2 - \dot{\rho} \right] \right) + \frac{h^2}{\tau} \ddot{p} + \frac{h\chi}{\tau} \left\{ \frac{6}{\rho} \dot{\rho} \ddot{\rho} - \frac{6}{\rho^2} \dot{\rho}^3 - \ddot{\rho} \right\}, \end{aligned} \quad (1)$$

where ρ is the density of a medium, u is the velocity, p is the pressure, $\gamma \rho^m$ is the external mass force, τ is the relaxing time, σ and h are parameters of spatial and temporal nonlocalities, the parameters \varkappa and χ are proportional to the squares of equilibrium and frozen speeds of the sound. The function $\Phi_1 = \varepsilon \Phi_1(\rho, p, \dot{\rho}, \ddot{\rho})$ describes hysteretic reaction of a medium under the deformation, ε is the scale parameter.

Previous investigations of the wave solutions of model (1) in the form [Danylenko, Skurativsky, 2009]

$$\rho = R(\omega), \quad p = P(\omega), \quad u = 2\xi t + U(\omega), \quad \omega = x - \xi t^2 \quad (2)$$

shown that accounting the spatial and temporal nonlocal effects in the dynamical equation of state ex-

pands substantially the class of solutions in comparison with the local model (τ, σ, h are infinitesimal). In particular, the set (2) contains periodic, quasiperiodic, multiperiodic, and chaotic regimes, which are connected with each other by means of bifurcational scenarios. The solitary waves with the oscillating asymptotics were discovered as well.

Thus, basing upon the results of investigations of models that do not take into account the hysteretic character of media deformations, we shall study the influence of the hysteretic function Φ_1 in the dynamical equation of state on the structure of solutions (2). The function Φ_1 describes the hysteretic loop in the plane $(\rho; p)$ under the harmonic loading. The form of this loop is defined by the following relation

$$\Phi_1 = \begin{cases} \exp(C_2\{\rho - \rho_0\}) - 1, & \rho \geq \rho_0, d\rho/dt \geq 0, \\ \left[\exp(C_2\{\rho_{\max} - \rho_0\}) - 1 \right] \frac{\exp(C_1\{\rho - \rho_0\}) - 1}{\exp(C_1\{\rho_{\max} - \rho_0\}) - 1}, & \rho \geq \rho_0, d\rho/dt < 0, \\ -\left[\exp(-C_2\{\rho - \rho_0\}) - 1 \right], & \rho < \rho_0, d\rho/dt \leq 0, \\ -\left[\exp(C_2\{\rho_{\max} - \rho_0\}) - 1 \right] \frac{\exp(-C_1\{\rho - \rho_0\}) - 1}{\exp(C_1\{\rho_{\max} - \rho_0\}) - 1}, & \rho < \rho_0, d\rho/dt > 0, \end{cases} \quad (3)$$

where $C_2 < C_1$. In the case, when $C_2 < C_1$, the area bounded by the loop is zero. We should note that the set of enclosed loops appears in the plane $(\rho; p)$ instead of one loop, if we use the loading, distinct from harmonic one. The elements of function (3) can be used for description of the simplest cases of enclosed loops (Fig. 1).

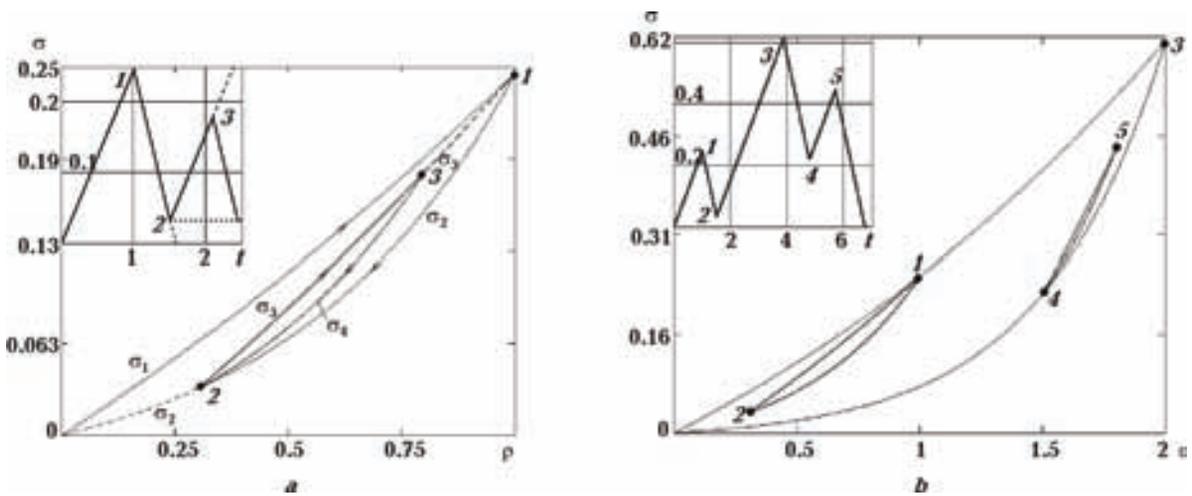


Fig. 1. The construction of two (a) and three (b) enclosed hysteretic loops in the plane $(\rho; p)$.

Substituting solution (2) into system (1), we obtain the quadrature $UR=S=\text{const}$ and the dynamical system:

$$R' = W, \quad P' = \gamma R^m - 2\xi R = \frac{S^2}{R^2} W, \quad W' = Z, \quad Z' = \frac{F + R^7 \varepsilon \Phi_1}{bS^3(\chi - S^2)R^2}. \quad (4)$$

Here $(\circ)' = d(\circ)/d\omega$, $F = F(R, P, W, Z)$ is the nonlinear function. The analytical expression for the function F is omitted due to its length. Note that analytical expression (3) for the hysteretic loop can also be written in terms of invariant variables (2).

Nonlinear dynamical system (4) is investigated by means of the qualitative and numerical methods. Equating the right parts of system (4) to zero, we get the coordinates of the fixed point

$$R_0 = (2\xi/\gamma)^{1/m-1}, P_0 = \alpha R_0^n, W_0 = 0, \quad (5)$$

which do not depend on the parameters of the hysteretic function Φ_1 . Analyzing the stability of fixed point (5) under the neglecting the hysteretic function Φ_1 we state that at $\chi = 16; S = 3.2; \tau = 1; \alpha = 0.9; \zeta = -0.5; h = 6.74; \sigma = 0; \gamma = -0.6; n = 4; m = 8; b = -3.8$ the fixed point changes the type from unstable node-focus to stable one. In the vicinity of fixed point (5) the unstable limit cycle appears at increasing the parameter h . Bifurcational analysis of dynamical system (4) in the case when $\Phi_1 \neq 0$ and zero area of hysteretic loop shown that for $C_2 = C_1 > 0$ fixed point (5) is a stable node-focus surrounded by both unstable and stable cycles. Consider the case, when $C_1 \neq C_2$ and function (3) describes the loop with nonzero area. Then as a consequence of hysteretic loop including the structure of the phase space of system (4) becomes more complicated at increasing the parameter C_1 . In particular, at $C_2 = 7.5; C_1 = 27$ there are several localized and separated regimes in the phase space of dynamical system (4), namely, fixed point (5), both stable and unstable cycles surrounding it (Fig. 2, a), and the chaotic attractor in addition (Fig. 2, b). Note that the chaotic attractor does not exist neither at $C_1 = C_2 = 7.5$ nor at $C_1 < C_2 = 27$. So that the chaotic attractor is created due to accounting the hysteretic loop.

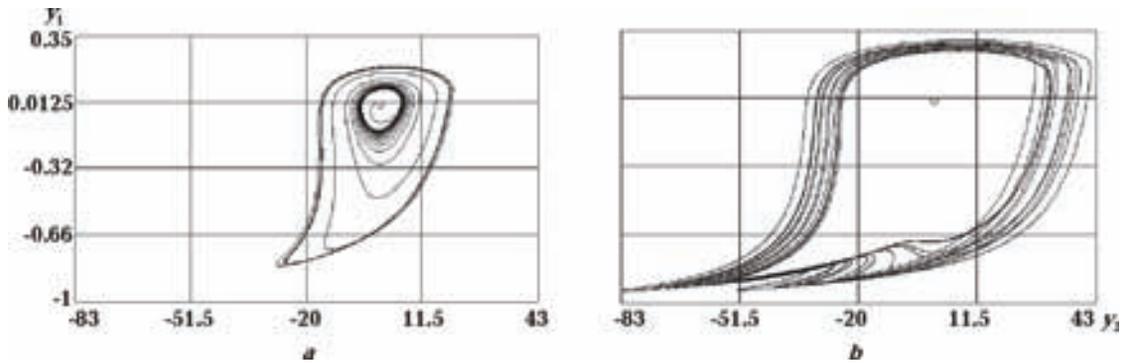


Fig. 2. The structure of the phase space of system (4) at $C_1=27$ (a) and $C_2=7.5$ (b).

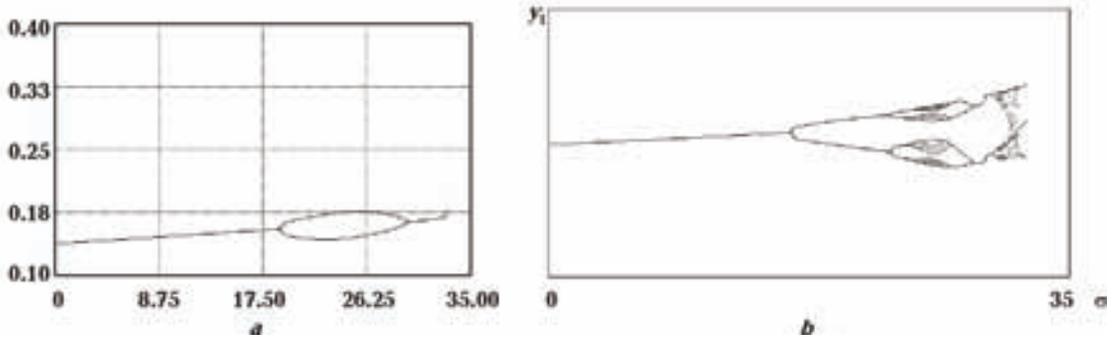


Fig. 3. Poincaré diagrams of the limit cycle development at increasing σ : a — $C_1=C_2=9.5$; b — $C_2=9.5, C_1=19$.

Another manifestation of the hysteretic function Φ_1 adding is regularization of chaotic oscillations. According to the numerical experiments, at $S = 3.8, C_1 = C_2 = 7.5, \varepsilon = 0, 1$ the complicated periodic trajectory exists in the vicinity of the fixed point. Analyzing the development of the periodic trajectory by Poincaré diagram we show that the period doubling cascade with chaotic attractor creation is actualized at $C_1 \in [7.5; 8]$. But if we fix $C_2 = 7.5$ and vary $C_1 \in [7.5; 8]$, then the periodic regime exists only instead of the chaotic attractor.

The existence of new wave regimes for model (1) is connected with the effects of spatial nonlocality, which are described by terms with the parameter σ . Analyzing the Poincaré diagram of the limit cycle

development at increasing σ and $C_1 = C_2$ (zero loop area) we see that the size of T -period cycle is growing and there is an interval of σ corresponding to the existence of $2T$ -period cycle (Fig. 3, a). The structure of solutions (2) is much more complicated in the case of nonzero hysteretic loop area at $C_2 = 9,5$, $C_1 = 19$. Studying the Poincare diagram in picture 3b we can distinguish several period doubling cascade, intervals of the different type chaotic attractor existence, moment of the hysteretic transition from one attractor to other.

Thus, the accounting a hysteretic loop in the dynamical equation of state causes new wave regimes creation. The hysteretic loop is the way of elastic energy utilization and in the same time it is the nonlinear element of media, which can cause unstability generation in media and produce localized dissipative structures.

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