

## РАДИОФИЗИЧЕСКИЕ ЯВЛЕНИЯ В ТВЕРДОМ ТЕЛЕ И ПЛАЗМЕ

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### ENHANCEMENT OF THE QUANTUM DOT LUMINESCENCE IN ALL-DIELECTRIC METAMATERIAL

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*We propose a simple design of an all-dielectric silicon-based planar metamaterial that manifests an extremely sharp resonant reflection and transmission at the wavelength of about 1550 nm due to both low dissipative losses and the trapped mode operating method particularities. The resonance Q-factor dozens of times exceeds that of resonances in the common infrared plasmonic structures. The designed metamaterial is considered for aggregation with a pumped gain medium to achieve the enhancement of luminescence and produce an all-dielectric analog of a “lasing spaser”. We report that an essential enhancement (above 500 times) of luminescence of a layer containing pumped quantum dots can be achieved by using the designed metamaterial. This value scores of times exceeds the known luminescence enhancement by the plasmonic planar metamaterials.*

*Key words:* all-dielectric metamaterials, trapped mode resonance, quantum dot, luminescence

#### 1. Introduction

Modern nanotechnologies enable to design optically thin layers of materials with a periodic subwavelength pattern in order to produce planar metamaterials. These latter are impressive objects-in-research, due to their some certain fascinating facilities, and have become mostly well-known over the last decade and are quite novel as well. Recent works report the results of aggregating laser materials with planar metamaterials to design the parametric gain systems [1–3] and develop the gaining or lasing devices, such as the spaser [4–6]. Some essential part of this development is studying the luminescence of a gain material hybridized with a planar metamaterial that can support a coherent high-Q electromagnetic oscillation.

Typically, a planar metamaterial for visible and near-infrared wavelengths is a plasmonic structure designed on the basis of either a periodic array of complex-shaped resonant metallic nanowires or slits in a metal slab. The main factor responsible for the spectacular properties of these metafilms is some

resonant interaction of light with the patterned layer. Moreover, numerous prospective applications of the planar metamaterials incorporating a pumped laser medium do require the high Q-factor resonance and strong confinement of electromagnetic fields.

The main factor extremely limiting the prospective applications of the infrared plasmonic metamaterials is high losses inherent to metals. Thus, typically, the Q-factor of the resonances excited in plasmonic structures is small because of radiation losses and a huge energy dissipation inherent to metals in the visible and near-infrared wavelength ranges.

Fortunately, there is some way to significantly decrease the radiation losses in the planar plasmonic structures. This way lies in using the so-called “trapped mode” resonances in the planar structures with a broken symmetry of periodic elements due to exciting of anti-phased currents in the metallic elements of a subwavelength periodic cell in microwave [7, 8], terahertz [9, 10] and in some optical bands [11–13].

Recently, it has been experimentally demonstrated that a hybridization of semiconductor quantum dots (QDs) with a plasmonic metamaterial supporting the trapped mode resonance leads to a multifold intensity

increase and narrowing of their photoluminescence spectrum [14]. The experiment shows that plasmonic metamaterials interact with QDs like a cavity, and the luminescence enhancement can be explained with the cavity quantum electrodynamic Purcell effect. This observation is an essential step towards understanding the mechanism of the radiation emission in plasmonic metamaterials and opens amazing opportunities for developing artificial metamaterial-enhanced gain media.

The trapped mode resonances have the typical peak-and-trough Fano spectral profile [15] and can be excited, for example, in the planar periodic array composed of twice asymmetrically-split metallic rings. Their specific character arises from the destructive interference of the radiation by the anti-phased current distribution. Theoretically, in a hypothetical lossless structure, the Q-factor of such resonance can be infinitely increased by the asymmetry degree decreasing, because the radiation loss tends to zero in this case. This property is the main difference of the trapped-mode systems from the nano-antenna structures intended for enhancement of Purcell effect.

Recently, a novel geometry of metallic strip particles and their arrangement within a periodic cell of trapped-mode structures has been proposed in [9, 10]. This suitable choice of the structure geometry results in an additional interference destruction and, as a consequence, in a radiation losses decrease.

The influence of a plasmon nanostructure with the Fano lineshape in absorption spectrum on the spontaneous emission spectrum and the lifetime of a single molecule of gain material was studied in [16, 17]. The opportunity of controlling the linewidths of atomic spontaneous emissions via the anisotropic Purcell effect was shown. Such interesting phenomena as the atomic spectral line rapid narrowing and the nanoscale line width pulsing were observed.

In practice, however, the Q-factor is strongly limited by the energy dissipation inherent to metals in the visible and near-infrared. Actually, the dissipative losses increase extremely as currents increase in metallic elements with decrease of radiation from nearly symmetry elements of an array. Thus the trapped mode resonances have a greater Q-factor than the regular resonance in a symmetric structure, though still a moderate one [13].

But there is a promising way to achieve a high-Q factor trapped mode resonance in the planar all-di-

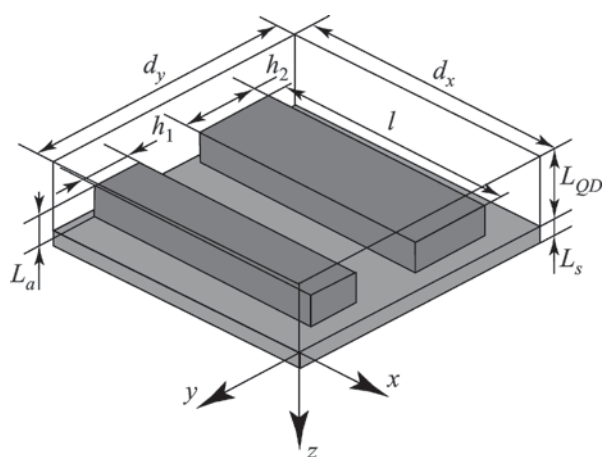
electric arrays with a broken symmetry of practically non-dissipative elements. It is well-known that the symmetry break in dielectric photonic band gap structures results in excitation of a high-Q defective mode resonance [18–20]. It was shown in [21] that a planar array with square periodic cell composed of two different-length semiconductor bars enables to achieve a trapped mode resonance with Q-factor much greater than that relating to this kind of resonances in plasmonic structures. For example, the Q-factor of the trapped mode resonance of the germanium array [21] can reach the value of about a thousand in the near-infrared transparency window.

In the case of this array, the dielectric bars work as open dielectric resonators, and the pair of the coupled resonators of each periodic cell can be considered as a metamolecule. The trapped mode resonance of a planar array is formed by a destructive interference of the fields scattered by such metamolecules outside the array. Besides the great Q-factor, a remarkable property of this all-dielectric arrays resonance is an essential red shift of their resonance frequency as against the resonance frequency of an array whose periodic cell consists of a single dielectric bar (see [21]). This property opens the way for dielectric materials with a relatively small refractive index in order to produce this kind of arrays.

We expect that the spectrum narrowing and intensity increasing of the QDs photoluminescence observed in [14] should rise with the Q-factor increase of the planar structure resonance. Thus, it is of high interest to study the features of the photoluminescence with a hybridization of QDs with a periodic all-dielectric array in the trapped-mode resonance regime.

## 2. Problem Statement and Solution Method

Now consider a double periodic array composed of dielectric bars placed on a substrate with thickness  $L_s$  (see Fig. 1). The periodic array of thickness  $L_a$  is immersed into QD layer (polymer layer in which QDs are dispersed) with total thickness  $L_{QD}$ . The unit cell of the array includes a pair of dielectric bars of different sizes. In [21], the asymmetry of a double periodic array of dielectric elements was achieved by using the dielectric bars of identical square cross-sections, whereas different in length. Though, it is more suit-



**Fig. 1.** A sketch of the unite cell of a double-periodic planar structure. The all-dielectric array composed of two dielectric bars per periodic cell is immersed into the QD layer

able in practice to use the dielectric bars different in width along the  $y$ -direction. If all the bars are identical both in length and thickness, then such a double periodic array is very convenient for manufacture. The dimensions of the square periodic cell are chosen as  $d_x = d_y = d = 900$  nm. The periodic cell is assumed to be symmetric as respects the line drawn through the cell center and parallel to the  $y$ -axis. The normal incidence of a linearly  $x$ -polarized plane wave is considered.

As a material for the dielectric bars, we propose the silicon as the most widespread material in electronics, with the transparency window within the wavelengths of  $1.2 \mu\text{m}$  to  $14 \mu\text{m}$ . The dimensions of the array elements are chosen to provide the trapped mode resonance of optically non-pumped structure at  $1550$  nm, the typical wavelength used in telecommunications. At  $1550$  nm, the silicon refractive index is approximately  $3.48$  at room temperature [22]. An extremely thin substrate used enables to exclude any undesired wave interference. The substrate is assumed to be a synthetic fused silica membrane. Its refractive index is approximately  $n_s = 1.45$  in the wavelength range under consideration [23]. The substrate thickness is  $L_s = 50$  nm. Note that there are no high non-evanescent diffraction orders in the substrate (the inequality  $n_s d < 1550$  nm is valid).

To study the intensity enhancement of the luminescence of QDs resulting from their hybridization with a dielectric array, we propose to use the solutions of plane wave diffraction of the structure with

and without an optical pump. In the proposed method, the photoluminescence emission intensity  $W_e$  is evaluated as a difference between the intensity of the dissipation (or emission) defined by the expression  $W = 1 - |R|^2 - |T|^2$ ,  $R$  and  $T$  are the coefficients of reflection and transmission, in the structure without optical pump and with it

$$W_e = W_0 - W_+, \quad (1)$$

where  $W_0$  and  $W_+$  are the intensities of the dissipative losses (or emission) in the structure without and with optical pump respectively. Note that the value  $W$  will be positive, if any dissipation is observed in the structure, and negative, if the total intensity of the reflected and transmitted waves exceeds the intensity of the incident wave. This fact is taken into account in (1), where the photoluminescence intensity  $W_e$  has a positive value.

The proposed method was used for calculating the enhancement luminescence of QDs layer combined with a plasmonic metamaterial, which was studied experimentally in [14] (see the Appendix). We use the experimental data on the luminescence spectrum presented in [14] to define the parameters for the adequate theoretical description of a certain actual composite material which is QDs dispersed in a host polymer layer. In addition, the comparison between the experimental results of that work and our theoretical ones demonstrates the validity of the proposed theoretical approach to study the photoluminescence. The proposed approximation provides good coincidence of the numerical results with the experimental ones, despite ignoring the quantum phenomenon inherent to the interaction of gain medium molecules and a strong local field. Besides, for simulation, we use the well-known model of gold and the parameters of QDs provided in this paper. Therefore, we may conclude that our simple approach can be effectively used for estimation of the enhancement of luminescence in the all-dielectric metamaterials.

Thus, to determine the photoluminescence intensity, it is required to solve the problem of plane wave diffraction by the considered structure. For this aim, the mapped pseudospectral time domain (PSTD) method proposed in [24, 25] is used. For the sake of simplicity, the dispersion of dielectrics is not taken into account in this paper. Such approxima-

tion is justified because the refractive index of silicon in the wavelength range under consideration (from 1500 nm to 1600 nm) varies within 3.48 to 3.47. It is evident that such an extremely weak dispersion has no effect on the properties of a trapped-mode resonance.

To take into account the gain in the optically pumped QDs, the method of additional differential equations (ADE) [26] is used along with the model of a negative frequency-dependent conductivity [27]. The expression of the corresponding time-domain conductivity  $\sigma(t)$  is chosen in the form to ensure that this value is real and causal,

$$\sigma(t) = \frac{\sigma_0}{\tau} \cos(\omega_0 t) \exp(-t/\tau) u(t). \quad (2)$$

Here  $u(t)$  is the Heaviside unit step function. The coefficient  $\sigma_0$  is proportional to the peak value of the gain set by the pumping level and the resulting population inversion. Such model assumes that the optical gain medium is homogeneously broadened and wherein the atoms of the gain medium are indistinguishable and have the same atomic transition frequency,  $\omega_0$ . Any variation of the QD sizes will cause broadening of the exciton line of the QDs. The time constant  $\tau$  permits to include the relaxation processes in a phenomenological manner (decay rate is  $1/\tau$ ), and shows that any phase coherence introduced into the system of atoms by the action of the electric field will be lost in the time interval  $\tau$ , once the electric field is turned off.

The Fourier transform of (2) reduces to the following frequency dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0(1+i\omega\tau)}{(1+\omega_0^2\tau^2) + 2i\omega\tau - \omega^2\tau^2}, \quad (3)$$

where the time dependence of the electromagnetic field is chosen in the form  $\exp(i\omega t)$ . It is clear from (3) that the gain coefficient is governed by a single Lorentzian profile with a width determined by  $\tau$ . The resonance frequency, that is the frequency at which the response is maximized, is given by expression  $\omega_d = \sqrt{\omega_0^2 + \tau^{-2}}$ . The peak of the gain curve is  $\sigma(\omega_d) = \sigma_0/2$  and the full-width-at-half-maximum band-width is  $\delta\omega = 2/\tau$ . Using (3) one can obtain a complex propagation constant of a plane wave and see that the wave amplification will be observed only for the case of  $\sigma_0 < 0$ . The chosen form of the

frequency-dependent conductivity can be easily included in the numerical scheme of time domain approaches by the use of ADE method. It results in the two additional first order differential equations in time in each grid node immersed into the gain medium [27].

A good agreement between our theoretical and the known experimental results of the study of the luminescence of a plasmonic metamaterial combined with QDs (see the Appendix) evidences of correctness of both the model of active QD medium and the proposed approach to study the luminescence in a resonant planar metamaterial.

Now let us apply the proposed method to designing a resonant silicon metamaterial combined with the semiconductor QDs to enhance the photoluminescence intensity with the spectral maximum at 1550 nm.

For numerical study, a laser medium based on semiconductor QDs was chosen with the following parameters:  $\omega_0 = 1.26 \cdot 10^{15} \text{ s}^{-1}$  which corresponds to wavelength  $\lambda_0 = 1550 \text{ nm}$ ,  $\tau = 4.85 \cdot 10^{-15} \text{ s}$ ,  $\epsilon_{QD} = 2.19$  which corresponds to refractive index  $n_{QD} = 1.48$  of non-pumped QD laser medium, and  $\sigma_0 = -500 \text{ S/m}$  corresponding to the emission factor  $\text{tg} \delta_e = -0.021$  in analogy to the lossy factor of the media. The small value of  $\tau$  results in a wide-band QD spectral line, and it enables to exclude from consideration the effects caused by displacement between the metamaterial dissipation peak and the maximum of exciton emission line of QDs. Note that the pump level (i.e. the value of the parameter  $\sigma_0$ ) is one order less than it was needed in the case of plasmonic metamaterials (see the Appendix), because of the low losses of the all-dielectric array.

The geometry parameters of the dielectric bars array immersed in the non-pumped QD layer ( $\sigma_0 = 0$ ) are chosen to provide a high Q-factor trapped mode resonance of the structure near the wavelength 1550 nm. The sizes of the silicon structure corresponding to this condition are the following:  $L_a = 100 \text{ nm}$ ,  $L_{QD} = 210 \text{ nm}$ ,  $L_s = 50 \text{ nm}$ , the dielectric bars length equals to  $l = 800 \text{ nm}$ , the width of the bigger and smaller bars are  $h_2 = 260 \text{ nm}$  and  $h_1 = 160 \text{ nm}$ , respectively, and the distance between the two different in width bars is 240 nm (see Fig. 1).

In order to take into account the energy dissipation in the bars, a model of constant conductive medium is used to describe the dielectric with the PSTD

method. This means that the dielectric of bars is modeled as a medium with both  $\epsilon_a = n_a^2$  and  $\sigma_a$  being some constants. Such an approach results in the frequency-dependent losses.

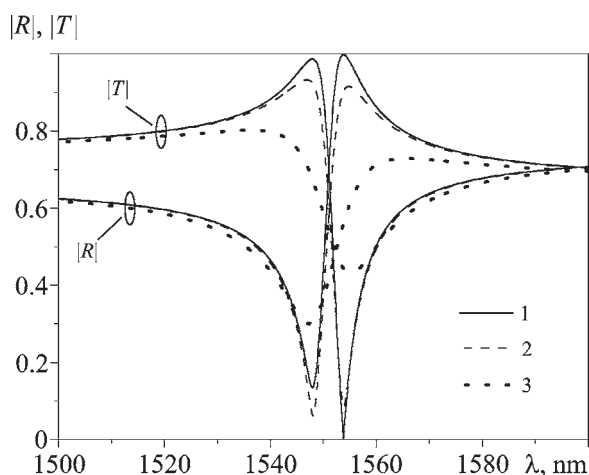
### 3. Analysis of the Luminescence of a Hybridization of QDs with a Low-Loss Planar All-Dielectric Metamaterial

The wavelength dependences of the reflection and transmission coefficients magnitudes of the designed structure are shown in Fig. 2. There is a trapped mode resonance near the wavelength 1550 nm which has the typical peak-and-trough Fano spectral profile. As was shown in [21], this resonance results from the anti-phased excitation of the dielectric bars behaving as an open half-wavelength dielectric resonators in this case. Lines 1, 2, and 3 correspond to  $\sigma_a = 0$ ,  $\sigma_a = 130.3$  S/m, and  $\sigma_a = 1303$  S/m, or equivalently, to  $\text{tg } \delta_a = 0$ ,  $\text{tg } \delta_a = 10^{-3}$ , and  $\text{tg } \delta_a = 10^{-2}$  at the wavelength 1550 nm. Note that actually the silicon lossy factor  $\text{tg } \delta_{Si}$  is less than  $10^{-3}$  at this wavelength.

We estimate the trapped mode resonance Q-factor by using the following formula proposed in [21]

$$Q = \frac{\lambda_1 \lambda_2}{\lambda_0 |\lambda_2 - \lambda_1|} = \frac{2\lambda_1 \lambda_2}{|\lambda_2^2 - \lambda_1^2|}, \quad (4)$$

where  $\lambda_1$  and  $\lambda_2$  are the wavelengths of the maximum and minimum of the reflection or transmission



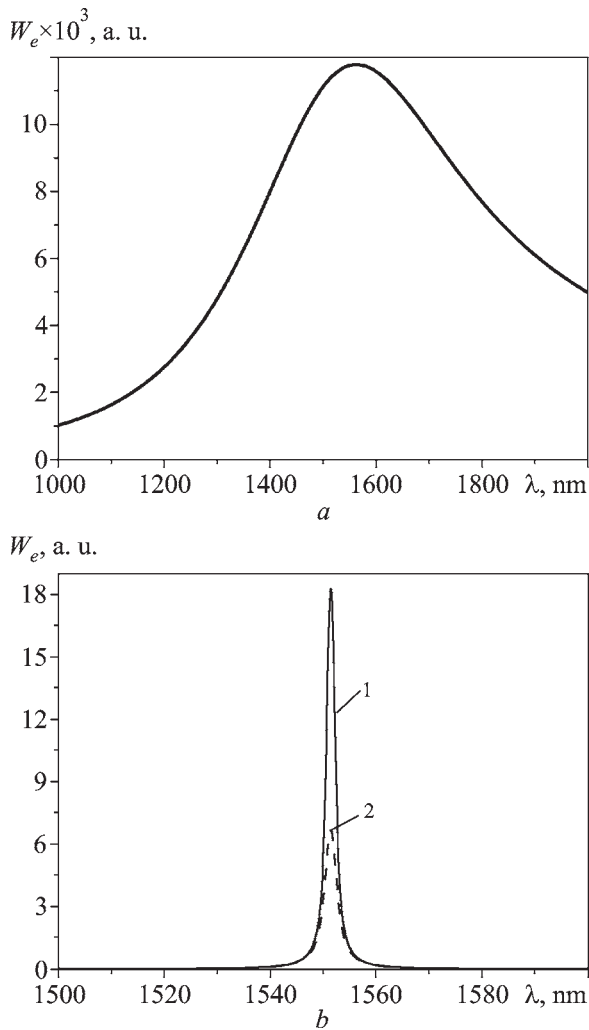
**Fig. 2.** Wavelength dependences of the reflection and transmission coefficients of the periodic array of silicon bars immersed in a non-pumped QD layer. Lines 1, 2, and 3 correspond to  $\text{tg } \delta_a = 0$ ,  $10^{-3}$  and  $10^{-2}$ , respectively

coefficient of the corresponding Fano spectral profile and  $\lambda_0 = (\lambda_1 + \lambda_2)/2$  is the average wavelength of the trapped mode resonance. Using (4), the following Q-factor values are obtained: 268, 232 and 83 corresponding to  $\text{tg } \delta_a = 0$ ,  $10^{-3}$  and  $10^{-2}$ , respectively. It is of interest that the wavelengths of the maximum and minimum of the reflection and transmission coefficients coincide only in case of a nondissipative structure. The reflection and transmission extremes shift with respect to each other if dissipation in the dielectric is taken into account. Then, the Q-factor of the trapped mode resonances is estimated as the average value  $Q = (Q_R + Q_T)/2$ . Here  $Q_R$  and  $Q_T$  are calculated from the wavelength dependences of the reflection and transmission coefficients by (4).

Note that the average trapped mode resonance wavelength  $\lambda_0 = ((\lambda_0)_R + (\lambda_0)_T)/2 = 1551$  nm is observed in all of the considered cases of dissipative losses.

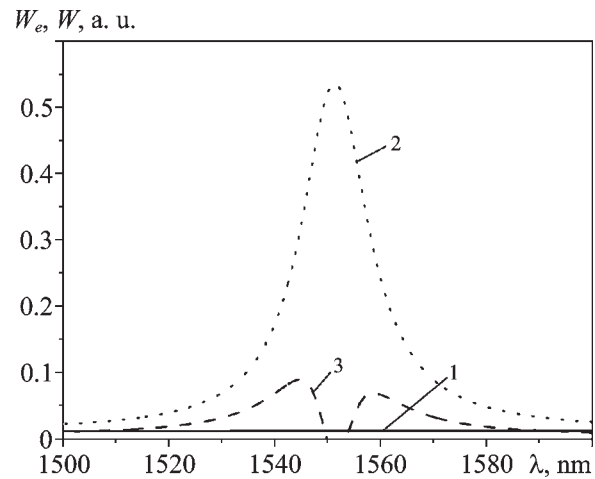
The wavelength dependences of the photoluminescence intensity  $W_e$  calculated by using (1) are presented in Fig. 3. It shows the photoluminescence intensity of the homogeneous QD layer with thickness  $L_{QD} = 210$  nm placed on 50 nm thick silica membrane (Fig. 3(a)), i.e. the intensity related to the layered structure without any array. The wavelength dependences of the photoluminescence of QD hybridization with non-dissipative and low-dissipative ( $\text{tg } \delta_a = 10^{-3}$ ) metamaterials are shown in Fig. 3(b). One can see a great enhancement in photoluminescence intensity. In the case of non-dissipative bars, the photoluminescence intensity enhancement makes 1560 times. If the dissipation of silicon, estimated as  $\text{tg } \delta_{Si} = 10^{-3}$ , is taken into account, this coefficient of the intensity enhancement equals to 560. These values mentioned are much more greater than the known orders of the photoluminescence enhancement inherent to plasmonic metamaterials combined with QDs (see [14] and the Appendix).

Up to this point, we consider the photoluminescence in the pumped all-dielectric metamaterial with its emission factor ( $\text{tg } \delta_e = -0.021$ ) which in absolute value exceeds essentially the lossy factor of the metamaterial ( $\text{tg } \delta_{Si} = 10^{-3}$ ). Now let us study the case of some pumped lasing medium with the intensity of energy dissipation in the dielectric array ( $\text{tg } \delta_a = 10^{-2}$ ), which is comparable with the gain of QDs. In Fig. 4, we present the wavelength depen-



**Fig. 3.** Wavelength dependences of the luminescence intensity of a QD layer with thickness 210 nm placed on the silica membrane (a), and QD layer aggregated with metamaterials (b): 1 –  $\text{tg} \delta_a = 0$ ; 2 –  $\text{tg} \delta_{Si} = 10^{-3}$ . The units of graphs (a) and (b) have the same scaling

dences of the photoluminescence intensity of a QD layer placed on the silica substrate (line 1), the power dissipation in the array of dielectric bars hybridized with non-pumped QDs (line 2), and the photoluminescence intensity of the optically pumped QDs aggregated with the same array (line 3). In the last case, the QD photoluminescence enhancement is also observed (see Fig. 4, line 3). However, in the trapped-mode resonance wavelength, we observe a decrease of the emission intensity until complete forbidding of the photoluminescence (see the region near 1550 nm, corresponding to  $W_e < 0$ , in Fig. 4). Such wavelength dependence of the photoluminescence intensity of QDs aggregated with the array is explained by the



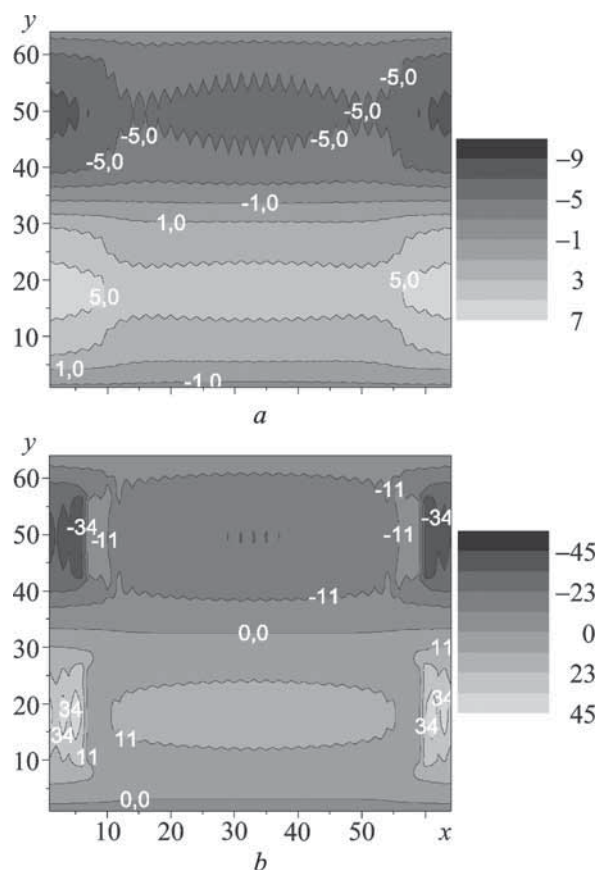
**Fig. 4.** Wavelength dependences of the photoluminescence intensity of a QD layer with thickness 210 nm (line 1), the energy absorption of the QD without optical pump aggregated with an array of dielectric bars,  $\text{tg} \delta_a = 10^{-2}$ , (line 2) and the photoluminescence intensity of metamaterials aggregated with QD (line 3)

fact that the QD gain results in an increase of the field amplitude inside the bars and the relating increase of energy dissipation in the array. In case of a low energy dissipation (e.g., the dissipation of silicon array with  $\text{tg} \delta_{Si} = 10^{-3}$ ), these losses result merely in decrease of the level of photoluminescence enhancement.

The increase of the local field in the silicon array is shown in Fig. 5. The distribution of  $x$ -component of the electric field in the plane  $z = -50$  nm corresponding to the half array thickness, is presented in Fig. 5(a) and 5(b) for the structure without optical pumping and for the gained QD layer, respectively. A homogeneous increase of the local field is the result of using a homogeneous-gain-medium approximation in our simulation. It is evident that the random distribution of quantum dots in the gained layer will result in nonhomogeneous enhancement of the local field, which will obviously reduce the level of photoluminescence enhancement. However, the value of the luminescence enhancement for the all-dielectric metamaterials is to be much higher than for the plasmonic ones.

#### 4. Conclusions

We have proposed a simple design of a planar silicon-based all-dielectric metamaterial that demonstrates its extremely sharp resonant reflection and transmission at the wavelength of about 1550 nm,



**Fig. 5.** Distribution of  $x$ -component of the electric field in the cross section of the array periodic cell ( $z = -50$  nm) for the nongained (*a*) and gained (*b*) QD layer aggregated with metamaterial at 1551.5 nm that corresponds to the maximum luminescence of the studied structure,  $\text{tg } \delta_{si} = 10^{-3}$

due to both the low dissipative losses and involvement of the trapped mode operating method. Accurate numerical estimations evidence that the Q-factor of the resonance of the proposed structure dozens of times exceeds the Q-factor of the known plasmonic structures.

The designed planar metamaterial is meant for aggregating with a layer of pumped active medium in order to achieve an enhancement of luminescence and to produce an all-dielectric analog of the lasing spaser [5].

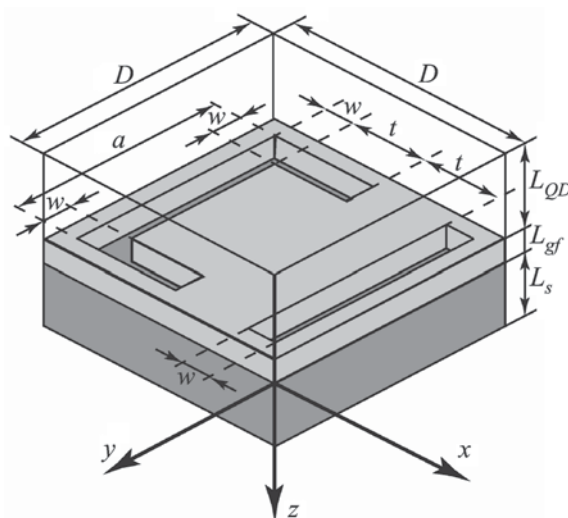
We have proposed the approach to study the luminescence intensity of pumped QDs aggregated with a planar metamaterial. The approach is based on the comparison of the results of the two diffraction problems solutions. They are the problems of a plane electromagnetic wave diffraction by the structure with and without optical pump. The validity of the method has been proven by its application to theoretical re-

production of the known experimental data relating to QD luminescence in plasmonic metamaterials [14].

We demonstrate that essential enhancement (above 500 times) of luminescence intensity of a layer containing pumped QDs can be achieved by using the designed resonant all-dielectric metamaterial. This value scores of times exceeds the known luminescence enhancement by plasmonic planar metamaterials.

### Appendix: Validation of the Diffraction Approach to Study Luminescence

For an evaluation of our approach to choosing the parameters describing the QDs immersed in a host polymer and the diffraction approach to study luminescence as well, let us consider one of the plasmonic metamaterials treated experimentally in [14]. The sample consists of a gold film periodically patterned by asymmetric split-ring slits and placed on a glass substrate (see Fig. 6). The film thickness is  $L_{gf} = 50$  nm. The square cell of the periodic array is  $D = 545$  nm on side. The slits of the periodic cell shown in Fig. 6 are as these:  $a = 470$  nm,  $t = 170$  nm, and  $w = 65$  nm. The slits are filled with a polymer material containing semiconductor QDs and the array is coated with a layer of the same material. The layer is  $L_{QD} = 180$  nm thick. In [14] the lead sulfide (PbS) semiconductor QDs from Evident Technologies was used with the luminescence peak around 1300 nm and the



**Fig. 6.** The geometry of the periodic cell of the aggregated with QD/PMMA metamaterial studied experimentally in [14]

mean core diameter of 4.6 nm. These QDs were dispersed in polymethylmethacrylate (PMMA).

Let us apply the diffraction approach based on the formula (1) to estimate the photoluminescence intensity of QD/PMMA hybridized with this plasmonic metamaterial.

The parameters of the gain medium model must be determined before luminescence intensity is estimated. As mentioned above, the parameters  $\omega_0$ ,  $\tau$ , and  $\sigma_0$  are required to determine the gain medium. Parameters  $\omega_0$  and  $\tau$  of QD/PMMA are determined from the photoluminescence peak frequency  $\omega_d = \sqrt{\omega_0^2 + \tau^2}$  and the full-width-at-half-maximum bandwidth of QD/PMMA photoluminescence  $\delta\omega = 2/\tau$ . We use the values  $\omega_0 = 1.47 \cdot 10^{15} \text{ s}^{-1}$  and  $\tau = 9.69 \cdot 10^{-15} \text{ s}$ . The refractive index of the non-pumped QD/PMMA is  $n = 1.48$  (see [14]). The value of  $\sigma_0$  depends on the level of population inversion in QD/PMMA, and determines the intensity of photoluminescence. Here, we use the value  $\sigma_0 = -5000 \text{ S/m}$  to describe our optically pumped QD/PMMA. It is a typical value used to describe a gain of semiconductor media, for example, in [27].

We consider the metamaterial aggregated with QD/PMMA placed on a semi-infinite silica substrate ( $L_s \rightarrow \infty$ ). Such a choice enables to exclude the interference in the substrate from the study. The model of gold permittivity proposed in [28] is used in our simulation of a plasmonic metamaterial.

The results of our numerical simulation of the wavelength dependences of the reflection, transmission, and dissipation coefficients of the metamaterial aggregated with non-pumped QD/PMMA are shown in Fig. 7. These theoretical results can be compared with the experimental ones presented in Fig. 1 of [14]. One can see a very good agreement between our numerical and the measured results. Some difference between the levels of reflection, transmission and absorption in theoretical and experimental results can be explained by distinctions between the actual structure and the perfect periodic one.

To estimate the enhancement of the luminescence intensity due to using metamaterial structure, we compare its value with the luminescence intensity of a homogeneous optically pumped QD/PMMA layer placed on the same substrate. The thickness of the layer is chosen the same as the total thickness of the metallic array and its QD/PMMA coating. The wavelength dependences of the photolumines-

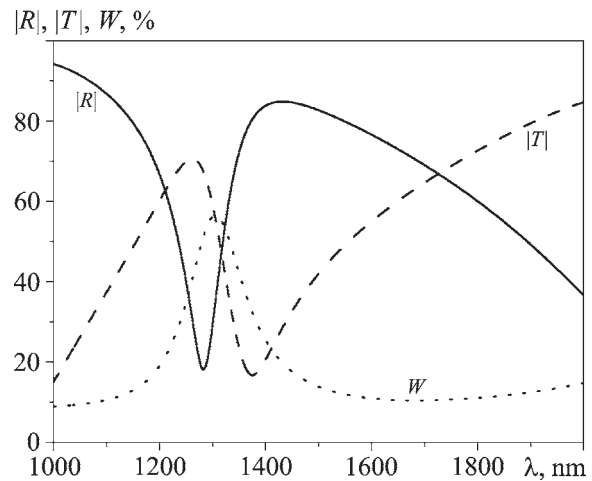


Fig. 7. Results of numerical simulation of wavelength dependences of the reflection, transmission, and dissipation coefficients of a metamaterial aggregated with non-pumped QD/PMMA investigated experimentally in [14]

cence intensity of the homogeneous QD/PMMA layer of 230 nm thickness (line 1) and the metamaterials aggregated with 180 nm-thick QD/PMMA layer (line 2) are shown in Fig. 8. In this case, we obtain 17 times enhancement of photoluminescence intensity. This value of the luminescence enhancement only twice exceeds that obtained experimentally in [14], that is quite a good result in the modelling of gain structures.

Thus, we have proven that both our model of gain QD medium and the diffraction approach for studying the luminescence intensity demonstrate that

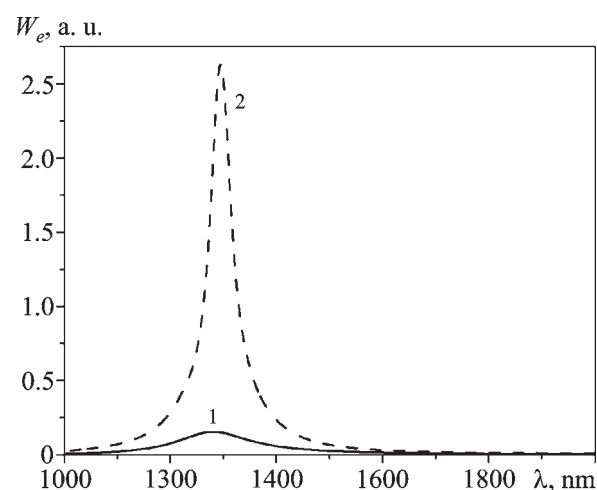


Fig. 8. Wavelength dependences of the photoluminescence intensity of homogeneous QD/PMMA layer (line 1) and plasmonic metamaterial aggregated with optically pumped QD/PMMA (line 2)



the numerical data obtained are in good agreement with the known experimental ones.

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#### УСИЛЕНИЕ ЛЮМИНЕСЦЕНЦИИ КВАНТОВЫХ ТОЧЕК С ПОМОЩЬЮ ПОЛНОСТЬЮ ДИЭЛЕКТРИЧЕСКОГО МЕТАМАТЕРИАЛА

Предложен полностью диэлектрический метаматериал на основе кремния, который поддерживает очень резкий резонанс отражения и прохождения вблизи 1550 нм. Такой характер резонансного взаимодействия метаматериала со светом обусловлен особенностями резонанса на запертой моде и низкими диссипативными потерями в диэлектрике. Добротность резонанса в десятки раз превосходит добротность резонанса в обычных плазмонных структурах инфракрасного диапазона. Рассматривается возможность объединения разрабатываемого метаматериала с активной средой с целью получения усиления люминесценции и создания полностью диэлектрического аналога устройства, известного как “lasing spaser”. Показано, что при использовании данного метаматериала удается достичь существенного усиления (более чем в 500 раз) люминесценции слоя с активными квантовыми точками. Значение коэффициента усиления люминесценции многократно превышает известный коэффициент усиления для планарных плазмонных метаматериалов.

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#### ПІДСИЛЕННЯ ЛЮМІНЕСЦЕНЦІЇ КВАНТОВИХ ТОЧОК ЗА ДОПОМОГОЮ ПОВНІСТЮ ДІЕЛЕКТРИЧНОГО МЕТАМАТЕРІАЛУ

Запропоновано повністю діелектричний метаматеріал на базі кремнію, який підтримує надзвичайно різкий резонанс відбиття і проходження поблизу 1550 нм. Такий характер резонансної взаємодії метаматеріалу зі світлом обумовлений особливостями резонансу на замкненій моді та низькими дисипативними втратами в діелектрику. Добротність резонансу в десятки раз перевищує добротність резонансу у звичайних плазмонних структурах інфрачервоного діапазону. Розглядається можливість поєднання розроблюваного метаматеріалу з активним середовищем з метою отримання підсилення люмінесценції та створення повністю діелектричного аналога приладу, відомого як “lasing spaser”. Показано, що, використовуючи даний метаматеріал, вдається досягти суттєвого підсилення (понад 500 разів) люмінесценції шару з активними квантовими точками. Значення коефіцієнту підсилення люмінесценції багаторазово перевищує відомий коефіцієнт підсилення для планарних плазмонних метаматеріалів.

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